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FLUCTUATIONS IN THE NUCLEAR DYNAMICS OF FISSION

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Abstract - First steps towards a dynamical theory of random neck rupture are described. Gaussian curvature is used to examine the dynamical behaviour of the fissioning nucleus.

The perhaps most important finding from so many experiments on fission and deep-inelastic reactions is the definite failure of statistical equilibrium and transport models /1,2/. This stimulates hope that nuclear physics might offer more than reformulations of ideas which have been developed for other branches of physics. An extraordinary characteristic of nuclear physics are the enormous fluctuations which are as large as the mean values; it is impossible to treat these fluctuations as small perturbations.

A principle of our approach is to consider nuclear fission and the deepest inelastic collisions as similar processes where the same mechanism applies. This mechanism is random neck rupture, i.e. prior to scission the nucleus forms a massive neck, which finally ruptures. The measured fluctuations are caused by variations of the rupture point. Despite of the simplicity of its present form, random neck rupture explains three types of hitherto poorly understood data:

a) Nuclear fission measurements show that heavy fragments evaporate disproportionately many nucleons /1/.

b) For the smallest measured total kinetic energies there is an extended Viola systematics which exceeds the range of validity of the familiar systematics by a factor five /2/.

c) Large variances of the mass and charge distribution are connected with very little drift /2/.

I want to give here first indications on the treatment of the dynamics. The problem is to find the reason why small thermal or quantal noise may induce those large fluctuations of the rupture point; one needs something like an amplifier of fluctuations.

If we consider the nucleus as a mechanical system, the dynamics comes out as a consequence of the structure of potential and kinetic energy. It turns out that the potential energy rather restricts fluctuations, but the kinetic energy destabilizes the position of the rupture point.
The concept of stability we need here is kinematical stability: A 
trajectory $x(t)$ (which in general is more than a point) is said to be 
stable if another trajectory $\tilde{x}(t)$ remains to be a neighbour for all 
times when it was a neighbour trajectory at some initial time. In 
other words: A diverging bundle of trajectories signalizes kinematical 
instability. Can one decide about kinematical instability without 
computing such bundles? The familiar procedure is to consider not 
kinematical but only static stability (i.e. stability in the vicinity 
of a point), to compute a potential energy surface, and to look for 
its extrema. But this does not suffice for the present problem. 
Rather one has to find some measure of stability which includes the 
kinetic energy. This measure is the Gaussian curvature, well-known 
in differential geometry /3/.

Suppose we have a mechanical system with two degrees of freedom $u$ 
and $v$, with potential $V(u,v)$, kinetic energy $2T=\epsilon(u,v)\dot{u}^2+2\phi(u,v)\dot{v}+$ 
g($u,v$)$\dot{v}^2$ and total energy $H$. Then one identifies

$$
E(u,v):=(H-V(u,v))\epsilon(u,v)
$$

$$
F(u,v):=(H-V(u,v))\phi(u,v)
$$

$$
G(u,v):=(H-V(u,v))g(u,v)
$$

where $E$, $F$, and $G$ denote Gauss' fundamental elements of the metric 
$ds^2=\epsilon du^2+2\phi du dv+g dv^2$. From this the Gaussian curvature $K$ follows as

$$
4(EG-F^2)K=\begin{vmatrix}
4\epsilon\phi-2(G\epsilon+\phi\epsilon) & \epsilon & 2\phi-\epsilon \\
2\phi-\epsilon & G & 0 \\
0 & G & \epsilon \\
\end{vmatrix}
$$

where subscripts are meant as partial derivatives /4/. The mechanical 
importance of $K$ is expressed by Jacobi's variational equation

$$
\frac{d^2\Delta}{ds^2}=-K\Delta
$$

where $\Delta$ stands for the distance between two neighbouring trajectories. 
Thus if $K>0$, the trajectories oscillate around each other forming a 
kind of plait. Small initial perturbations are amplified only if $K<0$.

The just mentioned ideas have been clearly spoken out by J.L. Synge 
/5/ while their importance for modern developments is stressed by 
D.V. Anosov and V.I. Arnold, see e.g. /6/.

To make the application of Gaussian curvature to nuclear fission a bit 
more precise, it is useful to discuss a scenario of nuclear fission:

1. Due to a large fluctuation in collective motion the compound state 
is left.

2. The nucleus surmounts the saddle. A shape forms with two heads and 
a massive neck.

3. The shape elongates while the heads get smaller. The neck telescopes out and straightens more and more.

4. The position of smallest neck diameter (the prospective rupture point) destabilizes.

5. When the neck becomes too long, it snaps. It snaps at the position of 
smallest neck diameter.

Point 5 was discussed in /1/. Here qualitative arguments shall be 
given for the assertions made in point 3 and 4.
Why does the shape elongate by diminishing its heads without strangulating its neck? This is a consequence of Rayleigh's jet instability as explained in /1/ and /2/: Strangulations of the neck are suppressed until the shape reaches a certain length. The material for elongation is taken from the heads since they move fastest into the outward directions. The material in the neck however moves very little, the less the closer it is to the center of mass. This unavoidably straights the neck.

Why does the rupture point destabilize? Suppose we can neglect for a moment the variability of the potential, i.e. set \( V = \text{const} \), and restrict the dynamics in phase 4 to two degrees of freedom: total length \( l \) of the shape and the position \( a \) of smallest neck diameter ("a" to indicate "asymmetry"). The kinetic energy reads

\[
2T = \mu \dot{l}^2 + v(l) \dot{a}^2 .
\] (4)

It is important to note that \( \mu \) is approximately the reduced mass, a constant, whereas \( v(l) \) decreases when \( l \) increases. This is because the neck straights when the shape elongates, the straighter the neck, the less mass is necessary to shift the position of smallest neck diameter. Furthermore \( v(l) \) is bounded from below, viz. \( v(l) > 0 \). Therefore \( v(l) \) must be a concave function, viz. \( v_{\|}(l) > 0 \) and also

\[
\sqrt{v(l)}_{\|} > 0 .
\] (5)

Identifying (4) according to (1) and inserting into (2) yields up to a positive constant

\[
k = -\sqrt{v(l)}_{\|}/\sqrt{v(l)} ,
\] (6)

and this is definitely negative. It follows from Jacobi's equation (3) that the stretching neck amplifies fluctuations.

Numerical calculations of the respective potential were presented in /7/; it is, as a function of \( a \), sufficiently flat. Neglecting however the characteristic dependence of the inertia parameter \( v(l) \) on the length \( l \) would lead to a positive Gaussian curvature.

I willingly admit that all these considerations are still only qualitative ones and may fail by closer inspection. But it is better to have an idea of the direction of search before one starts to torment the computer. Some discussion with S. Grossmann and the self-sacrifying help of Dr. Hahn with a picture which I could show only in the talk are gratefully acknowledged.

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