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DYNAMIC UNIAXIAL STRAIN MODEL FOR PLASMA-SPRAYED MATERIAL

R.E. Tokheim


Abstract - A theoretical model is described for plasma-sprayed materials under uniaxial strain shock loading along the sprayed direction, based on anisotropic elastic relations. Parameters can be obtained from simple acoustic tests and static tension or compression tests. A Mohr-Coulomb model is assumed for stress loading beyond yield, and a von Mises yield criterion with work hardening is assumed for stress unloading.

Plasma-sprayed materials are fabricated by impacting molten droplets of fine particles onto a cooled substrate. The resulting material consists of overlapping, flattened pancake-like particles of various shapes and typically has a porosity of 10% to 15%. This material usually has mechanical properties that are transversely isotropic with respect to the direction of plasma spraying. This paper describes a general anisotropic modeling approach to characterizing plasma-sprayed materials for use in stress wave propagation computations along the plasma-sprayed direction.

Anisotropic elastic relations were derived assuming hexagonal single crystal system symmetry. These provide a basis for a consistent interpretation of ultrasonic and static data, which can then be used to determine low stress elastic behavior. Using the standard matrix notation, the stress-strain relations are given by

\[ \sigma_i = C_{ij} \varepsilon_j \ (i, j = 1, 2, \ldots, 6) \]

where indices 1, 2, and 3 represent principal stress directions, with 3 as the axis of symmetry (and plasma-sprayed direction). Indices 4, 5, and 6 are for shear stress relations. For uniaxial strain loading, four of the five independent elastic constants are of interest: \( c_{11}, c_{12}, c_{13}, \) and \( c_{33} \). The initial porous loading modulus \( E_3^* \) is given by

\[ E_3^* = \frac{\partial \sigma_3}{\partial \varepsilon_3} = c_{33} \]

the initial porous pressure modulus \( K_3^* \) is given by

\[ K_3^* = \frac{1}{3} \frac{\partial \sigma_3}{\partial \varepsilon_3} = \frac{c_{33} + 2c_{13}}{3} \]
and the porous Grüneisen ratio $\Gamma^*$ becomes

$$\Gamma^* = \frac{\alpha_V K_3^*}{\rho C_p} = \frac{K_3^*}{\rho C_p} \Gamma_{so}$$

where $\alpha_V$ is the assumed isotropic volumetric expansion coefficient, $C_p$ is the specific heat, and $\rho$ is the density. The solid Grüneisen ratio $\Gamma_{so}$ is based on an estimated isotropic solid bulk modulus $C$. If we had assumed isotropy, the porous Grüneisen ratio would be

$$\Gamma = \frac{\alpha_V K}{\rho C_p}$$

where the porous bulk modulus $K$ is given by

$$K = \frac{C_{33}(C_{11} + C_{12}) - 2C_{13}^2}{2C_{33} + C_{11} + C_{12} - 4C_{13}}$$

Thus, if anisotropy were ignored and a model were constructed based on a presumed isotropy, the porous moduli and Grüneisen ratio could be significantly different. For the plasma-sprayed materials we have studied, we find that $K_{so}/K = 0.7$, based on acoustic and static data. Consequently, one of the most significant effects of anisotropy is to reduce the pressure and increase the deviator stress ($\sigma - p$).

Anisotropic elastic constants can be obtained from acoustic data and from static tension or compression tests. Simple acoustic tests give

$$C_{33} = \rho v_{x3}^2$$

$$C_{11} = \rho v_{x1}^2$$

$$C_{11} - C_{12} = 2\rho v_{s1,2}^2$$

in terms of longitudinal ($l$) and shear ($s$) wave velocities ($v$), where the first numerical subscript of the velocity refers to the propagation direction and the second subscript refers to the direction of particle motion. The remaining constant $C_{13}$ is most readily obtained by measuring Poisson's ratio statically. Static tests described in Figure 1 will give elastic moduli $E_2$ and $E_3$, and Poisson's ratios $\mu_2$ and $\mu_3$ in each of two planes. Elastic constants are related to these by

$$C_{11} = E_2 \left[ \frac{1 - \frac{E_2}{E_3} \mu_2^2}{(1 - \frac{E_2}{E_3} \mu_3^2)(1 + \mu_2)} \right]$$

$$C_{12} = E_2 \left[ \frac{\mu_2 + \frac{E_2}{E_3} \mu_3^2}{(1 - \frac{2E_2}{E_3} \mu_3^2)(1 + \mu_2)} \right]$$
Both static and acoustic data are useful and complementary. One advantage of taking static data is that we can obtain the yield behavior along directions 3 and 1 that we could not obtain from acoustic data. Of course, ultrasonic acoustic data have the advantage of being at a strain rate corresponding more to that of shock waves and are best for determining dynamic moduli.

\[
C_{13} = \frac{\mu_3 E_2}{1 - \mu_2 - \frac{2E_2}{E_3} \mu_3^2} \frac{1}{1 - \frac{2E_2}{E_3} \frac{\mu_3^2}{1 - \mu_2}}
\]

FIGURE 1 STATIC TESTS FOR DETERMINING PLASMA-SPRAYED MATERIAL STRESS-STRAIN BEHAVIOR

Now we discuss the remaining part of the uniaxial strain model. For loading stresses in the direction of propagation above the initial yield strength \(Y_o\), we assume a modified Mohr-Coulomb approach, whereby the yield strength \(Y\) increases with increasing compaction (surface) pressure \(p\) by

\[
Y = Y_o + p \tan \phi
\]

where \(\phi\) is the friction angle. For rough sliding plasma-sprayed particles, we expect \(\phi\) to be as high as 40 to 50 degrees. For unloading stresses, we assume that particles do not slide, and we use the von Mises yield criterion for elastic-plastic strain with work hardening.

To allow for significant energy transfer or for rapid energy deposition by radiation, we introduce a thermal softening factor \(F(e)\) to account for reduction of the yield strength, the compaction pressure equation-of-state surface, and the elastic unloading moduli at higher temperatures (internal energies). The relation of compaction pressure to density and energy is given by

\[
p = p \left(1 + \frac{\rho}{\rho_0} e^{C} \right) F(e)
\]
where density is referenced to zero energy at the ambient temperature. The low pressure shape of this compaction surface will be determined by $K_3$ and hence may be significantly lower than that expected based on a presumed isotropy.

We draw several conclusions from this work:

- Anisotropic effects can be significant in plasma-sprayed materials.
- Anisotropy significantly reduces pressure generation compared with what would be expected based on a presumed isotropy.
- Both acoustic and static tests provide useful data for determining anisotropic behavior.