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SURVEY OF FULLY ANTISYMMETRIC ROTATORS FOR THE ELECTRON RINGS HERA AND LEP

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Abstract - A systematic study of fully antisymmetric rotators made of six magnets each is presented for HERA and for LEP. The maximum polarization that could be obtained with them is lower than with horizontally symmetric minirotators as it results from the antisymmetry and from the space available for installing these antisymmetric rotators.

A search of 90° minirotators for the HERA electron ring has been achieved recently and is described elsewhere. Each of the four interaction regions is intended to be equipped with a pair of such minirotators. The first one bends the electron spin direction along the longitudinal axis and the other one restores the vertical spin direction in the arc. These minirotators consist of 6 or 7 magnets rotating alternatively the spin around the radial and the vertical directions.

The two minirotators of a pair produce two local vertical bumps which are antisymmetric with respect to the interaction point. However, in the horizontal plane, the corresponding horizontally bending magnets of the two minirotators are symmetric. Consequently the vertical spin direction in the arcs is only restored at the design energy. A correction is needed for restoring it when there is a small energy off-set. Therefore these horizontally symmetric minirotators can only operate in a small energy range. This limitation is not too serious for an e−p collider as HERA, but would be of great concern for an e+e− collider as LEP.

On the contrary, rotators, which are antisymmetric in the horizontal and vertical planes, can operate in a larger energy range.

A search for fully antisymmetric rotators is presented here in the case of LEP, but also HERA for comparison. The same analytic method has been used as for the horizontally symmetric minirotators [3].

Starting with a six-magnet minirotator, an additional horizontal bend $B_{H}$ is added before entering the rotator from the arc, in order to obtain an antisymmetric configuration of the rotator pair (fig. 1). It results a small angle of the beam line in the interaction region with respect to its geometrical axis.

The six-magnet minirotators depend on three parameters which have been chosen to be: the spin rotation angles $\alpha_{1}$, $\alpha_{2}$ in the two first horizontally bending magnets, and the total beam line rotation $\theta = (\alpha_{1} + \alpha_{2})/\nu$ in the rotator ($\alpha_{3}$ is the spin rotation in the last rotator magnet and $\nu = E(\text{GeV})/0.44065$ is the spin tune).
The optimum rotator scheme corresponds to the maximum polarization [5] \( P \) achievable if all other depolarization effects due to quantum fluctuations can be suppressed:

\[
\frac{P}{P_0} = 1 + \frac{\rho_i^{-2}}{1 + \frac{\rho_i^{-2}}{1 + \frac{\rho_i^{-2}}{2\pi} \frac{1}{\rho_0} \cos \phi_i}}
\]

where \( \rho_i^{-1} \) and \( \rho_0^{-1} \) are respectively the trajectory curvatures in the normal arc magnets and in the \( i \)th rotator magnet, and \( \phi_i \) is the beam line rotation produced by this rotator magnet. The angle \( \phi_i \) is the angle between the magnetic field and the polarization direction in this rotator magnet. A small term depending on the longitudinal polarization component is neglected here. \( P_0 = 92.376\% \) is the maximum polarization in a planar ring.

In the case of fully antisymmetric pairs of rotators, the \( \cos \phi_i \) terms in the numerator of the above formula cancel out. The depolarization \( \Delta P/P \) can be written:

\[
\frac{\Delta P}{P} = \frac{\lambda}{1 + \lambda} \quad \text{with} \quad \lambda = \frac{P_0}{P} - 1 = \frac{\rho_i^{-2}}{1 + \frac{\rho_i^{-2}}{1 + \frac{\rho_i^{-2}}{2\pi} \frac{1}{\rho_0} \cos \phi_i}}
\]

At a given energy and for fixed normal curvature \( \rho_0^{-1} \), the depolarization \( \Delta P/P \) scales like the inverse square of the rotator length \( L \). Therefore the product \( L^2 \Delta P/P \) measures the figure of merit of any rotator. In fact the strength of the additional horizontal bend depends slightly on the length of the rotator, and one observes a few percent variation of \( L^2 \Delta P/P \) when the length is varied. This small variation is neglected here.

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Fig. 1 — Schematic lay-out (top view) of a pair of fully antisymmetric rotators in an interaction region of a storage ring.

\( H_{\alpha}, H_{\alpha'}, H_{\alpha''} \): horizontally bending magnets of a six-magnet rotator.

\( V_{\alpha}, V_{\beta'}, V_{\gamma} \): vertically bending magnets of a six-magnet rotator.

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I — FULLY ANTISYMMETRIC ROTATORS FOR HERA

The three parameters \( \alpha, \alpha', \) and \( \phi \) for a six-magnet rotator have been varied for minimizing the figure of merit \( L^2 \Delta P/P \).

According to the ring geometry and taking into account the space needed for matching the quadrupole triplets on each side of a rotator, the available space for a rotator is about 40 m, starting at 173 m from the interaction point.
Consequently the curvature in the rotator magnets has been fixed at 1.5 times the normal curvature $\rho_0$ (apart the curvature of one horizontally bending magnet which is set at the minimum value compatible with the length available for this magnet in the rotator) \cite{[3]}. The minimum value $L^2 \Delta P/P = 449 \text{ m}^2$ has been found for the rotator parameters given in Table I. The total length of the rotator is 44.433 m and the maximum polarization is 71.36\% for eight rotators. The beam line is at a 5.695 mrad angle in the straight section.

<table>
<thead>
<tr>
<th>HERA</th>
<th>LEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{\alpha_0}$</td>
<td>$H_{\alpha_0}$</td>
</tr>
<tr>
<td>$V_{\alpha}$</td>
<td>$V_{\alpha}$</td>
</tr>
<tr>
<td>$H_{\alpha_1}$</td>
<td>$H_{\alpha_1}$</td>
</tr>
<tr>
<td>$V_{\beta}$</td>
<td>$V_{\beta}$</td>
</tr>
<tr>
<td>$H_{\alpha_2}$</td>
<td>$H_{\alpha_2}$</td>
</tr>
<tr>
<td>$V_{\gamma}$</td>
<td>$V_{\gamma}$</td>
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<tr>
<td>$H_{\alpha_3}$</td>
<td>$H_{\alpha_3}$</td>
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</table>

<table>
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<th>SPIN</th>
<th>ORBIT</th>
<th>LENGTH</th>
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<td>ROTATION</td>
<td>ROTATION</td>
<td>5.742 m</td>
</tr>
<tr>
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<td>2.704</td>
</tr>
</tbody>
</table>

| TABLE I |

The figure of merit $L^2 \Delta P/P$ is larger than the optimum value ($333 \text{ m}^2$) obtained with the proposed horizontally symmetric minirotators A2 for two reasons:

i) The additional horizontal bend $H_{\alpha}$ increases the rotator length and contributes to the depolarization since one of the two additional bends of a pair is bending the orbit in the direction opposite to the normal one. However, this effect can be suppressed if this opposite bend is obtained by shortening the last arc magnet by an amount equivalent to this opposite bend. In this case the polarization increases up to 72.56\% and the figure of merit decreases down to 395 $\text{ m}^2$.

ii) In horizontally symmetric minirotators, the horizontally bending magnets participate to the normal bending of the beam-line. They globally contribute to polarize the beam by the Sokolov-Ternov effect, even if one of them is bending in the opposite direction. On the contrary in a fully antisymmetric pair of rotators, if one horizontally bending magnet is bending in the normal direction, then the corresponding magnet in the other rotator of the pair is bending in the opposite direction. Their polarization effects cancel out, while their depolarization effects add together. This feature is exactly the quoted cancellation of the $\cos \theta$ terms in the numerator of the formula giving the maximum polarization $P$. The result is a lower maximum polarization than for horizontally symmetric minirotators.

Finally, one observes that the space available for installing rotators is larger for horizontally symmetric minirotators, since the arc can be made a little shorter as the minirotators participate also to the normal bending of the orbit. Therefore horizontally symmetric minirotators can be made longer and their maximum polarization larger.
II - FULLY ANTISYMMETRIC ROTATORS FOR LEP

The same optimization procedure has been performed for finding fully antisymmetric rotators for LEP.

A reasonable space of 100 m has been considered, starting at about 300 m from the interaction point (fig. 2). The curvature of the horizontally bending magnets has been fixed to 2.0 times the normal curvature \( \rho_0^{-1} \) (apart for one magnet as for HERA). The curvature of the vertically bending magnets has been fixed 10% lower than the curvature of the horizontally bending magnets. So one obtains a better balance between the contributions of each magnet to the depolarization and a slightly improved polarization degree.

Fig. 2 - General lay-out of the optimum fully antisymmetric rotator for LEP at 50 GeV: a) side view, b) top view.

Fig. 3 shows a bidimensional plot of \( L^2 \Delta P/P \) as a function of the spin rotation angles \( \alpha_x \) and \( \alpha_y \), at a fixed value of \( \theta \) corresponding to the minimum value of \( L^2 \Delta P/P \), and for only one pair of rotators.

The minimum value \( L^2 \Delta P/P = 554 \, m^2 \) has been obtained for the rotator parameters given in table I, and figure 2 shows the general lay-out of this optimum rotator. The total length of the rotator is 95.737 m and the maximum polarization is 73.47% for eight rotators (\( L^2 \Delta P/P = 1876 \, m^2 \)).

A comparison between LEP and HERA is obtained by remarking that the curvature and the orbit rotation in the rotator magnets scale like the inverse of the energy. Therefore one must compare the values of \( \lambda L^2 \rho_0^{-2} L^3 \) for eight rotators.

<table>
<thead>
<tr>
<th>HERA</th>
<th>LEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>hor. symmetric</td>
<td>fully antisym.</td>
</tr>
<tr>
<td>20.7</td>
<td>32.1</td>
</tr>
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</table>
Bidimensional plot of the figure of merit $L^2 \Delta P/P$ versus the spin rotation angles $\alpha$, $\alpha$ of the two first horizontally bending magnets $H_x, H_y$ for LEP at 50 GeV ($\theta = 7.592$ mrad, only one pair of rotators).
Hatched area: no solution.

In conclusion the optimum fully antisymmetric rotators for LEP and HERA have very close values of the invariant parameter $: AL^2 \rho^{-2} E^2$. It is an obvious consequence of the very close spin rotation angles in their magnets (see table I). Both of them have a worse figure of merit than the proposed horizontally symmetric minirotators A2 for HERA.

III - ACKNOWLEDGMENTS

The author is indebted to B. Montague and K. Steffen for many fruitful discussions.

IV - REFERENCES

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[2] STEFFEN K., DESY HERA report 83/09 (May 83)
[3] BUON J., LAL report LAL-RT/83-10 (June 83)
[4] BUON J., DESY HERA report 84/03 (Feb. 84)