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EXPLORING DYNAMICS THROUGH POLARIZATION

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Résumé - Nous présentons une revue du formalisme optimal qui est appliqué à la détermination de plusieurs amplitudes à partir des observables de la diffusion pp élastique. Un modèle de Regge élaboré ne réussit pas à expliquer ces amplitudes, mais le schéma OPE reste valable. On trouve une étonnante simplification dans le repère "de côté". Nous considérons la pertinence de QCD.

Abstract - The Optimal Formalism is reviewed and applied to determining various amplitudes from observables in pp elastic scattering. A sophisticated Regge model fails to explain these amplitudes, but OPE remains valid. Striking simplicity is found in the "sidewise" frame. The relevance of QCD is considered.

I - INTRODUCTION

Polarization experiments provide important insights into the dynamical structure of hadronic interactions. It has been demonstrated on many occasions that spin is not an "inessential complication" in understanding the underlying dynamics. The activity and excitement generated in these High Energy Spin Symposia by new polarization data attest to the importance of spin phenomena. Of particular interest has been the measurement and interpretation of two-body scattering. Such exclusive scattering provides the simplest examples of purely hadronic interactions while exhibiting sufficient complexity to test our theoretical understanding extensively.

A brief reminder of the history of one line of spin phenomenology will serve to orient the following discussion. Over 15 years ago it was realized that the single polarization asymmetry in \( \pi p \) charge exchange scattering was a clear signal of interfering exchanges(1) and therefore provided a probe of the then popular model, the Regge pole model(2). Experiments were performed in that and other systems(3) with somewhat ambiguous results. The models had to be complicated with the addition of branch cuts and the structure of those cuts was the subject of considerable debate(4). In a perfect example of the interplay between theory and experiment, more refined data led to more refined models which, in turn, led to more refined experiments until it was clear that basic changes in the theoretical structure were necessary. In particular, the measurement of the \( A \) and \( B \) double polarization correlations in \( \pi^+ p \) elastic(5) dealt such a blow to the prevailing theoretical notions that the models never recovered their widespread credibility.

The remarkable interplay between theory and polarization experiments led to further developments in experimental techniques. The most noteworthy program was undertaken at Argonne to measure a complete set of spin correlations from which the amplitudes for pp elastic scattering could be determined(6). As the results were gradually obtained over many years (long after the ZGS was shut down) many puzzles appeared. Some of the original motivation, however, was lost as notions about fundamental processes changed. At this point there exists more than a complete set of observables at 6 GeV/c and several angles(7). It behooves us to obtain as much information from these data as we can.
The spin dependence of the pp system at "medium" energies provides a testing ground for many theoretical ideas and specific models. In this talk we will review the results of our own analysis of the pp amplitudes(8) and present some striking new systematics(9). Using the very general framework of the "Optimal Formalism" for polarization phenomenology(10) has enabled us to explore hidden dynamical clues(8,9) as well as to test some model predictions(11,12) in novel ways.

There is one particular s-channel frame that is simply related to the t-channel helicity frame. That is the center-of-mass planar frame - the "Magic" frame - in which the quantization axes are in the directions given by the crossing angles.(18) Those angles depend on the energy, momentum transfer and external particle masses.(21) So for each kinematic point of a reaction there is a set of planar axes for which the corresponding Magic amplitudes are equal to t-channel helicity amplitudes. And for OPE of J the constraints of Eq.(1) apply to the magic amplitudes directly giving many zero amplitudes, in general.(See page 257).

The second type of constraint is "factorization". The couplings of the exchanged particle of definite J to the incoming and outgoing states (in the t-channel) are independent so that the non-linear relations of the form

$$D(t)(c',a;d,b) = D(t)(c,a;d,b) = (2)$$

hold, where the a,b,c,d refer to spin components along the Magic axes and the sign depends on the parity of the particles. These relations provide many constraints among the amplitudes but are of direct use only in special circumstances as we see in the relevant example of p-p elastic.

For p-p elastic amplitudes, when J=0 exchange is dominant, Eq.(1) forces all but aJ=0 and cJ=0 to be zero (henceforth in this section the Magic frame for quantization will be understood). Then factorization, Eq.(2), requires aJ = +/-cJ, the sign depending on the parity of the exchange. Finally, because the angular functions in Eq.(1) are the same for the J=0 contribution to a and c, the relation a = +/-c holds for the total amplitudes and b=d=e=0.

When J for OPE is 1 or greater there are no J-constraints for p-p elastic, but factorization gives

$$a_j = +/-c_j,d_j = +/-a_j, a_j e_j = +/-b_j \quad (3)$$

Since a and c involve the same angular function in Eq.(1), the remaining relation for the entire amplitudes from Eq.(3) is just

$$a = +/-c \quad (4)$$

for natural/unnatural parity exchange of any definite J.

We have tested for OPE in p-p elastic scattering using these Magic amplitude tests.(19) To establish some credibility for the results we first applied the tests to the "intermediate energy" amplitudes at 300,580,800 MeV. For each of these energies the phase shift analysis of the SAID group(22) was used to obtain amplitudes in the Magic frame. For the 580 MeV amplitudes, the amplitude analysis of the complete set of polarization data from the SIN group(23) was used also as a check along with our own Optimal determination. In no case does the relation of Eq.(4) hold over the angular range of the analyses. Given the expectation that many exchanges of different parities are important in this energy region and that the u-channel poles must contribute as well, it is not surprising to find a null result here.

On the other hand, the 6 GeV/c Optimal amplitudes (rotated to the Magic frame) do satisfy Eq.(4) with the + sign as Fig.6 shows. This is true, within uncertainties, for almost all four sets of solutions. This indicates that natural parity exchange, with at least some J > 0 (since b,d,e are non-zero), plays a significant
role in the p-p dynamics at 6 GeV/c. Why should this be if the standard ideas about Regge exchange seem to fail as we have seen? Perhaps there is some new dynamics that still involves OPE in an essential way. Or perhaps the OPE dominance reflects some QCD mechanism wherein one gluon exchange is somehow dominant. We will speculate on this last possibility later. But next we try to look for other clues to the dynamical structure using our ability to change planar frames.

II - OPTIMAL FORMALISM

Several years ago we developed a formalism with which to define amplitudes and observables in two-body scattering of particles with arbitrary spins(l0). In this scheme the amplitudes are defined (for each energy and momentum transfer) in terms of the spin projections of each particle along quantization axes defined separately for each particle. Because of various symmetry constraints the choices of axes are limited, but infinite nevertheless.

For the pp system, parity, time reversal and identical particle constraints(l3) restrict the axes to be all normal to the scattering plane or within that plane. Further, for the “planar” choice each particle’s axis must make the same angle with that particle’s momentum as every other particle (in the center-of-mass frame). See Fig. 1 below. That is, for example, if the beam quantization direction is 30° counterclockwise (in the scattering plane) from the beam momentum, then the target spin quantization axis must be 30° from the target momentum; and so on for the outgoing protons. Hence there is one arbitrary angle to choose for planar amplitudes to be fully specified. Note that the “planar angle” of 0° corresponds to choosing helicity amplitudes. For most models helicity or transversity amplitudes are the simplest but, a priori, there is no reason why some other planar amplitudes might not reveal simpler dynamics. We will make a case for this latter possibility later.

Once the axes are specified, observables can be defined in many different ways. The “Optimal” choice involves defining the simplest possible observables (consistent with hermiticity)(10). Using each particle’s density matrix to fix that particle’s polarization, the simplest observables are those for which the density matrices have the simplest forms:

\[
\begin{pmatrix}
0 & 1 & \cdots & 1 \\
0 & 0 & \cdots & 0
\end{pmatrix},
\begin{pmatrix}
0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 1
\end{pmatrix},
\begin{pmatrix}
0 & 1 & \cdots & 0 \\
0 & 0 & \cdots & 0
\end{pmatrix}.
\]

For details, refer to the review in this Symposium and Refs. 10 and 13.

The optimal observables so defined are not usually observed because they involve all particles being polarized. Symmetries reduce that requirement somewhat, e.g. in pp elastic the fourth particle’s polarization is fixed once the other three are prepared and measured(l3). However, to relate to actually measured observables certain averages and sums over spins must be made. In spite of this complication it is still very illuminating to formulate an amplitude analysis in terms of the Optimal Formalism. What is particularly important is the generality of relations between observables and amplitudes so that the frames can be chosen to fit the applications.
Since the pp elastic system is our primary concern, herein we will consider the transversity, helicity, and planar frames specified by one planar angle, as indicated above and defined in Table I.

**TABLE I. Amplitudes for p-p elastic scattering**

<table>
<thead>
<tr>
<th>General Form for ( A + B + C + D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D(c,a;d,b) ) where ( c = \langle s(c).z_c \rangle ),.....</td>
</tr>
</tbody>
</table>

**Transversity**

\[
\alpha = D^\ell(+;+;+), \quad \beta = D^\ell(-;--;-), \quad \gamma = D^\ell(+;++;-), \quad \delta = D^\ell(+;++;+), \\
\epsilon = D^\ell(+;++;+)
\]

**Helicity**

\[
a = D^h(+;+;+), \quad b = D^h(+;++;+), \quad c = D^h(+;++;-), \quad d = D^h(+;++;+), \quad e = D^h(+;++;+)
\]

**Planar**

\[
DP(c',a';d',b') = \sum_{a,..d} d^{(1/2)}_{a' a} d^{(1/2)}_{b' b} d^{(1/2)}_{c' c} d^{(1/2)}_{d' d} d^{(1/2)}_{e' e} D^h(c,a;d,b)
\]

The actual determination of the amplitudes can be done in any frame. However, an examination of the observable-amplitude bilinear relations makes it clear that the transversity frame provides the most direct connection and, thus, the least error propagation\(^8\). The magnitudes of the five transversity amplitudes are fixed by the differential cross section and the four polarization quantities \( P \), \( C_m \), \( D_m \), and \( K \).

The phases are then fixed by the quantities \( C_{11}, C_{SS}, C_{1S}, K_{SS}, K_{SS} \) which have larger uncertainties. Although there are 10 observables in this set there is still a fourfold ambiguity. Furthermore the additional observable (beyond the minimum of 9) is not quite compatible with the others at some of the scattering angles. The actual determination of the transversity amplitudes was accomplished by doing a least squares fit of two of the phase angles to several observables. At some scattering angles this gave rather large \( \chi^2 \) verifying possible inconsistencies.

When the final amplitudes were obtained and plotted in the complex plane no special features appeared. Transversity amplitudes do not appear to be especially simple or revealing of underlying dynamics. An example of two solutions at one momentum transfer is shown in Figure 2. Note that in going from one momentum transfer (or scattering angle) to another it was not always clear how the 4 solutions
continued into the 4 at the neighboring momentum transfer.

Some of the uncertainties and ambiguities are expected to be reduced when the additional observables recently analyzed and made available by the Argonne group are incorporated into our determination of transversity amplitudes. There is still no expectation that transversity amplitudes will have any particularly noteworthy structure.

Helicity amplitudes, however, are expected to indicate underlying exchange processes. These amplitudes are simply obtained from linear combinations of transversity amplitudes and exhibit certain systematics that can be compared with various models. We have done this for a Regge pole model as we will review next.

IV - TESTING REGGE POLE MODEL

As outlined in the introduction, the Regge pole model had to become more and more complicated to account for the considerable data in polarized two body scattering processes. Several refined models were constructed that could accommodate most of the relevant data by incorporating many poles and cuts with their accompanying residue parameters. Before the Argonne program was completed, these models could be used to make predictions for the pp amplitudes. We have tested one of the more sophisticated of the models; that of Berger, et al.

The Regge model in question uses Pomeron, $\rho$, $A_2$, $\varepsilon$, $\omega$, $\bar{\omega}$, as natural parity exchanges and the "poor man's absorption model" $\pi$, $B$, $A_1$, $Z$, as unnatural parity exchanges. With the dominance of the diffractive component – the Pomeron – the helicity non-flip natural parity combination of amplitudes is expected to dominate.

In Regge pole models the pole contributions are organized into combinations of helicity amplitudes that have definite naturality for high enough energies. For the pp system these are

$$N_0 = a+c, \quad N_1 = b, \quad N_2 = d-e,$$
$$U_0 = a-c, \quad U_2 = d+e,$$

for natural parity, and

for unnatural parity.

So it is the $N_0$ that should be dominant; the other definite naturality amplitudes are predicted to be about an order of magnitude smaller in magnitude. The phases have definite predictions as well. The diffractive part, and hence $N_0$, is primarily imaginary, while the pion contribution, $U_2$, is mostly real. These are the firmest predictions of most Regge models. The other details depend on particular parameterizations.

We have used the model to calculate the 6 GeV/c amplitudes at several momentum transfers or $t$ values. One of those sets, at $t = -0.2$ GeV/c, is shown in Fig. 3.

Fig. 2: Transversity Amplitudes at 6 GeV/c, $t = 0.6$ GeV$^2$/c$^2$

Fig. 3: Comparison of Regge Model of Ref. 15 with Optimal Amplitudes at $t=-0.2(\text{GeV}/c)^2$
To compare with the actual amplitudes that we have obtained from the data, the definite naturality combinations must be formed. However, there is an overall undetermined phase at each $t$ value. Such phases can only be measured by interference with a known process, e.g., Coulomb scattering. To facilitate the comparison with the definite phase prediction of the Regge model we assume that the phase of our $N_0$ amplitude is equal to the model prediction. All other phases are then fixed.

Recall that there is a fourfold ambiguity in the "actual" or Optimal amplitudes. At each $t$ value the four sets were compared with the predicted amplitudes. Fig. 3 shows the one Optimal set that is closest in relative magnitudes to the Regge prediction at that single $t$ value. The agreement is poor. Comparing other $t$ values in the same way does no better. In general the magnitude of $N_0$ is always relatively smaller than the model would like, although there is always at least one Optimal set for which the $N_0$ is the largest amplitude.

Another way to compare the Optimal amplitudes with the model predictions is to look at helicity amplitudes directly as a function of $t$. The agreement is no better but is revealing of the rather striking fluctuating of the Optimal amplitudes while the model amplitudes are quite smooth functions of $t$. An example of this is shown in Fig. 4 where $|c|$ is plotted. In Fig. 5 the relative phase of $e$ is shown to have similar large excursions compared to the model.

What can be concluded from this comparison? The sophisticated (Fourth generation?) Regge model for $pp$ elastic scattering amplitudes does not agree with those amplitudes obtained from the complete set of polarization data at 6 GeV/c. The major disagreement involves the dominant amplitude, $N_0$. The Optimal result does include a relatively larger $N_0$, but not an order of magnitude larger than all others. It is true that the phases of $a$ and $c$ are nearly equal, so that $N_0$ is an order of magnitude larger than $U_2$ in magnitude. This is not the case for the remaining three magnitudes, however. Perhaps such $N_0$ dominance is not indicated by the data.

The recent publication of more polarization observables from the ZGS experiments bears on this question of $N_0$ dominance. The experimenters enhance the observable set with the addition of $KLS$, $DSS$, $LSK$, $NLS$, $NSS$. In principle, this gives an overdetermined set. The amplitudes should be fixed uniquely-unambiguously. In practice that is not necessarily the case as we now report.
The experimenters attempt to determine the amplitudes from their data\(^7\). They use a \(\chi^2\) minimization to obtain those amplitudes that give the best fit to the new observables. To make this procedure tractable they assume that \(N_0\) is dominant. In fact, they are unable to obtain a reasonable \(\chi^2\) for the full observable set. The kob data must be excluded to obtain a meaningful \(\chi^2\) for a fit with \(N_0\) dominated amplitudes. Then the resulting definite parity helicity amplitudes are in fair but not close agreement with various Regge models\(^{15,16}\).

However, there is no reason to expect the kob data to be incompatible with the other observables\(^17\). Either the \(N_0\) dominance assumption is unwarranted or there are inconsistencies in the full data set. The former possibility is under study since our determination from the smaller data set casts doubt on \(N_0\) dominance at 6 GeV/c.

If indeed \(N_0\) dominance is incorrect, even at the smallest \(t\) values, as we found previously, then what can be said about the dynamics in this intermediate energy region? The Regge picture in its latest form\(^{15,16}\) certainly will have to be modified significantly. And that latest form has so many complications that the whole enterprise has few remaining proponents to modify the scheme further. What other dynamical approaches would be fruitful? We consider other points of view next.

V - ONE PARTICLE EXCHANGE TESTS

While the particular Regge pole approach to particle exchange may be inadequate or incorrect, the notion of particle exchanges dominating hadronic interactions is as old as Yukawa’s proposal that led to the pion. As complete explanations of dynamics, single particle exchange is long known to have theoretical problems – reality of amplitudes, unitarity, exploding energy dependences, etc. Those various problems have received considerable attention through the years. As a result, many solutions have been proposed that maintain some of the structure of one particle exchange (OPE) while modifying that structure to satisfy some of the necessary theoretical constraints. OPE has returned in many different guises and has usually provided at least qualitative understanding of scattering phenomena.

Can the basic notion of OPE dominance be tested, independent of particular models for such exchanges, e.g. Regge poles, complex poles, absorbed poles? We have recently shown how to accomplish this given some complete set of amplitudes for a reaction at some energy and momentum transfer\(^{18}\). The test is obtained fairly easily in the Optimal Formalism.

Consider the reaction \(A+B \rightarrow C+D\) with spins \(S_A...S_D\). Suppose a single “particle” of definite \(J\) and parity is exchanged in the crossed \(t\) channel. Then for planar amplitudes defined in the \(t\) channel, i.e. for the process \(A+C \rightarrow B+D\), two types of constraints operate. The first, the ”\(J\)-constraint”, requires that the net spin projection, \(S^z\), for the incoming state along some planar axis, \(z\), not exceed \(J\); similarly for the outgoing state. Hence for \(t\) channel helicity amplitudes for example, the helicities are constrained by

\[
| a-c | < J, \quad | b-d | < J
\]

since

\[
D(t)(c,a;d,b) = \int D_J(t)(c,a;d,b) \, d_{-c;d-} \, (\theta_t), \quad (1)
\]

where the \(a,...,d\) are the spin projections along the momenta (i.e. helicities)\(^{20}\) for particles \(A,...,D\) in the \(t\)-channel. For reactions with high spin particles these \(J\)-constraints can force many amplitudes to be zero. Of course when the physical \(s\)-channel amplitudes are formed as linear combinations of continued \(t\)-channel amplitudes\(^{21}\) those zeroes give rise to equalities or linear relations among the \(s\)-channel amplitudes. The form of the linear relations depends on the choice of frame again.
Fig. 6: Amplitudes a and c in the magic frame vs t; (a) magnitudes, (b) phases.

VI - SIDEWISE AMPLITUDES

Once the p-p elastic transversity amplitudes were obtained at 6 GeV/c we explored different choices of planar amplitudes to see whether or not simplifications in the structure of the amplitudes appear for particular quantization axes, other than the standard ones. We reported in the previous Symposium that one propitious choice of planar axes is transverse to the momenta, but in the scattering plane. In that planar-transversity, or "sidewise", frame the amplitudes become almost exclusively pure real or pure imaginary with respect to one another as Fig.7 illustrates. Since then we have looked at the lower energy data, 300 to 800 MeV using phase shifts and amplitudes there as well. The results are rather startling as Fig. 8 shows. These lower energies exhibit the same kind of simplicity in the sidewise frame as the 6 GeV/c case.

Why should the sidewise frame have particularly simple phases for the amplitudes? There seems to be no simple explanation in terms of exchanges since sidewise amplitudes are combinations of both naturalities. Furthermore the persistence over the large energy range considered suggests none of the "standard" pictures that apply only to the lower intermediate or higher medium energies could be operating herein. We continue to puzzle over this striking phenomenon. Is this new dynamics?

VII - QCD BASED MODELS

It is not at all obvious that QCD should have anything to say about 6 GeV/c p-p elastic scattering. The energy may be too low; the momentum transfer is too soft; the process is an exclusive one. Yet it is worth asking whether the OPE dominance found above can reflect a gluonic sub-process.

There are many versions of QCD inspired models for exclusive scattering in the hard region, i.e. \( s/2 = |t| >> m^2(\text{quark}) \) or \( m^2(\text{hadron}) \) or \( \Lambda^2(\text{QCD}). \) In such models the hadrons dissociate into constituents and the constituents interact via fundamental QCD perturbative diagrams like one-gluon-exchange, quark interchange, multiple gluon exchange, multiple quark exchange, etc. Because the helicity structure of the fundamental quark-gluon vector coupling is so simple for large momentum transfer, there are simple predictions for the helicity structure of the hadronic process itself. In particular we know that single overall helicity flip is difficult to produce by hard scattering (yet that is contradicted by the latest BNL data reported at the Symposium).
What becomes of this picture as the $|t|$ value starts decreasing? For one thing the running strong coupling constant starts increasing, although only logarithmically. Hence higher orders become relatively more important and multiple gluon exchange should start to dominate. The quark interchanges should be less important as the u-channel becomes further away. Now in the multiple gluon exchanges the loop integrations always include a region in which one of the gluons carries most of the momentum transfer. The other exchanged gluons will then be soft and serve only to "fix the color". Suppose these regions dominate the integrals at these intermediate $|t|$ values. Then the process will have the kinematic structure of "one gluon exchange" even though multiple soft gluons are involved in initial and final state interactions.

Adopting this speculative picture\(^{(12)}\) of "one gluon exchange" at moderate momentum transfers, we have a simple conclusion. The amplitudes for the p-p elastic process, in which the "OGE" is embedded, will have the structure of natural parity OPE. The simple helicity predictions for the hard region need not hold in this region, but the basic dominance of a single spin exchange will apply. So the near equality of $a$ and $c$ in the magic frame could confirm this point of view.

Certainly there is nothing conclusive or compelling about the argument presented here except the phenomenological necessity. OPE has been seen to dominate the 6 GeV/c data\(^{(19)}\) but Regge models fail to fit the amplitudes\(^{(11)}\). What can be the source of the OPE structure? We simply present a possible interpretation in terms of QCD based\(^{(12)}\) notions. Whether the proposed "OGE" mechanism can actually dominate such relatively soft kinematic regions remains to be seen.

We gratefully acknowledge very useful discussions with A. Yokosawa concerning the ANL data, the work of our collaborators N. Ghahramany and F. Arash, and the hospitality of J. Soffer and the Symposium organizers. This work was supported in part by grants from the U.S. Department of Energy.

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Fig. 7: Relative phases of sidewise amplitudes at 6 GeV/c.

Fig. 8: Relative phases of sidewise amplitudes at 579 and 800 MeV.
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14. See Ref. 6 and 13 for references to data.
17. We are indebted to A. Yokosawa for discussion of data analysis.
27. A. Krisch in these Proceedings.