SEMICLASSICAL CALCULATION OF THE NUCLEAR RESPONSE FUNCTION AT HIGH MOMENTUM TRANSFER

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Résumé - Nous calculons la fonction de la réponse nucléaire semiclassiquement. Il s'avère que les résultats obtenus pour des impulsions transférées $q \geq 2$ fm^{-1} sont en très bon accord avec un calcul exact.

Abstract - We calculate the nuclear response function in a semiclassic fashion. It is shown, that the obtained results are for high momentum transfers $q \geq 2$ fm^{-1} in a very good agreement with an exact calculation.

I - INTRODUCTION

Inelastic electron and proton scattering from nuclei at high momentum transfers necessitates on the full quantum mechanical level a big numerical effort. Since in the quasi-elastic peak region shell effects are absent a semiclassical approach may be sufficient. We will show in this work that this is effectively the case for momentum transfers of $q \geq 2$ fm^{-1}.

II - THE THEORY

The free response function $\pi_N^{(o)}(q,\omega)$ for an excitation operator $\hat{Q}$ can be calculated from the particle hole Greens function $\pi^{(o)}(r_1, r_2, r_1', r_2')$.

$$\Pi_N^{(0)}(q,\omega) = \int d^3r_1\, d^3r_2\, d^3r_1'\, d^3r_2'\; Q^*(\vec{r}_1,\vec{r}_2)\; \Pi_N^{(0)}(\vec{r}_1,\vec{r}_2,\vec{r}_1',\vec{r}_2')\; Q(\vec{r}_1',\vec{r}_2') \tag{1}$$

$\hat{Q}$ shall be the longitudinal excitation operator

$$Q(\vec{r}_1,\vec{r}_2) = e^{i\vec{q}\vec{r}_1}\,\delta(\vec{r}_1-\vec{r}_2) \tag{2}$$

In coordinate representation, the Greens function attains the following form:

$$\Pi_N^{(0)}(\vec{r}_1,\vec{r}_2,\vec{r}_1',\vec{r}_2') = 2\,\langle r_1 r_2' | \frac{\theta(h_1-\lambda)\theta(\lambda-h_2)-\theta(h_2-\lambda)\theta(\lambda-h_1)}{\hbar\omega - h_1 + h_2 + i\eta} | r_1' r_2\rangle \tag{3}$$

where h is the one particle hamiltonian and λ the Fermi energy. For the Greens function we take now a semiclassical approximation. This approximation is achieved by replacing the operators by their classical counterparts. This Thomas Fermi like approximation has been very successful in the case of particle hole densities /1/.

For the longitudinal response function one obtains with

$$\Pi_N^{(0)}(\vec{r}_1,\vec{r}_2,\vec{r}_1',\vec{r}_2') = 2\cdot\int \frac{d^3p_1}{(2\pi\hbar)^3}\frac{d^3p_2}{(2\pi\hbar)^3}\; e^{\frac{i}{\hbar}\vec{p}_1(\vec{r}_1-\vec{r}_1')}\, e^{\frac{i}{\hbar}\vec{p}_2(\vec{r}_2-\vec{r}_2')} \times$$

$$\times\; \frac{\theta(h_1'-\lambda)\theta(\lambda-h_2')-\theta(h_2'-\lambda)\theta(\lambda-h_1')}{\hbar\omega - h_1' + h_2' + i\eta} \tag{4}$$

$$h^c_{A(2)} = \frac{p^2_{A(2)}}{2m} + V\left(\frac{\vec{r}_{A(2)} + \vec{r}'_{A(2)}}{2}, \vec{p}_{A(2)}\right) \tag{5}$$

the local Fermi Gas (F.G.) derived previously in a somewhat different way by Rosenfelder /2/.

$$\Pi^{(0)}_N(q,\omega) = \int d^3R \; \Pi^{(0)}_N(\vec{R},q,\omega)$$

$$= \frac{2}{(2\pi\hbar)^3} \int d^3R d^3p \left\{ \frac{\theta(h(\vec{R},\vec{p}+\vec{q}) - \lambda)\,\theta(\lambda - h(\vec{R},\vec{p}))}{\hbar\omega - h(\vec{R},\vec{p}+\vec{q}) + h(\vec{R},\vec{p}) + i\eta} - \frac{\theta(h(\vec{R},\vec{p}+\vec{q}) - \lambda)\,\theta(\lambda - h(\vec{R},\vec{p}))}{\hbar\omega + h(\vec{R},\vec{p}+\vec{q}) - h(\vec{R},\vec{p}) + i\eta} \right\} \tag{6}$$

This result is valid for arbitrary potentials. If the potential is local, however, it cancels in the denominator and enters only in the arguments of the step functions.

For a spherical potential all but one integration can be performed analytically.

For the imaginary part one thus obtains for $\omega > 0$:

$$\mathrm{Im}\left\{\Pi^{(\omega)}_N(q,\omega)\right\} = -\frac{m}{\hbar^2 q} \int_0^\infty dR\, R^2\, \theta(\lambda - V(R)) \begin{cases} 0 & \text{for} \quad k_F^2(R) < C_-^2 \\ k_F^2(R) - C_-^2 & \text{f. } C_-^2 \le k_F^2(R) \le C_+^2 \\ 2\,\frac{m\omega}{\hbar} & \text{for} \quad k_F^2(R) > C_+^2 \end{cases} \tag{7}$$

where $k_F^2(R) = \frac{2m}{\hbar^2}(\lambda - V(R))$ is the local Fermi momentum

and $C_\pm = \frac{m\omega}{\hbar q} \pm \frac{q}{2}$

The real part can be calculated form eq. (7) by a dispersion relation

$$\mathrm{Re}\left\{\Pi^{(\omega)}_N(q,\omega)\right\} = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\mathrm{Im}\left\{\Pi^{(\omega')}_N(q,\omega')\right\}}{\omega' - \omega}\, d\omega' \tag{8}$$

or directly from eq. (6). One obtains

$$\mathrm{Re}\left\{\Pi^{(\omega)}_N(q,\omega)\right\} = -\frac{m}{\hbar^2 \pi q} \int_0^\infty dR\, R^2\, \theta(\lambda - V(R)) \left\{ (k_F^2(R) - C_-^2) \log\left|\frac{k_F(R) - C_-}{k_F(R) + C_-}\right| \right.$$

$$\left. - (k_F^2(R) - C_+^2) \log\left|\frac{k_F(R) - C_+}{k_F(R) + C_+}\right| + 2 k_F(R) q \right\} \tag{9}$$

For a square well potential eq. (7) leads directly to the Lindhard function /3/; for a harmonic oscillator potential $V(R) = \frac{1}{2} m \omega_0^2 R^2$, eq. (7) still can be given analytically.

$$S^{(\omega)}(q,\omega) = -\frac{1}{\pi} \mathrm{Im}\left\{\Pi^{(\omega)}_N(q,\omega)\right\}$$

$$= \frac{2\hbar}{15\pi m^2 \omega_0^2 q} \begin{cases} 0 & \text{for} \quad k_F^2 < C_-^2 \\ (k_F^2 - C_-^2)^{5/2} & \text{for } C_-^2 \le k_F^2 \le C_+^2 \\ (k_F^2 - C_-^2)^{5/2} - (k_F^2 - C_+^2)^{5/2} & \text{for} \quad k_F^2 > C_+^2 \end{cases} \tag{10}$$

III - COMPARISON WITH MICROSCOPIC CALCULATIONS

In fig. 1 the response function in a harmonic oscillator potential is compared for two momentum transfers with the exact quantum mechanical ones, calculated by S. Shlomo /4/. Because the semiclassical method employed is similar to the Strutinsky smoothing procedure /5/, the exact result had been smeared out by a step function of range $2\hbar\omega_0$.

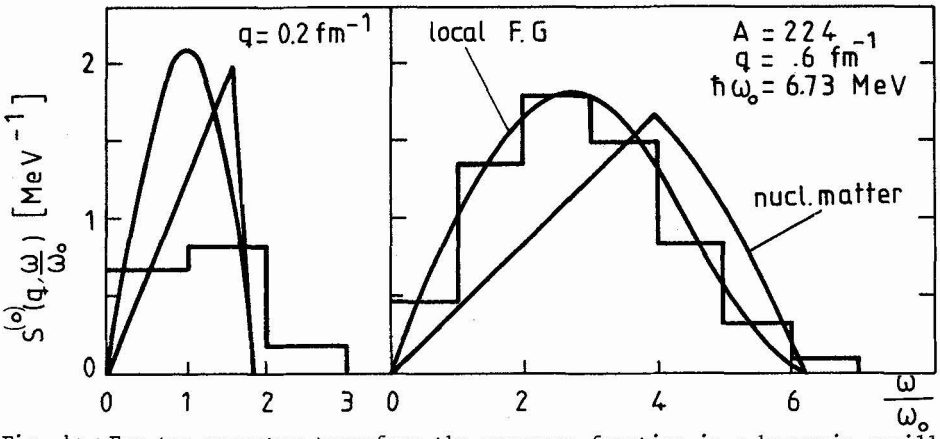

Fig. 1: For two momentum transfers the response function in a harmonic oscillation potential (k_F = 1.5 fm^{-1}) is compared to a quantum mechanical calculation /4/. For orientation the Lindhard functions (nuclear matter k_F = 1.36 fm^{-1}) are also shown.

The semiclassical method reproduces for momentum transfers q > 0.6 fm^{-1} the average values very well. For lower momentum transfers, the local Fermi Gas approximation fails, because only a few eigenvalues are excited and a local approximation is not able to account for single eigenstates which are a global property of the system. Energy integrated quantities may however still be quite accurate as can be deduced from the fact that the energy weighted sum rule is exactly fullfilled within the local F.G. approximation /2/.

In fig. 2 the local F.G. response function calculated for a Woods-Saxon potential

$$V(R) = \frac{V_0}{1 - e^{\frac{R-R_0}{a}}} \qquad \begin{aligned} V_0 &= -50 \text{ MeV} \\ a &= 0.5 \text{ fm} \\ R_0 &= 1.2\, A^{\frac{1}{3}} \text{ fm} \end{aligned} \tag{11}$$

is compared with the correspondent fully quantum mechanical calculation /6/. The result has been averaged with a Lorenzian of 3 MeV width.

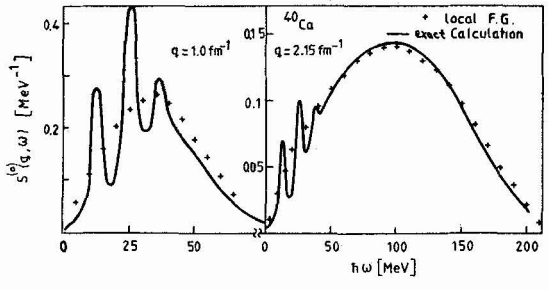

Fig. 2: The semiclassical response function in a Woods-Saxon potential compared with an exact calculation of N.van Giai /6/.

Our results, again, produce a good average in the region where shell effects are present. In the region where the continuum dominates, the exact result is very well reproduced. We thus can say that for q $\geq$ 2fm^{-1} the semiclassical approach is almost identical with the quantum result. The dependence of the response function on the

average potential is illustrated in fig. 3. One finds that the nuclear matter approximation (k_F = 1.36 fm) is better for heavy nuclei than for lighter ones. For light nuclei, surface effects are important and a harmonic oscillator (k_F = 1.5 fm^{-1}) response function (eq. (10)) becomes more realistic than the Lindhard function (nuclear matter).

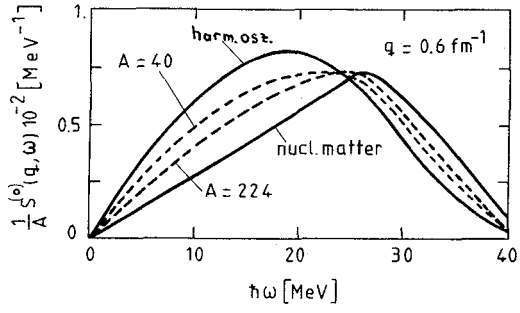

Fig. 3 : The response functions are compared for different potentials. The dashed lines belong to Woods-Saxon potentials for two different masses.

IV - THE DELTA RESONANCE

For inelastic proton scattering one can excite besides pure nucleon particle-hole pairs also Δ-hole pairs in the longitudinal channel /7,8,9/. Thereby a nucleon can be transformed by a π^o to a delta particle. We again want to study the difference between a pure nuclear matter calculation and our semiclassical approach. The total longitudinal free response is given by /7/.

$$\Pi^{(o)}(q,\omega) = f_N^2(q,\omega)\,\Pi_N^{(o)}(q,\omega) + f_\Delta^2(q,\omega)\,\Pi_\Delta^{(o)}(q,\omega) \qquad (12)$$

Here $f_N(q,\omega)$ and $f_\Delta(q,\omega)$ are the pion baryon vertex factors /7/:

$$f_{N_{(a)}}(q,\omega) = f_{N_{(a)}}\left(\frac{\Lambda^2 - (m_\pi c^2)^2}{\Lambda^2 - (\hbar\omega)^2 + (\hbar c q)^2}\right) \qquad (13)$$

Λ = 1300 MeV and $\frac{1}{4} f_\Delta^2 = f_N^2 = \frac{(\hbar c)}{(m_\pi c^2)}$ [MeV f_m^3]

In analogy to (6) one finds /9/:

$$\Pi_\Delta^{(o)}(q,\omega) = \frac{16}{9}\frac{1}{(2\pi\hbar)^3}\int d^3R\,d^3p\left\{\frac{\Theta(\lambda - h(\vec{R},\vec{p}))}{\hbar\omega + h(\vec{R},\vec{p}) - h^\Delta(\vec{R},\vec{p}-\vec{q}) + i\eta} - \frac{\Theta(\lambda - h(\vec{R},\vec{p}))}{\hbar\omega - h(\vec{R},\vec{p}) + h^\Delta(\vec{R},\vec{p}-\vec{q}) + i\eta}\right\} \qquad (14)$$

with $h^\Delta(\vec{R},\vec{p}) = \hbar\omega_\Delta + \frac{p^2}{2m_\Delta} + V(R)$, $\hbar\omega_\Delta$ is the difference between the Δmass (m_Δ) and, the nucleon mass. The delta width has been neglected.

For a spherical potential this gives

$$\mathrm{Im}\left[\Pi_\Delta^{(o)}(q,\omega)\right] = -\frac{16\,m_\Delta}{9\hbar^2 q}\int_0^\infty dR\,R^2\,\Theta(\lambda - V(R))\begin{cases} 0 & \text{for } k_F(R) \le \mathrm{Max}\{k_2^+, k_\Delta^-\} \\ k_F^2(R) - \mathrm{Max}\{k_2^+, k_\Delta^-\}^2 & \text{else} \\ (k_\Delta^+)^2 - \mathrm{Max}\{k_2^+, k_\Delta^-\}^2 & \text{for } k_\Delta^+ \le k_F(R) \end{cases}$$

$$k_{\Delta[2]}^{+(-)} = \frac{mq}{m_\Delta - m}\left\{{}^{+}_{[-]}\,1 \pm \sqrt{1 + \frac{2m_\Delta(m_\Delta - m)}{\hbar^2 q^2 m}\left(\hbar\omega_\Delta - \hbar\omega + \frac{\hbar^2 q^2}{2m_\Delta}\right)}\right\} \qquad (15)$$

And for the real part one obtains:

$$\mathrm{Re}\{\Pi_\Delta^{(0)}(q,\omega)\} = -\frac{16 m_\Delta}{9\hbar^2\pi q}\int_0^{\infty} dR R^2\,\theta(\lambda - V(R))\int_0^{k_F(R)} dk\, k\, \log\left|\frac{(1-\mu_0(q,\omega))(1+\mu_0(-q,-\omega))}{(1+\mu_0(q,\omega))(1-\mu_0(-q,-\omega))}\right|$$

$$\mu_0(q,\omega) = -\frac{m_\Delta}{\hbar^2 kq}\left\{\hbar\omega_\Delta - \hbar\omega + \frac{\hbar^2}{2m_\Delta}(q^2+k^2) - \frac{\hbar^2 k^2}{2m}\right\} \qquad (16)$$

In fig. 4 the total response function is drawn together with its components. In the imaginary part (full line) the nuclear and the delta contributions are clearly separated in energy. The delta response, however, contributes to the total response function through its real part (dashed dotted line) even for lower energies, and modifies the real part of the nucleon response (dashed line). For comparison, the delta part of the response function is also shown for the nuclear matter case (dotted line).

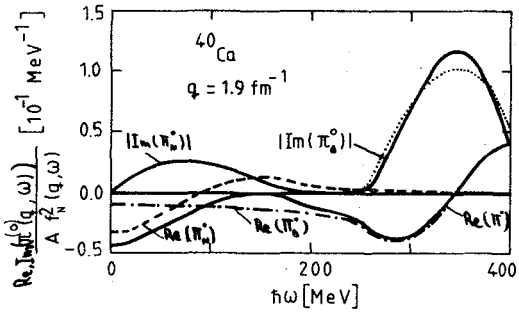

Fig. 4: The total response function and its components (dashed: the nucleon part dashed dotted: for the Δ part). The full lines give the sum of both. The calculation was done in a Woods-Saxon potential for ^{40}Ca. The points indicate the result for the nuclear matter case of the delta part.

IV - RESPONSE FUNCTION INCLUDING PARTICLE-HOLE INTERACTION

The Greens function including interactions is calculated from the free one by means of (neglecting the exchange part of the interaction):

$$\Pi(\vec{r},\vec{r}') = \Pi^{(0)}(\vec{r},\vec{r}') + \int d^3r_1 d^3r_2\, \Pi^{(0)}(\vec{r},\vec{r}_1)\, V(\vec{r}_1-\vec{r}_2)\, \Pi(\vec{r}_2,\vec{r}') \qquad (17)$$

One can show that to zero order of $\hbar$ one obtains for the response function:

$$\Pi(q,\omega) = \int d^3R\, \frac{\Pi^{(0)}(\vec{R},q,\omega)}{1 - V(q,\omega)\,\Pi^{(0)}(\vec{R},q,\omega)} \qquad (18)$$

For the special case of a one pion exchange potential (OPEP) plus Midgal parameter, representing the short-range repulsion,

$$V(q,\omega) = g' - \frac{(\hbar c q)^2}{(m_\pi c^2)^2 + (\hbar c q)^2 - (\hbar\omega)^2} \qquad (19)$$

m_π is the pion mass.

the results are represented in fig. 5, where g' = 0.6 has been used.

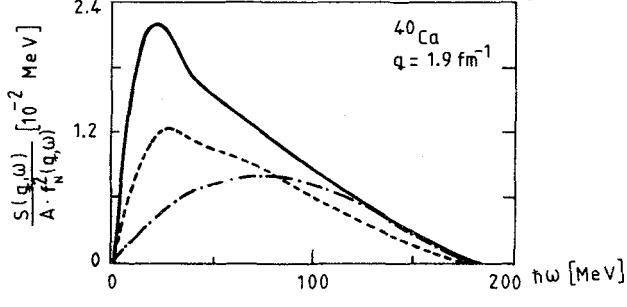

Fig. 5: The free response (dash-dotted line) is compared with the response including interaction (15) (full line). The dashed line representes the response where the Δ contribution was omitted.

The particle hole interaction causes a softening of the response function whereas the Δ contribution causes an enhancement. We omit here a detailed comparison with the pure nuclear matter result because we were so far unable to exactly reproduce the calculations of ref. /8/. The results presented in Fig. 5 should therefore at the moment only be considered as qualitative indicating the general trend of the effects.

In conclusion we can say that we have shown in this work that the semiclassical approach to the calculation of nuclear response functions works perfectly well for high momentum transfers ($q \gtrsim 2$ fm^{-1}) opening thus the possibility of quite precise and easy calculations even in the case of quite sophisticated linear response theories.

Nevertheless some further studies and refinements should be done in future:

i) One should include the first $\hbar$-correction to improve the far tail region of the cross section.

ii) A comparison of our result (Fig. 5) including residual interaction with an exact quantum calculation should be performed, though it is our strong belief that the same degree of accuracy holds in the interacting case as in the non interacting one.

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