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THE NUCLEAR VELOCITY FIELD WITH AND WITHOUT THE VORTEX SPIN*

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Résumé - L'existence du champ de vitesse défini dans le cadre d'une théorie collective $GL_+(3, \mathbb{R})$ est étudiée.

Abstract - The existence of the collective velocity field w.r.t. $GL_+(3, \mathbb{R})$ collectivity is studied.

We are interested in the extraction of the collective quantities, such as the collective velocity field, out of the many particle theory through some coordinate transformation for the system of $A-1 = n$ Jacobi vectors such that the collective coordinates belong to the set of the new $3n$ coordinates. For this ideas see refs. /1,2,3/.

First we define as the collective coordinates three rotational coordinates, angles $\varphi_i$, $i = 1,2,3$ and three vibrational coordinates $\alpha_j$, $j = 1,2,3$ which one obtains through the diagonalization of the coordinate quadrupole tensor,

$$ Q_{ij} = \sum_{s=1}^{n} x_{is} x_{js} $$

with $ij = 1,2,3$ the space index and $s = 1,..,n$ the Jacobi vector index. Such a coordinate transformation had been done in refs. /4,5/ in order to study the problem of separation of the collective kinetic energy. We did the coordinate transformation using the technique of an orbit analysis w.r.t. the $SO(3)xSO(n)$ group /6,7/. The type of collectivity related to this orbit analysis we call $SO(n)$ collectivity. The group $SO(3)xSO(n)$ act on the $3n$-dimensional Euclidean manifold parametrized by the Cartesian coordinates $x_{is}$ such that the "collective" group $SO(3)$ acts on the index $i$ and the "intrinsic" group $SO(n)$ on the index $s$. Through the orbit analysis one gets rid of the redundant coordinates without introducing constraints /6,7/, and as the result one writes the point of the configuration space as a function of the new coordinates in the unique fashion

$$ x_{is} = \sum_{l=1}^{3} \circ_{i1}(\varphi) \tilde{x}_l \circ_{is}(\beta) $$

where $\circ(\varphi) \in SO(3)/M$, the left coset space w.r.t. the group

$$ M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

(the restriction from $SO(3)$ to $SO(3)/M$ is not important in further

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analysis because M is a discrete group);
\[ \alpha'(\beta) \in \text{SO}(n)/\text{SO}(n-3) \equiv \mathbb{C}^R \text{ the right coset space.} \]

Both \( \phi \)'s and \( \beta \)'s are exponential parameters. The vibrational variables \( x'_{i1} \) \( l = 1,2,3 \) one obtains through the diagonalization of \( Q_{ij} \)
\[
Q_{ij} = \sum_l c_{il}(\phi)x'_{l1} \cdot c_{lj}(\phi) .
\]

Out of this relation one sees that the \( x' \)'s must be chosen as an ordered set, for example \( x'_{1} \geq x'_{2} \geq x'_{3} \). We restrict our analysis to \( x'_{1} > x'_{2} > x'_{3} \) related with the group M. Through this restriction, unimportant for the quantum system, for the classical system we are bound to one type of orbits /7/. The same restriction was used in ref. /8/.

The canonically conjugate momenta \( \pi_{\nu} \) \( \nu = 1, \ldots, 3n \) to the new coordinates
\[
\{ \xi_{\nu}|\nu=1, \ldots, 3n \} \equiv \{ \phi_{\mu}, x'_{1}, \beta_{t} | \mu, l = 1,2,3 \nu = 1, \ldots, 3n-6 \}
\]
are related to the Cartesian momenta \( p_{ij} \) by the inverse Jacobi matrix \( J^{-1} \),
\[
P_{ij} = \sum_{\nu=1}^{3n} J^{-1}_{is,\nu} \pi_{\nu},
\]
where
\[
J^{-1}_{is,\nu} = \frac{\partial \xi_{\nu}}{\partial x_{is}}
\]
The Jacobi matrix \( J \) is given by its matrix elements
\[
\frac{\partial x_{is}}{\partial \phi_{\mu}} = \sum_{i,j} c_{ij}(\phi) R_{\mu,ij}(\phi) x'_{i} \alpha_{is}(\beta)
\]
\[
\frac{\partial x_{is}}{\partial x'_{w}} = \alpha_{lw}(\phi) \beta_{ws}(\beta)
\]
\[
\frac{\partial x_{is}}{\partial \beta_{t}} = \sum_{i=1}^{3} \sum_{t=1}^{n} c_{il}(\phi) x'_{i} L_{t,\nu} \beta_{t}, \nu = \beta_{ts}(\beta)
\]
where \( R_{\phi,\theta}(\gamma) \) and \( L_{\phi,\theta}(\beta) \) are essentially given by the expressions
\[
R_{\phi,\gamma,1}(\gamma) = \sum_{k=1}^{3} \left( \frac{\partial}{\partial \phi_{\gamma}} \alpha_{ik}(\gamma) \right) \beta_{k1}(\gamma)
\]
\[
L_{\phi,\gamma,1}(\gamma) = \sum_{k=1}^{3} \left( \frac{\partial}{\partial \phi_{\gamma}} \alpha_{ik}(\gamma) \right) \beta_{k1}(\gamma)
\]
for any \( \text{SO}(j) \) group, compare ref. /7/ for the details. We state the following proposition without proof:

1. Proposition: The inverse Jacobi matrix \( J^{-1} \) is given by its matrix elements
\[
\frac{\partial \phi_{\mu}}{\partial x_{is}} = \sum_{\nu=1}^{3} \sum_{g=1}^{3} \phi_{\nu}(\phi) \frac{x'_{v}}{x_{v} - x_{g}} R_{\phi,\gamma,1}(\gamma) \beta_{\nu,\gamma}(\beta)
\]
where $R_{S}(q)$ is the inverse matrix to $R_{S}(q)$ and $O(B)$ is related to $L_{S}(6)$ (note that the $\beta_{S} \tau = 1, \ldots, 3n-6$ are the coset parameters and $\varphi_{M}, M = 1, 2, 3$ the group parameters up to the discrete group $M$) as

$$L_{\Theta, (6)}(\beta) L_{\Theta, \tau^{'}, \tau}(\beta) = \sum_{\tau} L_{\Psi, \sigma \tau}(\beta) L_{\Theta, \tau^{'}, \tau}(\beta) = \delta_{\sigma \tau}$$

$$L_{\Theta, (6)}(\beta) + \sum_{\tau} L_{\Theta, \tau}(\beta) L_{\Psi, \tau^{'}, \tau}(\beta) = 0 \ ,$$

$\sigma, \tau', \tau = 1, \ldots, 3n-6 \ , \kappa = 1, 2, 3$.

The expression

$$p_{i}s = \sum_{\nu} j_{is,v} \pi_{\nu}$$

one can use both to calculate the kinetic energy /7/ and the velocity field. For the velocity field we proceed as in ref. /8/. The Cartesian momenta become

$$p_{i}s = \frac{3}{j} \sum_{g=1}^{3} \sum_{v=g+1}^{3} (O_{Ig}(q) O_{jv}(q) + O_{Iv}(q) O_{jg}(q)) \frac{1}{x_{v}-x_{g}} L_{g v} x_{j} s$$

$$+ \frac{3}{j} \sum_{w=1}^{3} \sum_{v=g+1}^{3} O_{Iw}(q) O_{jw}(q) \frac{x_{w}}{x_{v}} \pi_{w} x_{j} s$$

$$+ \frac{3}{j} \sum_{g=1}^{3} \sum_{v=g+1}^{3} (O_{Ig}(q) \frac{x}{x_{v}} O_{jv}(q) + O_{Iv}(q) \frac{x}{x_{g}} O_{jg}(q)) \frac{1}{x_{v}-x_{g}} L_{g v} x_{j} s$$

$$- \frac{3}{j} \sum_{g=1}^{3} \sum_{v=4}^{n} O_{Ig}(q) x_{g} O_{jv}(q) L_{g v} x_{j} s$$

where the new classical momenta are defined as follows:

$$L_{g v} := \sum_{M=1}^{3} R_{g v, M}(q) \pi_{M} \quad g < v = 1, 2, 3 ,$$

$$L_{g v} := - \sum_{\tau=1}^{3n-6} L_{g v, \tau}(q) \pi_{\tau} \quad g < v = 1, 2, 3 ,$$

$$L_{g v} := \sum_{\tau=1}^{3n-6} L_{g v, \tau}(q) \pi_{\tau} \quad g = 1, 2, 3 \quad v = 4, \ldots, n$$

$\pi_{w}$ are the momenta canonically conjugate to the coordinate $x_{w}$,
The momenta $L_{gv}$ are angular momenta "referred" to the body /9/ and $L_{gv}$ is the vortex spin /10/.

By the collective part in Cartesian momenta we mean the part that can be expressed as

$$p_{is}^{\text{coll}} = \frac{3}{2} \sum_{j=1}^{3} X_{ij} x_{js}$$

with coefficients $X$ depending only on collective coordinates and new momenta related to the collective manifold. For local Hamiltonians we have

$$m v_{is} = p_{is}.$$ 

We extract $v_{is}^{\text{coll}}$ as

$$v_{is}^{\text{coll}} = \frac{1}{m} \sum_{j=1}^{3} (X_{ij}^{\text{vib}} + X_{ij}^{S}) x_{js}$$

where

$$X_{ij}^{\text{vib}} = \sum_{w=1}^{3} O_{iw}(\varphi) O_{jw}(\varphi) \pi_{w}^{-1}, \quad X_{ij}^{S} = X_{ji}^{\text{vib}};$$

$$X_{ij}^{S} = \sum_{v=g+1}^{3} \sum_{g=1}^{3} (O_{ig}(\varphi) O_{jv}(\varphi) + O_{iv}(\varphi) O_{jg}(\varphi)) \frac{1}{\pi_{v}^{2} - \pi_{g}^{2}} L_{gv}, \quad X_{ij}^{S} = X_{ji}^{S}.$$ 

As the authors in ref. /8/ we define the velocity field $\vec{v}$ at the point $\vec{x}$ to be the collective component of the velocity $\vec{v}_{S}$ for the relative vector $\vec{s}$ at $\vec{x}_{S} = \vec{x}$

$$v_{i} = \frac{1}{m} \sum_{j=1}^{3} (X_{ij}^{\text{vib}} + X_{ij}^{S}) x_{j} \quad i = 1,2,3.$$ 

It is clear that the $SO(n)$ (collective) velocity field separates completely from the intrinsic part and is irrotational, $\text{rot} \vec{v} = 0$.

In order to study more realistic type of collectivity in which for example the velocity field is such that rot $\vec{v} \neq 0$, Weaver et al. /11,10/ and Rowe et al. /8,12/ introduced the phenomenological CM(3) or MQC model (Mass Quadrupol Collective Model) based on the idea of spectrum generating algebras. These authors introduced 9 collective degrees of freedom: the six ones already mentioned for the vibrations and the rotations and three $SO(3)$ degrees of freedom which belong to the vortex spin and should yield the collective flow which is not irrotational.

The collective coordinates in the MQC model are the $GL_{+}(3, \mathbb{R})$ parameters. This type of collectivity we call the $GL_{+}(3, \mathbb{R})$ collectivity. Note that the vortex spin we already introduced in $SO(3) \times SO(n)$ orbit analysis, but w.r.t. the $SO(n)$ collectivity it was an intrinsic quantity. In ref. /8/ the authors tried to introduce the MQC model through the coordinate transformation which was similar to the one of Villars /3/ (redundant coordinates), introducing 9 $GL_{+}(3, \mathbb{R})$ collective coordinates and nine constraints from which 6 were holonomic, but three nonholonomic which we are going to discuss later on. Due to these 9 constraints they extracted the collective $GL_{+}(3, \mathbb{R})$ velocity field which was no longer irrotational and also obtained the total separation of the kinetic energy into a $GL_{+}(3, \mathbb{R})$ collective part and the intrinsic one. Other attempts were done in order to obtain the microscopically based MQC model: Rowe and Rosensteel /6,13/ introduced the coordinate transformation based on the orbit analysis which has to be seen as the transformation of momenta rather than coordinates. Buck, Biedenharn and Cusson /14/ did similar work. All these authors obtained
actually the results already found by Zickendraht /4/ and Dzyublik et al. /5/ in a frame of an SO(n) collectivity, but they interpreted them from the point of view of the GL+(3, IR) collectivity. They discussed particularly the problem of the separation of the GL+(3, IR) collective kinetic energy from the intrinsic one. Let us turn back to our expression for $p_1$ given above. The first three lines of $p_1$ are the "candidates" for the (collective) velocity field w.r.t. the GL+(3, IR) collectivity: the momenta $l_{x'y}$ can still be seen as related to the SO(3) submanifold of the right coset space $SO(n)/SO(n-3) \cong CR$, but the part proportional to $l'$'s is related to the whole $CR$ and can not be defined as purely intrinsic w.r.t. $GL+(3, IR)$ collectivity. Similar problems appear if one studies the problem of the separation of the collective kinetic energy /7/.

We claim that the coordinate transformation introduced for the SO(n) collectivity is not the adequate one for $GL+(3, IR)$ collectivity, and for $GL+(3, IR)$ collectivity we introduce the coordinate transformation (c.t.) based on the orbit analysis w.r.t. the $GL+(3, IR) \times SO(n)$ group /7/. Through the orbit analysis w.r.t. the $GL+(3, IR) \times SO(n)$ group one concludes that the whole 3n-dimensional Euclidean space is given by the only orbit parametrized as

$$x_{is} = \sum_{j=1}^{3} x^i_j \Theta_j^s(\beta) \quad i = 1, 2, 3 \quad s = 1, \ldots, n$$

where $x^i \in GL+(3, IR)$ and $\Theta^s(\beta) \in SO(n)/(SO(3) \times SO(n-3))$, right coset space. This decomposition is unique. The coordinates $x^i_j$ are the 9 collective coordinates for the $GL+(3, IR)$ collectivity. The c.t. $\phi_1$ we define by

$$(x_{is} | \ i=1, 2, 3 \ s=1, \ldots, n) \phi_1^t (x^t_{i1}, \beta_1 | \ i, l=1, 2, 3 \ t=1, \ldots, 3n-9) \equiv (\xi_\mu | \mu=1, \ldots, 3n)$$

The c.t. $\phi_1$ is only a first step: we would like to obtain some collective coordinates which we can interpret as rotational coordinates, coordinates for vibrations and deformations and coordinates for vorticities. These coordinates can be easily introduced formally using the result of the coordinate transformation related to the SO(n) collectivity only for $n = 3$. So we introduce c.t. $\phi_2$, and $\phi$ which is defined as $\phi := \phi_2 \circ \phi_1$. The c.t. $\phi_2$ is introduced through the unique decomposition of $x'$

$$x^t_{ik} = \sum_{l=1}^{3} O_{il}(\phi) \bar{x}_l \Theta^t_{ik}(\phi')$$

where $O(\phi) \in SO(3)/M$ the left coset-space, $O'(\phi') \in SO(3)$. We restrict again as for SO(n) collectivity to the orbits of the type $x_1 > x_2 > x_3 > 0$ related to the group M /6,7/.

Finally the c.t. $\phi$ becomes

$$(x_{is} | \ i=1, 2, 3 \ s=1, \ldots, n) \phi (\xi_\mu, x, \xi_\nu, x', \beta_1 | \mu, \nu, i=1, 2, 3 \ t=1, \ldots, 3n-9)$$

and is related to the following unique decomposition of x

$$x_{is} = \sum_{l=1}^{3} \sum_{k=1}^{3} O_{il}(\phi) \bar{x}_l \Theta^t_{ik}(\phi') O^t_{ks}(\beta).$$
The Cartesian momenta are expressed through the momenta canonically conjugate to the new coordinates after the c.t. \( \phi \) as

\[
p = J_1^{-1} J_2^{-1} \pi
\]

where the \( J_1^{-1} \) and \( J_2^{-1} \) are inverse Jacobi matrices for the coordinate transformations \( \phi_1 \) and \( \phi_2 \) respectively. The matrix \( J_2^{-1} \) we already determined in proposition \( \dagger \) and we state without proof /7/: 2 Proposition: The inverse Jacobi matrix \( J_1^{-1} \) for the coordinate transformation \( \phi_1 \) is given by its matrix elements

\[
\frac{\partial x_{i1}}{\partial x_{is}} = \delta_{ij} C_{1s} (\beta) + \sum_{m=1}^{3} \sum_{k=1}^{3} \sum_{r=1}^{n} \chi_{m}^{-1} o_{k's}(\beta) L_{m'k'}mk(\beta)_{x'r} j m'k'l
\]

\[
\frac{\partial \beta_{s}}{\partial x_{is}} = \sum_{m=1}^{3} \sum_{k=1}^{n} \chi_{mi}^{-1} o_{k's}(\beta) L_{mk},(\beta).
\]

The matrix block \( L_{mk},(\beta) \) for \( m=1,2,3 \) \( k=4,\ldots,n \) and \( \tau=1,\ldots,3n-9 \) is of the type \( L_{mk},(\beta) \) in proposition \( \dagger \) and the block \( L_{mk},(\beta) \) for \( m'=1,2,3 \) \( k'=4,\ldots,n \) and \( m,k=1,2,3 \) is of the type \( L_{mk},(\beta) /7/ \).

Now we can try to define the collective velocity field. Making use of \( J_1^{-1} = J_1^{-1} J_2^{-1} \) we obtain the candidate for \( p_{is}^{coll} \)

\[
p_{is}^{coll} = \frac{3}{2} \chi_{i1} x_{is}^{s}
\]

\[
+ \frac{3}{2} \chi_{i1} x_{is}^{A}
\]

\[
- \frac{3}{2} \sum_{g=1}^{3} \sum_{v=g+1}^{n} \sum_{m=1}^{3} \sum_{k=1}^{n} \sum_{r=1}^{n} \chi_{x1} x_{v}^{-1} o_{il}(\beta) o_{1v}(\beta) o_{il}(\beta) L_{mk},(\beta)_{x'r} j m'k'l
\]

where

\[
\chi_{i1}^{S} : = \frac{3}{2} \sum_{g=1}^{3} \sum_{v=g+1}^{n} \frac{(o_{ig}(\beta) o_{1v}(\beta) + o_{1g}(\beta) o_{i1}(\beta))}{x_{v}^{-1}} \frac{1}{k_{v}^{-1} k_{v}^{-1}}
\]

\[
\chi_{i1}^{A} : = \frac{3}{2} \sum_{g=1}^{3} \sum_{v=g+1}^{n} \frac{1}{x_{v}^{-1} k_{v}^{-1} k_{v}^{-1}} \frac{1}{k_{v}^{-1} k_{v}^{-1}}
\]

and

\[
\chi_{i1}^{S} = \chi_{i1}^{A}.
\]
The momenta $\pi_w$ are canonically conjugate to the vibrational coordinates $x_w$, 

$$L_{\gamma \nu} = \sum_{\mu=1}^{3} R_{\gamma \nu, \mu} (\phi) \pi_\mu$$

with $\pi_\mu$ canonically conjugate to $v_\mu$ represent the angular momenta referred to the body and

$$L_{\gamma \nu} = \sum_{\mu=1}^{3} L_{\gamma \nu, \mu} (\psi') \pi_\mu$$

with $\pi_\mu$ canonically conjugate to $v'_\mu$, is the vortex spin.

The last line in "Pf.111", including $L_{\alpha \beta} (\phi)$ related to the matrix block $L_{\alpha \beta} (\beta)$ which depends on the intrinsic coordinates $\beta$, spoils the separation. If we would postulate that $L_{\alpha \beta} (\beta) = 0$, what is equivalent to the constraints $L_{\alpha \beta} (\beta) = 0$ would obtain the separation. If this condition is compared with ref. /8/, one finds that the relation (2.9) postulated there may be written as

$$\sum_{s=1}^{n} \left( o''_{\nu s} (\beta) \frac{\partial o''_{\nu s}}{\partial \beta T} - o''_{\nu s} (\beta) \frac{\partial o''_{\nu s}}{\partial \beta T} \right) = 2 L_{\gamma \nu} (\gamma), b, \nu = 1, 2, 3.$$ 

Actually in this reference $o''$ is some $3 \times n$ matrix which is not identified as $o'' \in SO(n)/(SO(3) \times SO(n-3))$, and the relation (2.9) means three nonholonomic constraints from altogether 9 constraints on $o''_{\nu s}$ there denoted by $x_{\alpha \beta} n = 1, \ldots, N (N=A-1)$ and $\alpha = 1, 2, 3$. These constraints define $3n-9$ intrinsic coordinates $\beta_T$ denoted there by $\xi_\sigma$. It should be stressed that the constraints $L_{\alpha \beta} (\beta) = 0$ do not follow from the theory of the Lie groups involved. Similar conclusion follows if one studies the problem of the separation of the $GL+(3, \mathbb{R})$ collective kinetic energy /7/, which is also possible under the same conditions.

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