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ANALYSIS OF ENERGY BROADENING IN CHARGED PARTICLE BEAMS

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Résumé

L'anomalie de distribution énergétique d'un faisceau d'ion monomères ou d'électrons est décrite par un modèle de file indienne à un paramètre, régi par l'émission de bruit de grenaille et par les interactions coulombiennes de paires. La dépendance de la largeur $\langle\Delta E\rangle$ de la distribution et du décalage $\langle\phi\rangle$ de l'énergie moyenne en fonction du courant, de la masse et de la tension est calculée. Une unification de plusieurs autres théories d'élargissement énergétique est faite à partir de la variation d'un paramètre libre qui représente des effets de champ liés à la géométrie. Une caractéristique unique du modèle présent est le fait que $\langle\Delta E\rangle$ est inversement proportionnel à la charge.

Abstract

The anomalous energy distribution of a monomer ion or electron beam is described by a one parameter, single file model governed by shot noise emission and pairwise Coulomb interactions. The dependence of the width of the distribution $\langle\Delta E\rangle$ and the shift in the mean energy $\langle\Phi\rangle$ on current, mass, and voltage is calculated. A unification of a number of other theories of energy broadening is made by variation of a free parameter that represents field-geometrical effects. A feature unique to the present model is that $\langle\Delta E\rangle$ can be inversely related to charge.

1. Introduction

Since the work of Boersch [1] the anomalous energy distribution in charged particle beams has attracted the attention of a number of investigators. The distribution provides an insight to the characteristics of the emission process that govern its behavior as a function of current (I), mass (m), and charge (q=ne). Additional motivation for studying this property arises from the variance of the distribution being directly proportional to the chromatic image blur diameter [2]:

$$d_c = C_c\,\alpha\langle\Delta E\rangle / eV_0,$$

where C_c is the chromatic aberration coefficient of the lens, α is the aperture half angle subtended at the image, V_0 the beam potential, and $\langle\Delta E\rangle$ is taken as the full width half maximum of the total energy distribution regardless of its nature. Therefore understanding the mechanisms that effect the distribution may allow one to minimize the chromatic aberration limit on the beam size in microprobe applications. This limit is particularly important in liquid metal ion source (LMIS) applications.

Here we consider an analysis of the energy distribution in charged particle beams with the hope of elucidating the fundamental physical mechanisms that effect its behavior. A classical statistical model of charged particle emission is presented in which the process is governed by shot noise. This is consistent with the measurement of the power spectra of Ga and Bi LMIS that were found to be shot noise limited [3]. By further assuming that pairwise Coulomb interactions dominate, expressions for $\langle\Delta E\rangle$ and the shift in the mean energy $\langle\Phi\rangle$ are calculated. Results are compared to LMIS measurements and a number of other theories concerned with the behavior of the distribution.

2. Model

Our analysis assumes a one dimensional (single file) current regime of charged particle emission. This regime can be defined as those currents whose average inter-particle separation $<\delta>$ is greater than the minimum beam radius Δ [4].

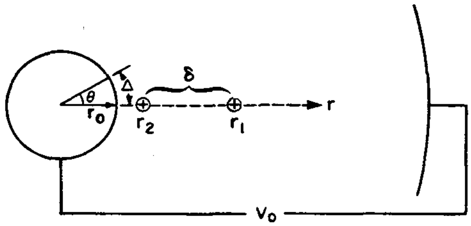

Figure 1. Diagram depicts the sphere emitter model with single file emission of particles 1 and 2. The emission half angle ϑ determines the beam width Δ near the emitting surface.

According to Figure (1) this implies that at the emitter surface:

$$<\delta_0> \; > \Delta \approx \vartheta_0 r_0 \ , \tag{1}$$

where ϑ_0 and r_0 are the emission half angle and emitter radius respectively and δ_0 is the initial position of particle 1 when particle 2 is just emitted. This qualitatively describes the boundary between pairwise and multiparticle, multidimensional interactions. Assuming for the LMIS that $r_0 = 3nm$ and $\vartheta = 15^0$ then $<\delta_0> \; > 0.8nm$ delineates the condition for single file behavior. The values for r_0 and ϑ_0 are believed to be correct within a factor of two [5,6].

Assuming shot noise appropriately describes the emission process the charged particles can be considered as being emitted randomly with the number per unit time following a Poisson distribution.

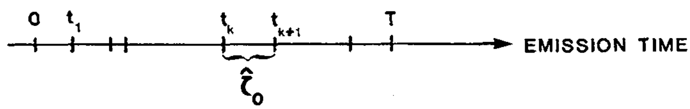

Figure 2. Diagram representing Poisson distributed emission times. The random variable τ_0 is exponentially distributed and corresponds to the interevent time of emission.

The probability density function (PDF) for the interevent time τ_0 is then given by [7]:

$$f(\tau_0) = \lambda \exp(-\lambda \tau_0) \ . \tag{2}$$

where $\lambda = I/q$ is the average rate of emission.

The electric field outside a sphere is :

$$F(\delta_0) = V_0 r_0^{-1} (1 + \delta_0/r_0)^{-2} \ . \tag{3}$$

When $\delta_0 \ll r_0$ this reduces to a uniform field :

$$F_0 = V_0/r_0 \ . \tag{4}$$

It is shown below that during the interval τ_0, the emitted particle interacts, on the average, with a nearly uniform field for the range of currents and masses used in the analysis of LMIS data. Therefore the following relation holds:

$$\delta_0 = (qF_0 \tau_0{}^2)/2m \ . \tag{5}$$

Using Eqs.(2) and (5) the PDF for δ_0 is given by:

$$f(\delta_0) = (k/2)\delta_0^{-1/2} \exp(-k\, \delta_0{}^{1/2}) \ . \tag{6}$$

where $k = I(2m/q^3 F_0)^{1/2}$. The mean initial spacing is:

$$<\delta_0> = q^3 F_0/mI^2 \ . \tag{7}$$

For example, if one assumes the evaporation field for Al^+ is $F_0 = 20\,V/nm$, then a current of $1\mu A$ implies, $<\delta_0>_{Al^+} = 1.8nm$. This is within the single file regime defined earlier for $r_0 = 3nm$ and also satisfies the uniform field requirement, Eq.(1). One also finds that for typical ranges of current the uniform field approximation is reasonable when considering field electron emission.

The current, mass, and charge dependence of the shift in the mean energy $<\Phi>$ can be obtained as follows. The shift in the average energy per particle due to the pair-wise Coulomb interaction is:

$$\Phi = q^2/2\delta_0 \ , \tag{8}$$

i.e., only half the initial potential energy can be converted to kinetic energy at the anode. Equations (6) and (8) lead to the PDF for Φ:

$$f(\Phi) = (I^2 m/4qF_0\Phi^3)^{1/2}\exp(-I\sqrt{m/qF_0\Phi}) \ . \tag{9}$$

The mean value of the shift is defined as:

$$<\Phi> = \int_0^{\Phi_c} \Phi f(\Phi)\, d\Phi \ , \tag{10}$$

where $\Phi_c \equiv q^2/\delta_c$ and δ_c is taken as the minimum possible value of the initial separation δ_0. Without such an upper bound on the integration Eq.(10) would diverge. Evaluated in the low current limit this integral leads to:

$$<\Phi> = I\sqrt{mq/F_0\delta_c} \tag{11}$$

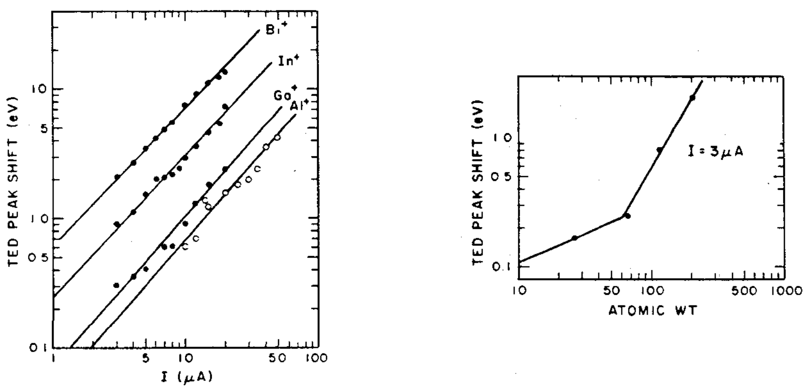

Figure 3. Plots of experimental shifts in the TED peak position as a function of current (left) and mass (right) for the indicated LMIS species.

According to Eq.(11) a shift to larger kinetic energy with increasing current is expected. This prediction is borne out experimentally for Al^+, Ga^+, In^+, and Bi^+ as shown in Fig. (3). For a given current the predicted relation $<\Phi> \propto m^{1/2}$ agrees with the data for low mass ions ($\leq$ m(Ga)). For high mass ions only a qualitative agreement is obtained, possibly due to an implicit mass dependence of δ_c in Eq.(11).

Exact evaluation of $<\Delta E>$ requires detailed knowledge of the individual trajectories of the particles. This appears to be possible only by numerical integration of the equations of motion. In order to retain a closed expression for $<\Delta E>$ with the hope of understanding how the observable quantities I, m, q, and V_0 effect this variable we note that ΔE is a monotonic decreasing function of δ_0 [8]. We approximate this relationship by:

$$\Delta E = b_\alpha/\delta_0^\alpha \ , \tag{12}$$

where α is a real positive parameter. Dimensional analysis indicates that, $b_\alpha = qV_0 r_0^\alpha$, where the tip radius has been chosen as the length measure. A choice of any other length would only introduce a numerical constant that will not effect the functional dependence of the distribution. The PDF of $<\Delta E>$ is then calculated using Eqs.(9) and (12). The result is:

$$f(\Delta E) = [d_\alpha/2\alpha(\Delta E)^{1+1/2\alpha}]\exp[-d_\alpha(\Delta E)^{-1/2\alpha}] \tag{13}$$

and $d_\alpha = k\, b_\alpha^{1/2\alpha}$. The expected value of $<\Delta E>$ is then found to be:

$$<\Delta E> = d_\alpha{}^{2\alpha} \int_0^\infty s^{-2\alpha} e^{-s}\, ds \quad . \tag{14}$$

For the case $\frac{1}{2} > \alpha > 0$, Eq.(14) becomes:

$$<\Delta E> = d_\alpha^{2\alpha}\, \Gamma(1-2\alpha) \quad , \tag{15}$$

where $\Gamma(1-2\alpha)$ is the gamma function [9]. We may allow $\alpha \geq \frac{1}{2}$ if a limit is imposed on the maximum possible value of the energy spread, $<\Delta E>_{max} \equiv E_c$. Again this is equivalent to imposing a limit δ_c on the initial separation. This type of approximation is typical of all classical theories dealing with charged particle interactions [4,8,10-16]. Assuming $\frac{1}{2} \leq \alpha$ then $<\Delta E>$ is expressed as:

$$<\Delta E> = d_\alpha^{2\alpha}\, \Gamma(1-2\alpha, z) \quad , \tag{16}$$

where $\Gamma(1-2\alpha, z)$ is an incomplete gamma function and $z = d_\alpha E_c^{-1/2\alpha}$. In the limit of low currents Eq.(16) is given by:

$$<\Delta E> = I m^{1/2} (V_0 / q)^{(1-\alpha)/2\alpha}\, r_0\, \delta_c^{(1-2\alpha)/2\alpha} \quad . \tag{17}$$

Variation of the parameter α results in a rather interesting unification of a number of energy broadening theories. To see this we first define a rescaled length, $l_q = q / V_0$, which will not effect the current or mass dependence of $<\Delta E>$. One then finds that the theoretical expressions for the energy broadening, Eqs.(15) and (17), have the general form:

$$<\Delta E> = (I^2 m)^\alpha\, V_0{}^{h(\alpha)}\, G(\alpha) \quad , \tag{18}$$

where $G(\alpha)$ is a product of the geometrical factors only and $h(\alpha)$ is some function of α.

Table I lists seven theories that predict an energy broadening of the form:

$$<\Delta E> = I^{1/2} m^{1/4} q^{3/4} V_0^{1/4}\, G_i \quad . \tag{19}$$

Here the parameter is $\alpha = 1/4$ and G_i (i=a,b,c,d) indicates the various forms the geometrical factor assumes in the different studies.

REFERENCE	GEOMETRICAL FACTOR	MODEL
PRESENT STUDY (α=1/4)	$G_a=(r_0/l_q)^{1/2}$	SINGLE FILE CLOSED FORM
ROSE and SPEHR (1980)	$G_b = 1$	CROSSOVER
VAN LEEUWEN and JANSEN (1983)		CROSSOVER
GESLEY, LARSON, SWANSON, and HINRICHS (1984)		SINGLE FILE W/ NUMER. CALC.
MASSEY, JONES, PLUMMER (1981)	$G_c=(l_q D^2/R_0^3)^{1/3}$	PARALLEL BEAM
KNAUER (1979)	$G_d=(l_q d_0/R_0^2)^{1/2}$	CROSSOVER
LOEFFLER (1969) $I \longrightarrow \infty$		CROSSOVER

Table I. List of various theories that predict an energy broadening of the form Eq.(19). Various lengths are: d_0=truncated emitter diameter, r_0=emitter radius, D=beam length, R_0=beam radius, $l_q = q / V_0$.

Setting $\alpha = 3/16$ leads to:

$$<\Delta E> = I^{3/8} m^{3/16} V_0{}^{5/4}\, G_i \quad . \tag{20}$$

Table II compares the results of the present study with the two cases considered by de Chambost and Hennion [12].

REFERENCE	GEOMETRICAL FACTOR	MODEL
PRESENT STUDY (α=3/16)	$G_a=(r_0^6\, l_q^7)^{1/16}$	SINGLE FILE
DE CHAMBOST-HENNION (1979)	$G_b=(l_q^{17} D^6/R_0^{12})^{1/16}$	PARALLEL BEAM
	$G_c=(l_q^{17}/R_0^4)^{1/16}$	CROSSOVER

Table II. List of theories corresponding to Eq.(20).

Setting $\alpha = 1/3$ yields:

$$<\Delta E> = I^{2/3} m^{1/3} V_0^{2/3} G_i \quad . \tag{21}$$

This reproduces Knauer's result concerning a diverging point source [4], Table III.

PRESENT STUDY	(α=1/3)	$G_a = r_0^{2/3}$	SINGLE FILE
KNAUER	(1981)	$G_b = (t_b/r_0^{1/3})$	DIVERGING POINT SOURCE

Table III. Theories corresponding to Eq.(21).

When $\alpha = \frac{1}{2}$ the energy broadening is given by:

$$<\Delta E> = I\, m^{1/2} G_i \tag{22}$$

This is also the result obtained in the low current limit of a circular crossover [14] and in a model of single Coulomb deflections [17], Table IV.

PRESENT STUDY	(α=1/2)	$G_a = (r_0/t_q^{1/2})$	SINGLE FILE
LOEFFLER $I \longrightarrow 0$	(1969)	$G_b = (t_q^{1/2} d_0^2/R_0^2)$	CROSSOVER
CREWE	(1978)	$G_c = t_q^{1/2}$	SINGLE DEFLECTION

Table IV. Theories corresponding to Eq.(22).

Figure (4) plots LMIS data of $<\Delta E>$ as a function of $(Im^{1/2})$ for singly and doubly charged monomer ions using data from [17-19]. This corresponds to the behavior predicted by Eq.(22), $\alpha = \frac{1}{2}$.

The present work diverges with the other theories when the charge dependence of the energy broadening is considered. Figure (4) and [18] indicate that monomer ions in the low current regime exhibit the following behavior:

$$<\Delta E>_{LMIS} \propto I\sqrt{m/q} \quad . \tag{23}$$

By setting $\alpha = \frac{1}{2}$ we find Eqs.(17) and (22) yield precisely this relationship. Interestingly, no other theory (including the others listed in Table IV) predict an inverse relationship between $<\Delta E>$ and q.

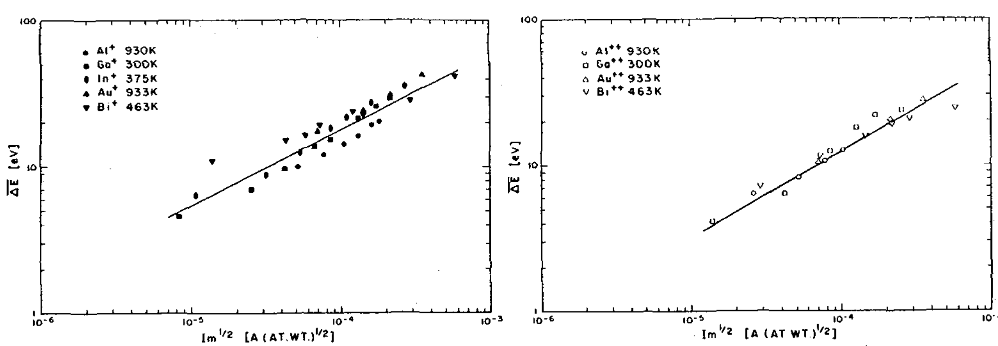

Figure 4. Plots of experimental values of $<\Delta E>$ according to the predictions of Eq.(22) for singly (left) and doubly (right) charged monomer ions.

3. Conclusion

Analysis of a model of one dimensional pairwise Coulomb interactions in a charged particle beam governed by shot noise has led to closed expressions for the anomalous energy broadening $<\Delta E>$ and shift in mean energy $<\Phi>$. The theoretical current dependence of <Φ> is in agreement with LMIS measurements of Al, Ga, In, and Bi monomer ions. For low masses the predicted mass dependence of <Φ> also agrees with the LMIS data. The use of a free parameter in the expression for <ΔE> has unified a number of energy broadening theories. The unique feature of the present study is its ability to explain that <ΔE> has an inverse relationship with charge as found in monomer ion emission from LMI sources.

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