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RELATIVISTIC ELECTRONS AND COHERENT RADIATION

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Résumé - Nous décrivons dans cet article trois manières d'obtenir une émission cohérente de radiation dans le domaine de l'ultraviolet sous vide et des rayons X mous : toutes sont basées sur l'existence d'un faisceau d'électrons relativistes circulant dans un anneau de stockage et un système magnétique appelé onduleur.

Abstract - In this paper we describe three ways to obtain coherent radiation in the vacuum ultraviolet and the soft X rays: all of them are based on relativistic electrons circulating in a storage ring and a magnetic device called undulator.

INTRODUCTION

In this paper I will describe some recent work done at Orsay to obtain coherent radiation in the vacuum ultraviolet. Three ways are considered: they are based on a storage ring and a magnetic device called undulator.

In the first part of the paper I will give the main characteristics of the storage ring and the undulator and then describe the different methods.

When an electron (or a positron) is accelerated in a magnetic field there is emission of radiation. The wavelength of this radiation will depend essentially on the energy E of the electrons. For E of the order of few GeV one can reach the hard X rays, for E \( \approx 500 \) MeV the spectrum is concentrated in the vacuum ultraviolet and the soft X rays. This emission is called synchrotron radiation and is emitted in any storage ring.

A schematic of a storage ring (A.C.O.) is shown in Fig. 1. It is made with 8 bending magnets and some "optics" to keep the beam to a reasonable size (quadrupoles and hexapoles). A radio frequency cavity allows to keep constant the energy of the electrons. One of the straight sections has been used to install a magnetic undulator.

Let see what are the important parameters for a storage ring. We suppose that the electrons have a gaussian distribution. The position of an e\(^{-}\) and its angle (measured relative to the equilibrium orbit) are correlated. The size of the electron beam in the \( x(z) \) plane is characterized by \( \sigma_x(\sigma_z) \) which is the standard deviation of the e\(^{-}\) displacements and its divergence \( \sigma^x(\sigma^z) \), the standard deviation of the direction angle. At focus, the product \( \sigma_x \times \sigma^x(\sigma_z \times \sigma^z) \) is equal to the horizontal (vertical) emittance \( \varepsilon_x(\varepsilon_z) \).

The horizontal emittance is the result of a balance between the oscillations induced by the quantum emission of radiation and the damping which is created by the average loss of energy. In an ideal storage ring there is no radiation in the verti-
cal plane and the vertical emittance should be very small. However in a real machine, imperfections cause coupling of the motions between the two planes.

At relativistic energies the dipole pattern radiated by the electrons is sharply peaked in the direction of motion of the electrons with a typical half-angle opening of the order of $\gamma^{-1}$ where $\gamma = E/mc^2 = 1.96 \ E(\text{MeV})$. This is important because it shows that synchrotron radiation has, at least in the vertical plane, a very good spatial (or transverse) coherence.

I. UNDULATOR

The term undulator is used for a periodic transverse magnetic field (quite often quasi-sinusoidal) with many periods (10 to 100) intended to produce a spectrum which is composed of one or several narrow lines (harmonics). These are due to the interference from electromagnetic fields emitted by the same electron at different points of its trajectory. The main properties /1/ can be summarised as follows:

a) the wave length of the lines is given by (along the axis)

$$\lambda_n = \frac{\lambda_0}{2\gamma_n^2} \left(1 + \frac{K^2}{2}\right)$$
where \( \lambda_0 \) is the magnet period, \( \gamma = E/mc^2 \), \( K \) is a parameter which defines its field strength (\( K = 93.4B\lambda_0 \), MKSA)

b) if the emittance of the storage ring is small enough, the increase in spectral brightness, compared with a bending magnet, is \( N^2 \) where \( N \) is the number of periods.

c) the bandwidth of the lines is \( \sim \frac{1}{nN} \)

d) the power in the \( n \)th harmonic which passes through a pinhole selecting a band \( \frac{\Delta y}{\gamma} \ll 1 \) is given by

\[
P_n = 109\epsilon^2 I N \frac{\Delta y}{\gamma} F_n(K)/\lambda_0 \quad \text{watts}
\]

where \( I \) is the current and \( F_n(K) \) a combination of Bessel functions /2/.

As an example with \( E = 5\text{GeV} \) it is possible to obtain easily \( P_1 \simeq 30 \) watts around 1 or 2 \( \AA \).

II. FIRST METHOD

Let try to see now how we can obtain coherent radiation with an undulator. We have a complete coherence for an optical beam if we have spatial (or transverse) coherence and temporal (or longitudinal) coherence.

The spatial coherence is related to the divergence of the source, the temporal to its monochromaticity.

The undulator is the only source with the laser having a remarkable spatial coherence, of the order of \( \frac{1}{\gamma} \simeq 0.1 \text{ mrad for } E = 5 \text{ GeV} \).

It is easy to understand that the divergence of the source is also important. If we are limited by the diffraction we must have

\[
\varepsilon_x \varepsilon_z = \varepsilon_x \varepsilon_z \lambda^2
\]

This condition shows that with increasing photon energy smaller emittances are required. As examples for the European project \( E = 5 \text{ GeV} \)

\[
\varepsilon_x = 6 \times 10^{-9} \text{ mrad}
\]

\[
\varepsilon_z = 6 \times 10^{-10} \text{ mrad}
\]

For \( \lambda \simeq 19 \text{ Å} \) the source has a spatial coherence. It is possible to go down to 1 or 2 \( \AA \) by inserting pinhole, with a loss of intensity proportional to the reduction in \( \varepsilon \). That means that powers of the order of 1 watt with total spatial coherence should be available around 1 or 2 \( \AA \).

At the exit of the undulator the optical beam has a poor temporal coherence. But this can be corrected easily by adding a monochromator. Around 10KeV resolution of 1meV are already possible which gives a coherent length

\[
L_{\text{coh}} = \frac{\lambda^2}{\Delta \lambda} \sim 1 \text{ or } 2 \text{ mm}.
\]

It is clear that with the appearance of storage ring with very low emittance as Bessy (Berlin), Super ACO (Paris) or the European project they will be in the next few years a lot of excitement in the field of microscopy, holography and protein crystallography.
III. ULTRAVIOLET GENERATION FROM AN OPTICAL KLYSTRON

A - Today's experiment

We will see in the last part of this talk that it is not very easy for the moment to make a free electron laser for \( \lambda < 1550 \, \text{Å} \). However, coherent radiation will be very useful for spectroscopy of atoms and molecules, microscopy and holography. We have seen in the first part of this paper that by using an undulator and a monochromator it was possible to obtain coherent radiation with a good average power. However, if one is interested by peak power, this is not a good technique. This is the reason of the experiment that I will describe now: it is based on the fact that in an undulator it is possible to create a bunching of the electrons by an external powerful laser.

Let take an undulator at resonance:

\[
\lambda = \frac{\lambda_0}{2\gamma^2} (1 + K^2/2)
\]

as seen previously.

If the electrons are uniformly distributed over a large number of optical wavelengths, \( \lambda_0 \), (which is the case in any storage ring where \( \lambda < 1 \, \text{μm} \) and \( \sigma_x \), the FWHM bunch length, is between 1 cm and 10 cm), then the radiation fields of two individual electrons are not correlated. Thus, for an uniform distribution of electrons, the total radiated power is only proportional to the number of electrons in the bunch. On the other hand, if the electron spatial distribution is no longer uniform but is modulated with a periodicity corresponding to the resonant wavelength, the average radiation field for a given harmonic from the whole bunch is no longer zero. In fact, the emitted power \( P \) is proportional to the square of the number of electrons times the square of the Fourier coefficient for this harmonic. In this case, the spontaneous emission of the undulator is strongly enhanced, and, in addition, the coherence properties of the radiation are modified. The coherence length of the radiation (which is \( N \lambda \) in the former case, where \( N \) is the number of periods of the undulator) is, in the latter case, given by the coherence length of the electron bunch modulation. This effect was studied several years ago for the microwave range and allowed for the development of klystron devices /3/.

Several authors have published theoretical proposals, applying this idea to the optical range /4/. These authors proposed (Fig. 2) to illuminate an electron bunch travelling along an undulator with laser light the wavelength of which is equal to the resonant wavelength of the undulator. According to the FEL theory /5/, this results in an energy modulation of the electron beam (and possibly to some spatial modulation).

![Fig. 2 - Schematic of the harmonic generation experiment using an optical klystron.](image-url)
This energy modulation can be converted to a spatial modulation using a drift section (as in microwave tubes where electrons are non relativistic) or using a dispersive magnetic section (the optical klystron configuration) /6/ for ultra relativistic electrons (Fig. 2). At the end of this section, the modulated electron beam enters into a second undulator; the spontaneous emission of this undulator at wavelength $\lambda/n$ is therefore modified as described above. This technique avoids the use of mirrors, as in the free electron laser case, to produce UV light. It should be efficient on most of the existing storage ring to produce light of wavelength between about 100 and 2000 Å by starting with a visible or U.V. commercially available laser. Although this process is often called "multiplication" or "up-conversion", it is different from usual harmonic production since the coherent output power is taken from the electron energy and not from the pumping laser.

Let us only recall that, in the case of the O.K., the ratio, $R_n$, of the coherent over the incoherent (spontaneous) emission, for the harmonic $n$ of the laser frequency, for a given laser power and within the bandwidth of the coherent emission, is proportional to:

$$R_n = N.I. f_n^2,$$  

where $N$ is the number of periods of the radiator and $I$ the ring current. $f_n$ is the spontaneous emission modulation rate, resulting from the interference of the two undulators constituting the O.K., at wavelength $\lambda/n$. This interference is driven by the strength of the dispersive section and the energy spread of the electrons and:

$$f_n = \exp \left( -2 \sqrt{2} N_d \left( \frac{\sigma_x}{\gamma} + \frac{\sigma_z}{\gamma} \right) \right)^2$$  

where $N_d$ is the number of wavelength of the YAG laser passing over an electron in the dispersive section and characterizes the dispersive section strength /6/ and $\sigma_x/\gamma$ is the relative energy dispersion of the beam. Thus this dispersion, which does vary much with $I$ on A.C.O., is a very crucial parameter.

The goal of the A.C.O. experiment was to demonstrate the feasibility of the harmonic production. Although there is no theoretical limitation in going into the V.U.V. spectral range (by using a higher electron energy), we chose to work in the visible part of the spectrum for convenience of the detection. We used the 1.06 μ fundamental line of a pulsed Nd:YAG laser (Fig. 3) focused into our optical klystron on the storage ring A.C.O. working at 166 MeV and looked at the $3^{rd}$ harmonic at 355 nm. At this energy, the modulation rate, $f_n$, is much smaller than one. This is due to the anomalous bunch lengthening on A.C.O. which makes the energy spread to be much larger than the nominal energy spread at 166 MeV ($1.4 \times 10^{-4}$ for $I \approx 0.01$ mA). Also there is an additional energy spread due to the interaction with the YAG pulse, since the ring energy damping time is 180 msec at 166 MeV and the YAG repetition rate was 20 Hz in our case.

The low laser repetition rate (20 Hz) makes the long time average coherent emission very weak compared to the incoherent spontaneous emission whose repetition rate is 13.6 MHz. Thus the coherent power has to be measured on a fast time scale.

The coherent emission pulse was sent on a box-car averager in order to obtain an output proportional to the coherent power and integrated over many YAG pulses.

The total angular divergence of the coherent emission has been found to be close to 1 mrad in good agreement with the diffraction limit ($\lambda/\sqrt{\pi/\sigma_x\sigma_y}$, where $\sigma_x, \sigma_z \approx 200\mu m$ are the bunch RMS transverse dimensions). The spectral width of the coherent emission was too small to be measured by our detection system. By using a monochromator of spectral resolution $\Delta \lambda \approx 0.3$ Å we could only set an upper limit of
0.1 Å. Since the spontaneous emission is very broad ($\Delta \lambda \sim 200$ Å), the measured values of $R_3$ depend linearly on the spectral resolution used and an absolute value can be set only by assuming a given value for the coherent emission spectral width. The value of $R_3$ (corresponding to a small solid angle), for $\Delta \lambda \sim 0.7$ Å, has been measured for ring currents ranging from 0.1 to 10 mA. The modulation rate and the electron bunch dimension have also been recorded in the same range of current (Fig. 4) in order to allow a comparison with the theory. The theoretical curve corresponding to formula (1) is also drawn on Fig. 4. It can be seen that the variations of $R_3$ with the ring current are qualitatively well explained by two opposite effects: the increase of the numbers of electrons and the strong decrease of the modulation rate ($f_0 \sim 10^{-3}$ for $\frac{\Delta \lambda}{\lambda} = 12.10^{-4}$ at $I \sim 10$ mA). The combination of these two factors produces the maximum observed at about 1 mA. The maximum measured value of $R_3$ in approximately $4.10^2$ (for $\Delta \lambda = 0.7$ Å) although the theoretical value is about 5 to 6 times more. We explained this effect by considering the pulse to pulse fluctuations of the coherent emission when recorded on a fast scope.

The maximum number of coherent photons emitted per pulse is about $10^5$ theoretically and $2.10^4$ experimentally /7/. This rather small number is due to various losses: (i) due to the large energy dispersion of the ring at low energy the maximum is reached at 1 mA of ring current where $f_3^2$ is only $10^{-2}$. (ii) we worked at a lower laser power ($P_L \sim 15$ MW). This accounts for a loss of a factor $\sim 10^2$ ($P_L \sim 100$ MW and optimized dispersive section). (iii) we have a limited number of periods in the "radiator" section ($N = 7$). Thus a factor $10^6 - 10^7$ is lost when we compare with an optimized klystron placed on a storage ring exhibiting no anomalous bunch lengthening. In our case the O.K. had been optimized for free electron laser studies and the parameter $N_d$ is too strong for this experiment. Moreover the energy of 166 MeV is very far from the nominal working energy of A.C.O. (540 MeV).

B - Future of the experiments

On ACO we expect to use a shorter pulse (3 nanosec) Nd:YAG laser as a pump.
Fig. 4 - Experimental value of the coherent emission ratio $R_3$ (points with error bars), modulation factor of the third harmonic incoherent emission with the laser off (a) and on (b). The three curves are plotted versus the stored current.

Therefore the available power, for about the same input energy, will be higher. By working between 240 and 350 MeV the numerical calculations show that the region 10-20 eV can be reached and that $10^7 - 10^8$ coherent photons/pulse will be produced for an input power of 50 MWatt.

Super ACO is a synchrotron radiation dedicated 800 MeV storage ring currently under construction at Orsay. It will be achieved in 1986. In that ring the anomalous bunch lengthening has been minimized. Also, the electron density will be rather high, in particular when using a 500 MHz RF cavity (120th harmonic of the ring frequency). Straight sections 3 m long will be available for the undulator and the free electron laser, or for frequency multiplication experiments.

Although super-ACO working energy will be 800 MeV, the machine has been designed in order to be able to run at lower energies. We have calculated the number of protons produced at 400 MeV (where the energy spread $\sigma_{\gamma}/\gamma$ is only $2.5 \times 10^{-4}$) by taking reasonable figures for the parameters ($\rho_e = 1.3 \times 10^{12}$ el/cm$^3$, $i = 7$ mA/bunch, $\sigma_1 = 0.7$ cm). Figure 5 shows the results obtained for an undulator optimized for FEL studies rather than for this experiment. However, one can see that typically $10^{10}$ photons/pulse can be obtained down to at least 500 A, corresponding to coherent peak powers close to 1 kWatt. These high peak powers should allow multiphoton excitation and non-linear processes studies in the V.U.V. spectral range. Therefore this technique will open a new field of investigation in solid state and molecular physics.
Fig. 5 - Expected number of photons on the ring Super ACO on various harmonics of a pump laser at 226 nm.

IV. THE FREE ELECTRON LASER

A free electron laser on a storage ring is, in principle, a simple device. It is made with an undulator and an optical cavity (see Fig. 1).

How the system works? It is out of the scope of this paper to give a detailed description of the F.E.L. /8/. We will just summarize the important parameters.

- When an electron travels in a storage ring it has only a longitudinal velocity: then it cannot couple to an electromagnetic field (which is transverse). The purpose of the undulator is to create a transverse velocity of the electron allowing coupling to the transverse electric field (transfer of kinetic energy of the electron beam to the photon). Acceleration of the electron beam is made at the expense of the electromagnetic field.

- The first laser oscillation has been obtained at Orsay in June 1983 /8/ showing the feasibility of the experiment. The emission was obtained around 6508 Å and allowed a very detailed study of the mechanism of the laser. The main problems can be summarized as follows:

  - the small signal gain per pass \( G \) is \( \alpha \lambda^{3/2} N^3 I_p / \Sigma \)

where \( I_p \) is the peak current, \( \Sigma \) the transverse dimension of the beam. This shows immediately why on A.C.O. it is only possible to have a laser in the range 4000-6000 Å due to the fact that the gain varies as the cube of the number of periods. The gain optimization demands long undulator (3 to 5 meters instead of 1.2 m on A.C.O.). The \( \lambda^{3/2} \) dependence gives also an idea of the limit in the short wavelengths (≈ 500 Å).

We are presently building an optical klystron for super A.C.O. The main characteristics are given in Table I. The lower limit of the expected wavelength range is determined mainly by the mirror reflectivity which drops rapidly below 120 nm. The output power of the laser is limited to few % of the synchrotron power emitted all around the ring. Then for super A.C.O. the expected power should be of the order of few watts in the visible and the ultraviolet.
TABLE I

a) Super ACO electron beam characteristics at 400 MeV
- Emittance : $\varepsilon_x = 2.8 \times 10^{-8}$ mrad
- Energy spread : $2.6 \times 10^{-4}$
- $I_p$ : 50 A
- RF freq. : 500 MHz
- Injection : $e^+$
- Bunch length : 2.7 mm

b) Optical klystron
- Length : 3.3 m
- Magnetic period $\lambda_0$ : 10 cm
- Number of periods $N$ : 15 (per undulator)
- $N_d$ : 0 to 300
- Expected wavelength range : 120-700 nm
- Gain at $\lambda=150$ nm : 20%
- Maximum $K$ : 6

I should mention that a 1GeV storage ring specially designed for free electron laser (with 25 meters undulator) has been proposed by J. Madey /9/ and it is under construction now at Stanford. In that case, rather high gain should be obtained.

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