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NONLINEAR SURFACE AND GUIDED POLARITONS OF A GENERAL LAYERED DIELECTRIC STRUCTURE

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Abstract - The surface and guided modes of a general structure, consisting of a nonlinear plane slab bounded by dissimilar nonlinear media, are analysed for both TE and TM modes. It is shown that all structures, independently of whether they are supporting TE or TM waves, are governed by a compact generic dispersion equation with the same appearance. This is proved by emphasising that the physical entry parameter to the dispersion equations is the electric field amplitude at one slab boundary. The alternative approach that corresponds to the current literature is also developed for comparison. Specialisations are then made to generate some numerical examples through a power flow equation. This equation also shows, in physical terms, the three classes of solution that are possible for layered systems.
1 - INTRODUCTION

The general principles underlying the propagation of strongly nonlinear surface waves on dielectric media were first established for s and p-polarised waves i.e. (TE and TM modes) travelling along the interface between two semi-infinite media /1-6/. In most cases one of the media is a vacuum. The immediate and surprising feature that emerges from these calculations is the fact that in the nonlinear state TE modes are supported in the vicinity of the interface. As the nonlinearity is shut down they disappear. These modes are, perhaps, not surface waves in the true sense because they are localised at a small distance from the surface. They are, in fact, self-trapped in that the interface could be removed to ±∞/2/. The TM (or p-polarised) nonlinear modes are more conventional in that they do have a linear limit.

A natural extension of this early work is to include guided and surface modes of a layered structure /7-10/. The study of such systems is motivated by the search for useful integrated optics devices /11/ that may exhibit optical bistability /7,8,11/ or other optical information processing capabilities.

The calculations that exist in the literature deal with both TE and TM modes. In the symmetric TE /7/ and the asymmetric TE /8/ cases, configurations consisting of a linear layer bounded by semi-infinite nonlinear media, have been considered in detail for frequency independent dielectric functions. In these calculations an analysis of the power flow leads to suggestions of optical bistability.

In this paper we consider a more general structure that can embrace all the known results. It consists of a nonlinear layer bounded by different nonlinear semi-infinite media. We show that a compact generic form of the dispersion curve can be obtained that has the same appearance for both TE and TM modes, provided that the parameters are calculated accordingly. The current conventional approach is also discussed, as are some general power flow formulae. The theory is then illustrated through the development of specific analytical and numerical examples.

II - GENERAL THEORY FOR THREE NONLINEAR MEDIA

We consider a uniaxial nonlinear dielectric layer, with boundaries that lie in the x-y plane, that is bounded by dissimilar semi-infinite nonlinear uniaxial dielectrics. In general the electric and magnetic field in each medium are

\[ \mathbf{E} = (E_x(z), E_y(z), E_z(z))e^{i(kx - \omega t)} \]
\[ \mathbf{H} = (H_x(z), H_y(z), H_z(z))e^{i(kx - \omega t)} \]

These fields represent a surface or guided wave that is propagating along the x axis with wave number k and angular frequency ω. In each medium it
is assumed that the dielectric tensor has the form

\[ \varepsilon(\omega) = \begin{bmatrix} \varepsilon_{\perp}(\omega) + \alpha (|E_x|^2 + |E_y|^2), & 0, & 0 \\ 0, & \varepsilon_{\perp}(\omega) + \alpha (|E_x|^2 + |E_y|^2), & 0 \\ 0, & 0, & \varepsilon_{11}(\omega) \end{bmatrix} \]  

(2.2)

where \( \varepsilon_{\perp}(\omega), \varepsilon_{11}(\omega) \) are the usual linear dielectric functions and \( \alpha \) is a frequency independent nonlinear coefficient.

In each medium the following basic equations are then obtained

**TE Modes:**

\[ \frac{d^2 E_y}{dz^2} - \left[ \kappa^2 \frac{E_y}{c^2} - \frac{\omega^2}{c^2} E_y \right] = 0 \]  

(2.3)

where \( \kappa^2 = k^2 - \omega^2 \varepsilon_{\perp} \)

**TM Modes:**

\[ \frac{d^2 E_x}{dz^2} - \frac{\kappa^2}{c^2} \left[ \varepsilon_{\parallel} + \alpha E_x^2 \right] = 0 \]  

(2.4)

where \( \kappa^2 = k^2 - \omega^2 \varepsilon_{11} \)

All the development for the TE case now uses only the \( E_y \) field component and for the TM case only the \( E_x \) field component. We now consider the TE and TM modes separately.

**TE Modes**

As stated above only the \( E_y \) field component of field enters into the problem. This will now be denoted by \( E_i \) where \( i = 1,2,3 \) refer to the lower medium, the slab and the upper medium respectively. Differentiation with respect to \( z \) will be written as \( E_i \). Equation (2.3) then integrates to

\[ \dot{E}_i^2 - (\kappa_i^2 - \lambda_i E_i E_i^2) = c_i^2, \quad i = 1,2,3 \]  

(2.5)

where \( \lambda_i = \frac{\omega^2 \alpha_i}{2c^2} \) and \( c_i \) is the constant of integration. First \( E_1, \dot{E}_1 \to 0 \) as \( z \to \pm \infty, \quad i = 1,3 \), therefore for the outer semi-infinite nonlinear media

\[ \dot{E}_i^2 - (\kappa_i^2 - \lambda_i E_i^2)E_i^2 = 0, \quad i = 1,3 \]  

(2.6)

Suppose now that the nonlinear slab has its boundaries at \( z = 0 \) and \( z = d \) and that the electric field at \( z = 0 \) is \( E_0 \) and at \( z = d \) is \( E_d \). The continuity of \( E_i \) and \( \dot{E}_i \) at a boundary therefore ensures, from equations (2.5) and (2.6), that

\[ E_0^2(\kappa_1^2 - \lambda_1 E_0^2) = c_2 + E_0^2(\kappa_2^2 - \lambda_2 E_0^2) \]  

(2.7)

\[ E_d^2(\kappa_3^2 - \lambda_3 E_d^2) = c_2 + E_d^2(\kappa_2^2 - \lambda_2 E_d^2) \]  

(2.8)

Hence, after setting \( \varepsilon_\perp = \varepsilon_\parallel \) we obtain

\[ c_2 = \frac{\omega^2}{c^2} E_0^2(\varepsilon_2 - \varepsilon_1 + \frac{(\alpha_2 - \alpha_1)E_0^2}{2}) \]  

(2.9)
The fields inside the slab are found by solving equation (2.5) with \( c_2 \neq 0 \). The particular solution chosen depends upon the signs and magnitudes of \( \alpha_2, \alpha_3 \) and \( c_2 \). The solution, in the general case \( c_2 \neq 0 \), is a Jacobi elliptic function. This need not be specified for the moment and the various options are kept open by writing it as \( \text{\textit{J}}(u|m) \) where \( u \) is the argument and \( m \) the modulus. The general solution of (2.5) in the slab is, therefore,

\[
E_2 = \pm p \left[ q(z + z_{02}) \text{\textit{J}} \right]
\tag{2.11}
\]

where \( p, q \) and \( u \) are expressions containing \( \alpha_2, \alpha_3 \) and \( c_2 \) and, since \( z=0 \) at \( E_2 = E_0 \),

\[
z_{02} = \pm \frac{1}{q} \text{\textit{J}}^{-1} \left( \frac{E_0}{p} \right)
\tag{2.12}
\]

If, as is common practice, the \( u \) is dropped from the argument of the Jacobi function and in addition \( \text{\textit{J}}^{-1} \) is defined as the inverse of the Jacobi function then the field in the slab becomes

\[
E_2 = \pm p \left[ q^2 \pm \text{\textit{J}}^{-1}(E_0/p) \right]
\tag{2.13}
\]

Now equation (2.10) is a quartic in \( E_d \) so that it has the following solutions

\[
E_d = \left( \epsilon_3 - \epsilon_2 \pm \sqrt{(\epsilon_2 - \epsilon_3)^2 + 2(a_2 - a_3)c_2^2} \right) / (a_2 - a_3) = a+/-
\tag{2.14}
\]

Equations (2.13) and (2.14) then imply that the dispersion equation for TE modes is in general,

\[
p^2 \text{\textit{J}}^2 \left[ q^2 \pm \text{\textit{J}}^{-1}(E_0/p) \right] = a+/-
\tag{2.15}
\]

Here the dispersion equation is expressed entirely in terms of the initial parameters \( (a_1, a_2, a_3, \epsilon_1, \epsilon_2, \epsilon_3, k, E_0) \). Formally there are four solutions to equation (2.15) but the relative values of \( a_i, \epsilon_i, E_0 \) may render some of them inadmissible. For instance \( E_d^2 \) must be real so the expression under the root sign must be \( > 0 \) and also the whole right-hand side of (2.14) must be \( > 0 \) to keep \( E_d^2 > 0 \).

In the semi-infinite media the field solutions are

\[
E_1 = \pm \sqrt{\frac{2}{\alpha_1}} \frac{c}{\omega} \kappa_1 \left[ \cosh[\kappa_1(z + z_{01})] \right]^{-\frac{1}{2}}, \ z < 0, \ \alpha_1 > 0, \ \kappa_1 \ \text{real}
\tag{2.16}
\]

\[
E_3 = \pm \sqrt{\frac{2}{\alpha_3}} \frac{c}{\omega} \kappa_3 \left[ \cosh[\kappa_3(z + z_{03})] \right]^{-\frac{1}{2}}, \ z > d, \ \alpha_3 > 0, \ \kappa_3 \ \text{real}
\tag{2.17}
\]

where \( z_{0i} \) are integration constants. The values of these constants
determine the profile of the field in the semi-infinite media e.g. if

\( z_{01} > 0 \) then there is a maximum of \( E_1 \) at \( z = -z_{01} \) and so on.

Suppose, for example, that in the centre layer \( \alpha_2 > 0, \beta_2 > 0 \) then

\( 2(\alpha_2 - \alpha_1) > (\alpha_1 - \alpha_2)E_0^2 \). Hence if \( E_0 \) is real \( \varepsilon_1 > \varepsilon_2 \Rightarrow \alpha_2 > \alpha_1 \).

In particular

\[
E_z = \pm \sqrt{\frac{q^2 + \kappa_2^2}{2\Lambda_2}} \text{cn}[q(z + z_{02})i]\mu
\]  

(2.18)

where \( z_{02} = \pm \frac{1}{q} \text{cn}^{-1}\left(\frac{A_2E_0^2}{q^2 + \kappa_2^2}\right), \mu = \frac{1}{2} + \kappa_2/2q^2 \) and \( q = \sqrt{\kappa_2^2 + 4\Lambda_2\beta_2} \).

Note now that (2.18) is a complete solution of equation (2.5). There is, apparently, another solution \( \text{sd}[q(z + z_{02})i]\mu \), say, but this can be expressed in terms of (2.18) through \( z_{02} = z_0 - K/p \) where \( K \) is the complete elliptical integral of the first kind. All the fields in this general TE case are now known, since

\[
Z_{01} = \pm \left[ \cosh(E_0^2/\kappa_1^2) \right]^{-1}, \quad Z_{02} = \pm \frac{1}{q} \text{cn}^{-1}\left(\frac{2\Lambda_2E_0^2}{q^2 + \kappa_2^2}\right),
\]

\( Z_{03} = \pm \frac{1}{\kappa_3} \left[ \cosh\left[\frac{2\kappa_3}{q^2 + \kappa_3^2} \cdot \frac{\alpha_2}{\alpha_3} \text{cn}^2q(d + z_{02})\right] \right]^{-1} \)  

(2.19)

and the specific form of the dispersion equation is

\[
\frac{(q^2 + \kappa_2^2)}{2\Lambda_2} \text{cn}^2q(d + z_{02}) = \varepsilon_3 - \varepsilon_2 \pm \sqrt{(\varepsilon_2 - \varepsilon_3)^2 + 2(\alpha_2 - \alpha_1)\beta_2^2\rho^2} 
\]

(2.20)

**TM Modes**

In this case both \( E_x \) and \( E_z \) are involved but, in practice, because of the uniaxial form of the dielectric function the problem reduces to working with \( E_x \) along. It is this that is now designated as \( E_i \) where, in general,

\[
E_i^2 - \frac{\kappa_1}{\varepsilon_\parallel(i)} \left[ E_{\parallel(1)}^2 + \frac{\alpha_1}{2} E_{\parallel(2)}^2 \right] E_i = C_i
\]  

(2.21)

where \( \kappa_1 = k^2 - \varepsilon_\parallel(i) \). Once again \( c_\parallel(i) = 0 \) for \( i = 1 \) and 3 corresponding to the outer semi-infinite media. The boundary conditions now are

\[
z = 0: \quad E_1 = E_2 = E_0, \quad \dot{E}_1 = \frac{\varepsilon_\parallel(2)}{\kappa_1} \dot{E}_2
\]

(2.22)

\[
z = d: \quad E_2 = E_3 = E_d, \quad \dot{E}_3 = \frac{\varepsilon_\parallel(2)}{\kappa_3} \dot{E}_2
\]  

(2.23)

The application of these gives

\[
c_2 = \left( \frac{\kappa_2}{\kappa_1} \right) \left[ \frac{\varepsilon_\parallel(1)}{\kappa_1^2} + \frac{\alpha_1}{2} E_0^2 \right] \left( \frac{\alpha_1}{2} E_0^2 - (\varepsilon_\parallel(2) + \frac{\alpha_2}{2} E_0^2) \right)
\]  

(2.24)
Equation (2.21) has formally the same structure as equation (2.5) in that its solution is, once again, a Jacobi elliptic function. Therefore even in the TM case the field component in the nonlinear slab is simply

\[ E_2 = \pm p' \mathcal{J}(q'(z + z_0'), 1u') \]  

(2.26)

Here \( p' \), \( q' \) and \( u' \) are, of course, different from those that arose in the TE case. Nevertheless, for TM modes, the general dispersion equation has the same generic form as for TE modes i.e.

\[ p'^2 \mathcal{J}^2 q'd \pm \mathcal{J}^{-1}(E_0/p') 1u' = a+/\]

(2.27)

where \( a+/\ ) is \( E_d^2 \) obtained from equation (2.25). This is a very interesting formal equivalence of appearance for both the TE and TM dispersion equations. Indeed all the known cases in the literature should emerge from the same generic formula.

For the three nonlinear media system analysed here with, for example, \( \alpha_2 < 0, \) \( \gamma_2 > 0 \) and the TM forms of \( p', q' \) and \( u' \), the specific form of the dispersion equation is

\[ \left( \frac{\varepsilon_{\perp}^{(2)} \kappa_2 + \varepsilon_{\parallel}^{(2)} q'^2}{\varepsilon_{u}^{(2)} \kappa_2} \right) c_2 q' \left[ d + z_{02}, l u' \right] = B \pm \sqrt{B^2 - D}, \quad u' = \frac{1}{2} + \frac{\varepsilon_{\perp}^{(2)} \kappa_2^2}{2 \varepsilon_{u}^{(2)} q'^2} \]

(2.28)

where

\[ B = \frac{\kappa_2^2 e_u^{(3)} - \kappa_3 e_u^{(2)} c_u^{(2)}}{\kappa_2^2 e_u^{(3)} c_u^{(2)} - \kappa_3 e_u^{(2)} c_u^{(2)}}, \quad D = \left\{ \frac{\varepsilon_{\perp}^{(2)} \kappa_2^2}{\kappa_2} \right\} c_2^2 \left[ \kappa_2^2 e_u^{(3)} c_u^{(2)} \right] \]

(2.29)

\[ q' = \left[ \left( \frac{\varepsilon_{\perp}^{(2)} \kappa_2^2}{\varepsilon_{u}^{(2)} \kappa_2^2} \right)^2 + 2 \varepsilon_{u}^{(2)} \kappa_2^2 \right] \left[ \kappa_2^2 e_u^{(3)} c_u^{(2)} \right] \]

III - ALTERNATIVE FORM OF DISPERSION EQUATIONS

The method developed in section II has the merit of producing a generic form of dispersion equation that is applicable to all structures and both TE and TM modes. It is a very convenient algorithm for computational purposes in which the appropriate candidate from any of the twelve Jacobi functions can be readily inserted. In the literature, however, a more formal method of enquiry has been adopted resulting in separate and apparently very dissimilar developments for TE and TM modes. This approach...
will be used in this section for comparison with the above analysis and to make contact with the existing literature.

**TE Modes**

For $\alpha_i > 0$ the field components are those given by equations (2.16), (2.17) and (2.18). The application of the boundary conditions then gives the formal dispersion equation

$$\kappa_3 \tanh \kappa_3 (d - z_{03}) = qsn(qd - qz_{02})dn(qd - qz_{02})/cn(qd - qz_{02})$$  \hspace{1cm} (3.1)

where $z_{0i}$ have been defined earlier in the text.

**TM Modes**

For TM modes, using $\alpha_i < 0$, $\epsilon_i < 0$, $\kappa_2 \frac{\epsilon_1^{(2)}}{\epsilon_0^{(2)}} > 0$, the application of the boundary gives the formal dispersion equation

$$\frac{\epsilon_1^{(3)}}{\epsilon_0^{(2)}} \kappa_2 \left\{ \frac{\epsilon_1^{(3)}}{\epsilon_0^{(3)}} \right\} \tanh \left[ \frac{\epsilon_1^{(3)}}{\epsilon_0^{(3)}} \kappa_3 (d - z_{03}) \right] = \frac{q'sn(q'd - q'z_{02})dn(q'd - q'z_{02})}{cn(q'd - q'z_{02})}$$  \hspace{1cm} (3.2)

Equations (3.1) and (3.2) reduce to those that exist in the literature.

For example, if $\alpha_1 \rightarrow 0$, $\alpha_3 \rightarrow 0$ so that $z_{01} \rightarrow \infty$ and $\tanh(\kappa_1 z_{01}) \rightarrow 1$ and $\tanh \kappa_3 (d - z_{03}) \rightarrow 1$ as $\alpha_3 \rightarrow 0$ then for $\epsilon_{\epsilon 1}^{(1)} = \epsilon_{\epsilon 1}^{(1)} = \epsilon_1$ and $\epsilon_{\epsilon 1}^{(3)} = \epsilon_{\epsilon 1}^{(3)} = \epsilon_3$

the TM dispersion equation becomes

$$\frac{\epsilon_1 \kappa_3}{\epsilon_3 \kappa_1} = \frac{sn(q'z_{02})dn(q'z_{02})cn(q'd - q'z_{02})}{sn(q'd - q'z_{02})dn(q'd - q'z_{02})cn(q'z_{02})}$$  \hspace{1cm} (3.3)

as obtained by Fedynanin and Mihalache /10/.

**IV - THIN SLAB APPROXIMATE FORMULAE : TM MODES**

For TM modes in a thin nonlinear slab between two dissimilar linear media, provided that $s_1 = \kappa_1/\epsilon_1$, $s_3 = \kappa_3/\epsilon_3$, and $s_2 = \kappa_2^2/\epsilon_2$, the thickness of the slab is given by

$$d \approx \frac{p'^2 s_2 (s_1 + s_3)/s_1 s_3 (p'^2 (1 - 2u'') + 2u' E_0^2) - p'^2 s_2^2}{\epsilon_2 (s_2)}$$  \hspace{1cm} (4.1)

In the event that the layer also becomes linear $s_2 = \kappa_2/\epsilon_2$, $q' = \sqrt{\frac{s_2}{\epsilon_2}} \kappa_2$, $sn(q'z_{02}) \rightarrow 1$ and $u' \rightarrow 1$ so that

$$d \rightarrow - s_2(s_1 + s_3)/(s_1 s_3 + s_2^2)$$  \hspace{1cm} (4.2)

This is exactly the small $\kappa_2 d$ limit of

$$e^{2\kappa_2 d} = \frac{(s_2 - s_3)(s_2 - s_1)}{(s_2 + s_3)(s_2 + s_1)}$$  \hspace{1cm} (4.3)

as derived by Kliewer and Fuchs /12/.

For the three nonlinear media system
where $q'$ and $\mu'$ refer to the inner nonlinear layer and

\[
A = 2q' \sum_{n=1}^{2} \frac{e_n^{(2)}}{e_n^{(1)}} \kappa_1 \sum_{k=2}^{3} \left( \frac{e_k^{(1)}}{e_k^{(1)}} \right)^{\frac{1}{2}} \frac{\gamma}{\kappa_1 E_0} , \quad G = \sum_{n=1}^{2} \frac{e_n^{(3)}}{e_n^{(1)}} \kappa_3 \sqrt{\frac{e_n^{(3)}}{e_n^{(1)}} \frac{e_n^{(3)}}{e_n^{(1)}}} ,
\]

\[
t_1 = \tanh \left[ \frac{e_n^{(1)}}{e_n^{(1)}} \kappa_1 z_{d1} \right] \quad (4.5)
\]

\[
\gamma = (e_n^{(3)})^{\frac{1}{2}} \kappa_3 , \quad t_3 = - \tanh (e_n^{(3)})^{\frac{1}{2}} \kappa_3 z_{d3} , \quad z_{d2} = \frac{cn^{-1}(E_0/p^*)}{q^*} (4.6)
\]

Thin slab formulae for TE modes can be similarly derived.

**V - POWER FLOW**

For a linear guide there is a unique $(\omega,k)$ plot so if we imagine a three-dimensional plot of power flow as a function of $\omega$ and $k$ the loci in all $\omega$, $k$ planes for constant powers would be identical. As $P$ the power level increases, however, the optical nonlinearity expressed through $\alpha_i$ causes the $(\omega,k)$ relationship to change. An interesting surface in $(P,\omega,k)$ space then develops and various sections through it will give vital information. For instance, if fixed $P$ contour maps of the $P,\omega,k$ terrain are produced then these are, in fact, power dependent nonlinear system dispersion curves. A section through $(P,\omega,k)$ for fixed $\omega$ may give information on possible bistable behaviour /7,8/.

The TM mode power flow through a nonlinear layer bounded by linear media has already been derived /10/. If this is written as $P=P_{out}+P_{in}$, where $P_{out}$ is the power flow in the outer semi-infinite media and $P_{in}$ is the power flow in the inner media, then the only general modification needed to describe TM modes is a redefinition of $P_{out}$ to account for, in our general structure, the existence of outer nonlinear media.

Since our numerical examples concern TE modes attention will now be restricted to them.

For TE modes, then, with $\alpha_1, \alpha_3 > 0$ the power flows in medium 1 and 3 are

\[
P_1 = \frac{2k_1 c_1^4}{8\pi \omega^2 \alpha_1} \left[ \tanh (\kappa_1 z_{d1}) + 1 \right] , \quad P_3 = \frac{2k_3 c_3^4}{8\pi \omega^2 \alpha_3} \left[ 1 - \tanh \kappa_3 (d + z_{d3}) \right] \quad (5.1)
\]

It the fields at the boundaries of the layer are $E_{b1}$ at $z=0$, and $E_{b2}$ at $z=d$ then the total outer power flow becomes $P_{out}=P_{out}+P_{in}$ where

\[
P_{out} = \frac{2k c_1^4}{\alpha_1 8\pi \omega^2} \left[ \kappa_1 \pm \sqrt{k_1^2 - A_1 E_{b1}^2 + \frac{A_1}{A_2} (\kappa_3 \pm \sqrt{\kappa_3^2 - A_3 E_{b2}^2})} \right] \quad (5.2)
\]
Note here that equation (5.2) does not reduce, directly, to the case where the outside media are linear. For \( \alpha_1 \to 0, \alpha_3 \to 0, \) and \( \Lambda_1 \to 0 \)
the two negative signs must be taken so that \( P_{\text{out}} \) remains finite. This makes (5.2) indeterminate. Also in equation (5.2) the + sign is chosen for increasing fields at the boundary and the -ve sign for decreasing fields.

In the inner nonlinear layer the power flow is

\[
P_{\text{in}} = \frac{k c^2}{8 \pi \omega} \int_0^d p^2 \sigma^2 [q + j - j] (p) dz
\]

(5.3)

For computational purposes (5.3) may be used directly, or it may be expressed in terms of \( E \), elliptical integrals of the second kind, e.g.

\[
P_{\text{in}} = \frac{k c^2}{8 \pi \omega} \int_0^d p^2 c n^2 [q + G02] dz = \frac{k c^2 p^2}{8 \pi \omega} \left[ E(qd + G02) - E(qG02) - (1 - \omega)qd \right]
\]

(5.4)

In order to illustrate the theory we now specialise to a linear layer, confined between two nonlinear media. The relationship between \( E_d \) and \( E_0 \) is given, in general, by equation (2.10). It is clear from this that a number of possibilities exist. The three choices are \( E_d = \pm E_0 \) and \( E_d \neq E_0 \). This can be seen more emphatically, and perhaps more surprisingly, from the case \( \varepsilon_1 = \varepsilon_3, \alpha_1 = \alpha_3 \) that reduces equation (2.10) to

\[
(E_o^2 - E_d^2) [\varepsilon_1 - \varepsilon_2 + \frac{(\alpha_1 - \alpha_2)}{2}(E_o^2 + E_d^2)] = 0
\]

(5.5)

Here the three possibilities are clearly displayed and furthermore imply that \( E_d^2 + E_o^2 = 2(\varepsilon_1 - \varepsilon_2) \), if \( E_o^2 = E_d^2 \). These are the symmetric, anti-symmetric and asymmetric solutions derived in a completely different manner by Akmediev /7/.

The existence of the \( E_o^2 = E_d^2 \) solution is due to the nonlinearity and can be seen from a 'cross-term' that arises in the power flow formula for a linear layer between two dissimilar nonlinear media. This formula is /8/

\[
p = \frac{kc^2}{\alpha_1 \omega 8 \pi \omega} \left\{ \begin{array}{l}
\frac{2c^2 \kappa_1}{\omega} [1 + \frac{\kappa_2}{\kappa_1} (E_o + E_d) \tanh(\kappa_2 d) - \frac{\kappa_2}{\kappa_1} (E_d - E_o) \coth(\kappa_2 d)] \\
+ \frac{2c^2 \kappa_2 \kappa_3}{\omega} [1 + \frac{\kappa_2}{\kappa_3} (E_o + E_d) \tanh(\kappa_2 d) + \frac{\kappa_2}{\kappa_3} (E_d - E_o) \coth(\kappa_2 d)] \\
+ \frac{(E_d + E_o)^2}{4} \frac{\alpha_1}{\kappa_2} \left[ \frac{\kappa_2 d}{\cosh^2(\kappa_2 d)} + \tanh(\kappa_2 d) \right] + \frac{(E_d - E_o)^2}{4 \kappa_2} \left[ \coth(\kappa_2 d) - \right. \\
\left. \frac{\kappa_2 d}{\sinh^2(\kappa_2 d)} \right] \\
+ \frac{\alpha_1}{2 \kappa_2} (E_d + E_o)(E_d - E_o) \tanh(\kappa_2 d) 
\end{array} \right\}
\]

(5.6)
Here setting $E_0 = \pm E_d$ gives even and odd modes power flows and the final term is eliminated. If $E_d^2 / E_0^2$ the final term remains and we obtain the third kind of mode. As the nonlinearity disappears this 'cross-term' disappears and for guided modes only the conventional even and odd modes are possible.

Fig. 1 shows a plot of TE mode power, in arbitrary units, against $\Omega$ and $K$ for a linear centre layer with a dielectric function $\varepsilon(\omega) = \varepsilon_\infty + (\varepsilon_0 - \varepsilon_\infty) / (1 - \omega^2 / \omega^2)$ that corresponds to a diatomic cubic crystal. The two outside media are nonlinear dielectrics with frequency independent dielectric functions. The frequency is $\Omega = \omega / \omega_T$ and the wave number is $K = c / \omega_T k$. This figure shows a clear division between any surface and guides modes. A surface wave region is the smooth part of the plot beyond the 'mountain peak' region. This latter region becomes very pronounced as $\omega \rightarrow \omega_T$, as would be expected. The guided wave region shows the allowed modes carrying more power. Fig. 2 is a plot for the same data but reaches down to much lower $\Omega$. Here it can be seen that the surface mode surface is actually the dominant feature of the total power plot.

**Fig. 1** - Even TE modes. Linear central frequency dependent layer. $E_0 = 2.5$, $E_0 = 2.0$, $\varepsilon_\infty = 1.92$, $\varepsilon_0 = 9.27$, $\lambda_0 = 1.5$, $\omega_0 k_0 = 1.2$. Power is normalised with $4 \pi d^2 / c$. 
Fig. 2 - Even TE modes. Data as for Fig. 1.

Fig. 3 - Even TE modes. Linear central frequency independent central layer. \( \varepsilon_{02} = 4.0 \). Other data as Fig. 1.
If the central linear layer is replaced by a linear layer with a constant dielectric function /7,8/, then the peaked region of Fig. 1 disappears to be replaced by a much smaller structure in the lower corner of the plot. This is shown, in detail, in Fig. 3. Here the peaks are quite small but the surface waves are still a dominant feature. The cross-section near to $\Omega \sim 1$ corresponds to /8/ and are similar to /7/. Note that in the corner of Fig. 3 the waveguide cut-off locus is shown, corresponding to $\kappa_1$ and $\kappa_3$ becoming complex. Finally Fig. 4 shows the contour plot that arises from Fig. 1. This plot can give directly the dispersion curves of the nonlinear system. It can be seen that they would be practically linear.

VI - CONCLUSION
A general theory of surface and guided polariton propagation in a layered structure has been derived. It is shown that a compact formula exists that accounts in formal terms for both TE and TM modes. Specific formulae for TM and TE modes for a number of systems are given and the theory is illustrated with selected numerical examples that refer to TE modes on special layered structures. In particular a linear layer,
with a frequency dependent dielectric function, between two nonlinear dissimilar media is considered in terms of three-dimensional power diagrams. These are then contoured to get the dispersion equation as a function of power level.

VII - REFERENCES