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ISOSPIN STRUCTURE OF GIANT RESONANCES

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Résumé - Nous examinons le problème de la structure en isospin des résonances géantes isovectorielles. Nous proposons une théorie fondée sur des règles de somme pour la détermination des centroïdes des différents fragments à partir d'un Hamiltonien donné. Plusieurs opérateurs d'excitation sont examinés et les formules explicites dérivées.

Abstract - We analyze the isospin structure of isovector giant resonances. We set-up a theory, based on sum rules, to predict the centroid energies and the strengths of the various fragments, for a given Hamiltonian. Different operators are discussed and explicit examples are worked out.

I - INTRODUCTION

An interesting feature of the isovector giant resonances is that the various isospin fragments split. This phenomenon was predicted long ago /1/ and many experimental efforts were done in the late sixties and in the early seventies in order to detect the isospin splitting mainly of the giant dipole resonance /2/. Theories were also proposed to predict that splitting /3/. At that time, however, rather few data (if any) on the lowest fragment were available and the identification of the various components was somewhat ambiguous. Today, because of the beautiful measurements on charge exchange reactions /4/ and the pioneer experiments on other excitations than the electric dipole ones /5/, the field is again rather promising and the situation can be reexamined. The aim of this talk is not to review the various detailed works on the subject or the results of many experiments, but to summarize the many aspects that the different isovector excitations have in common and to sketch out the main characteristics of the ingredients which force the various isospin fragments of an isovector excitation to split.

II - THEORY

At the very beginning let us introduce the necessary quantities, notations and conventions. When dealing with an isovector excitation it is now traditional to distinguish between its various charge exchange components \( \Delta T = 0,1 \). Let us suppose, as the most common case, that the various isovector excitation operators act on a \( T_3 = T > \frac{1}{2} \) nuclear target. Then a schematic situation of the various excited isospin

* The unpublished matter presented in this talk is part of a detailed paper to be published in collaboration with E. Lipparini and S. Stringari.
fragments is summarized by the familiar scheme of Fig. (1).

\[ \begin{array}{c}
\Delta T_3 = +1 \\
\Delta T_2 = 0 \\
\Delta T_3 = -1 \\
T_2 = T \\
Z-1, N+1 \\
N, Z \\
Z+1, N-1 \\
\end{array} \]

Fig. 1 - The different fragments excited by an isovector excitation operator acting on a target \( T_3 = T \). The isogeometrical factors are indicated.

Once the Coulomb energy is removed, the fragments with the same isospin are essentially degenerate (Fig. (2)).

\[ \begin{array}{c}
E_{T+1} \\
E_+ \\
E_T \\
E_0 \\
E_{T-1} \\
E_- \\
O \\
\end{array} \]

Fig. 2 - Once the Coulomb energy is removed the fragments with the same isospin are degenerate. We have labelled the fragments with various energy centroids.

In the following we will indicate with \( g_+ \), \( g_0 \), \( g_- \) the total strengths of the \( \Delta T = +1, 0, -1 \) excitation operator \( \hat{\mathcal{F}} = \sum_i f_i \hat{T}_i \) respectively and with \( E_+ \), \( E_0 \), \( E_- \) the centroids of the strengths. Owing to the isospin geometry, \( g_- \) is composed of three parts, having isospins \( T+1, T, T-1 \). \( g_0 \) is composed of two fragments with \( T+1 \).
and $T$; finally $\sigma^+$ is composed only of a $T+1$ fragment (Fig. (1)). Guided by the geometry it is convenient to introduce three "reduced" (via Wigner-Eckart) quantities $\sigma_{T+1}, \sigma_T, \sigma_{T-1}$ connected to $\sigma_-, \sigma_0, \sigma_+$, through the isogeometrical factors. A convenient definition of these "reduced" strengths leads to the following expressions /6/.

\[
\sigma_- = \frac{1}{(T+1)(2T+1)} \sigma_{T+1} + \frac{1}{T+1} \sigma_T + \frac{2T-1}{2T+1} \sigma_{T-1}
\]
\[
\sigma_0 = \frac{1}{T+1} \sigma_{T+1} + \frac{T}{T+1} \sigma_T
\]
\[
\sigma_+ = \sigma_{T+1}
\]

Note that this procedure factorizes out only the geometrical dependence on the isospin. A further (dynamical) dependence of $\sigma_T$ on $T'$ has to be expected. ($T'=T+1, T, T-1$). The goal of a fully detailed study of the isospin fragmentation of an isovector excitation is the prediction (or the measure) of the strength distribution of the various fragments. In the following we will restrict ourselves to a schematic study of the total isospin strength and of the centroids energies $E_{T+1}, E_T, E_{T-1}$ of the isospin fragments, within the assumption that the strengths are mainly concentrated in some narrow energy regions. Furthermore we will focus on large $T$ nuclei. For these nuclei, from rel. (1) the bulk of the excitation of $\sigma_-$ is a $T$-1 fragment, whereas the bulk of $\sigma_0$ is a $T$ fragment. For these nuclei one expects that $E_- = E_{T-1}$, $E_0 = E_T$ ($E_0 - E_{T+1}$), so that we can focus our attention on $E_+, E_0, E_-$. In order to express $E_+, E_0, E_-$ some convenient quantities can be introduced as follows: let $\Delta_T$ be a quantity such that

\[
(E_+ - \Delta_T) - E_0 = E_0 - (E_- - \Delta_T)
\]

i.e. $2\Delta_T = E_+ + E_- - 2E_0$

and let $\Delta_T$ be such that

\[
2\Delta_T = E_+ - E_-
\]

Then

\[
E_+ = E_0 + \Delta_ + \Delta_T, E_- = E_0 - \Delta_ - \Delta_T
\]

One expects that the leading contribution to $\Delta_T$ is a "first order effect" in the interaction responsible for the splitting and the leading contribution to $\Delta_T$ is a "second order effect", /7/ so that

\[
\frac{\Delta_T}{E_0} < \frac{\Delta_T}{E_0} < 1.
\]

Furthermore, as far as the dynamical dependence of $\Delta_T$ and $\Delta_T$ on the isospin is rather smooth we can study $E_{T-1}, E_T, E_{T+1}$, using the appropriate geometry. In particular

\[
E_{T+1} - E_T = \frac{\Delta_T}{T} + (T+1) (2T-1) \frac{\Delta_T}{2T}
\]
\[
E_T - E_{T-1} = T \frac{\Delta_T}{T} - (2T+3) T \frac{\Delta_T}{2T^2}
\]

In the following we will illustrate a simple theory, based on sum rules, to study
the gross behaviour of $\Delta_V$, $\Delta_T$, $\sigma^{+}$ and $\sigma^{-}$.

In order to get equations involving $\Delta_V$, $\Delta_T$, $\sigma^{+}$, $\sigma^{-}$ etc. we will use the following important sum rules /8/:

$$
\langle 0 \mid [F_{r}^{\pm}, F_{r}^{\pm}] \mid 0 \rangle = \sigma_{-} - \sigma_{+}
$$

$$
\langle 0 \mid [F_{r}^{\pm}, [F_{r}^{\pm}, H]] \mid 0 \rangle = \sigma_{-} (1) + \sigma_{+} (1)
$$

$$
\langle 0 \mid [F_{r}^{\pm}, H], [F_{r}^{\pm}, H]] \mid 0 \rangle = \sigma_{-} (2) - \sigma_{+} (2)
$$

(5)

where $\sigma (1)$ and $\sigma (2)$ are energy weighted and square-energy-weighted total strengths respectively. Since we assume that the strength is concentrated in a narrow energy region, $\sigma (2) = E_{0} (1)$ where $E$ is the centroid of the strength and

$$
\sigma (2) = E_{0} (1) = E^{2} \sigma.
$$

In this case our sum rules become

$$
\langle 0 \mid [F_{r}^{\pm}, F_{r}^{\pm}] \mid 0 \rangle = \sigma_{-} - \sigma_{+}
$$

$$
\langle 0 \mid [F_{r}^{\pm}, [F_{r}^{\pm}, H]] \mid 0 \rangle = E_{+} \sigma_{+} + E_{-} \sigma_{-}
$$

$$
\langle 0 \mid [F_{r}^{\pm}, H], [F_{r}^{\pm}, H]] \mid 0 \rangle = (E_{-})^{2} \sigma_{-} - (E_{+})^{2} \sigma_{+}
$$

(6)

with

$$
E_{+} = E_{0} + \Delta_{V} + \Delta_{T} \quad , \quad E_{-} = E_{0} - \Delta_{V} + \Delta_{T}.
$$

Later we will comment on the possibility of evaluating these sum rules: let's just now distinguish "first order" and "second order" contributions in the sum rules: we have

$$
\sigma_{-} - \sigma_{+} = \langle 0 \mid [F_{r}^{\pm}, F_{r}^{\pm}] \mid 0 \rangle = f_{0}
$$

$$
\sigma_{-} E_{-} + \sigma_{+} E_{+} = \langle 0 \mid [F_{r}^{\pm}, [F_{r}^{\pm}, H]] \mid 0 \rangle = f_{1} + f_{1} (V)
$$

$$
\sigma_{-} (E_{-})^{2} \sigma_{+} (E_{+})^{2} = \langle 0 \mid [F_{r}^{\pm}, H], [F_{r}^{\pm}, H]] \mid 0 \rangle = f_{2} + f_{2} (V) + f_{2} (T).
$$

(7)

The first sum rule does not involve the Hamiltonian so that we have only a zero order contribution $f_{0}$. The second one has a first order (if any) contribution $f_{1} (V)$. The third sum rule involving twice the Hamiltonian has in principle both a first order ($f_{2} (V)$) and a second order ($f_{2} (T)$) contribution, the latter originating from terms containing the interaction twice. The practical prescription to separate out the various contributions is the following:

$$
O_{1} (V) = \langle 0 \mid [F_{r}^{\pm}, [F_{r}^{\pm}, H]] \mid 0 \rangle - \langle 0 \mid [F_{r}^{\pm}, [F_{r}^{\pm}, H]] \mid 0 \rangle
$$

$$
O_{2} (V) + O_{2} (T) = \langle 0 \mid [F_{r}^{\pm}, H], [F_{r}^{\pm}, H]] \mid 0 \rangle - \frac{T}{\Lambda} \langle 0 \mid [F_{r}^{\pm}, H], [F_{r}^{\pm}, H]] -
$$

$$
\{[F_{r}^{\pm}, K], [F_{r}^{\pm}, K] \}_{i \neq j} \mid 0 \rangle.
$$

(8)

Where $K$ is the kinetic energy part of $H = K + V$.

Inserting $E$ and $E^{-}$ from (2) in (7) one can use an iterating procedure in order to extract $\Delta_{V}$ (first order effects from rel. (4)) and $\Delta_{T}$ (second order) and finally $\sigma_{+}$, $\sigma_{-}$ up to second order contributions. This simple exercise leads to the following results:

$$
\Delta_{V}^{2} = \frac{E_{0}^{2} f_{0} - f_{2} - O_{2} (V)}{2 f_{1}}
$$

(9)
Furthermore, if one notes that \( f \) is just 
\[
\langle \text{CF}^+, \text{CH}, \text{F}^- \rangle \langle \text{lo} >
\]
(see rel.8) and 
\[
\sigma_0 = \langle \text{O} \mid [\text{FOFO}^+, \text{O}] \rangle \exp
\]
by definition of \( E_0 \) (centroid) one has
\[
2\sigma_0 = \frac{f_1}{E_0}
\]

From formulas (9,10,11,12) some important properties become evident:

a) the tensor contribution could be rather large: note that \( \Delta_T \) contains a contribution of the 
kind \( \Delta_v \langle \text{F}^+, \text{H} \rangle \langle \text{F}^- \rangle \rangle \langle \text{lo} >
\) so that a large splitting between \( E_+ \) and \( E_- \), i.e. a large \( \Delta_v \), gives a 
large contribution to \( \Delta_n \). The so called blooking effect, which favours \( \sigma_- \) with res-
pect to \( \sigma_+ \), is displayed in formulas 11 and 12 in a clear way. These expressions 
can be then compared with formula 13. Furthermore these general relations can be 
easily worked out for simple operators \( F \) and simple 
Hamiltonians \( H \). In order to il-
lustrate how quickly one can enter the heart of a practical problem let's just work 
out some simple cases, starting from schematic forces of the kind suggested by Bohr 
and Mottelson /9/ and Lipparini and Stringari /10/.

III - APPLICATIONS: DIPOLE

For the dipole excitation we use 
\[
\vec{F} = \sum_i \vec{F}_i \cdot \vec{z}_i
\]
and the schematic force of ref./9/, which ignores exchange effects. In this case one obtains
\[
f_0 = \langle \text{O} \mid [\text{DF}^+, \text{F}^+] \rangle \langle \text{O} \rangle = \frac{4}{3} T \langle r_v^2 \rangle
\]

where \( \langle r_v^2 \rangle = \frac{N \langle r^2 \rangle_n - Z \langle r^2 \rangle_p}{N - Z} \) is the isovector m.s. radius and \( \langle r_n^2 \rangle \) and \( \langle r_p^2 \rangle \)
are the m.s. radii of the neutrons and protons respectively.

\[
f_1 = -\frac{A}{m}, \quad f_2 = \frac{\omega_0}{E_0} f_0, \quad E_0 = \sqrt{\omega_0^2 \frac{v_T^2}{4A} t_1 \langle r^2 \rangle}
\]

\[
0_1(V) = 0 \quad 0_2(V) = 0 \quad 0_2(T) = 0.
\]

Where \( V_T \) is the symmetry energy potential (100 - 130 MeV) and \( \omega_0 \) is the harmonic 
oscillator energy. With these inputs our formulas 9,10,11,12 give
\[
\Delta_v = \frac{1}{2} \langle \text{F}^+, \text{H} \rangle \langle \text{F}^- \rangle \rangle \langle \text{lo} >
\]

These results are identical to those obtained within an exact diagonalization of our 
schematic Hamiltonian in a R.P.A. basis /11/.

From these formulas one can easily work out numerical estimates for \( \Delta_v \), \( \Delta_T \)
and \( \sigma_+ \) and \( \sigma_- \). Using \( V_T = 120 \text{ MeV}, \quad r_n^2 = r_p^2, \quad E_0 = 78 A^{-1/3} \text{ MeV} \) one obtains

\[
\text{(14)}
\]
Note that the splitting
\[ E_{T+1} - E_T = (T+1) \frac{60}{A} + (T+1) \frac{23}{A^{5/3}} \text{ (MeV)} \] (15)
has contributions not only from the "first order" terms (the famous 60 MeV law) but also from "second order" terms which are significant in size and systematic. The latter has been referred in the past as the isotensor energy \(^{7}\). Data \(^{4a}\) are now available for several nuclei and they show-up clearly the "isotensor" energy; our schematic predictions are in line with the data, as the following table shows:

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>( \Delta E_+ )</th>
<th>( \Delta E_- )</th>
<th>( \frac{\Delta E_+}{T} )</th>
<th>( \frac{\Delta E_-}{T} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^{90}Zr )</td>
<td>3.9(4.3)</td>
<td>2.2 (2.9)</td>
<td>51 (60)</td>
<td>36 (23)</td>
</tr>
<tr>
<td>( ^{116}Sn )</td>
<td>4.0(5.3)</td>
<td>2.5 (3.5)</td>
<td>40 (60)</td>
<td>22 (23)</td>
</tr>
<tr>
<td>( ^{120}Sn )</td>
<td>5.5(6.3)</td>
<td>3.6 (4.1)</td>
<td>53 (60)</td>
<td>19 (23)</td>
</tr>
<tr>
<td>( ^{124}Sn )</td>
<td>6.3(7.4)</td>
<td>4.6 (4.6)</td>
<td>54 (60)</td>
<td>13 (23)</td>
</tr>
<tr>
<td>( ^{208}Pb )</td>
<td>11.2(8.1)</td>
<td>4.5 (4.8)</td>
<td>73 (60)</td>
<td>23 (23)</td>
</tr>
</tbody>
</table>

Table 1. Data and Theoretical Predictions (in parenthesis): everything is expressed in MeV.

As to the results on \( \sigma_+ \) and \( \sigma_- \), data are rather scarce; we can compare our results with those obtained within an R.P.A. framework by Auerbach and Klein \(^{12}\) using a Skyrme III force. For example for \(^{208}Pb\) we obtain:

\[ \sigma_- (fM) = 1057 \quad \sigma_+ (fM) = 22 \]

where the first row corresponds to \( V_T = 120 \cdot r = 5.55, r = 1.15 r^2 \) and the second to \( V_T = 100 \cdot r = 5.55 \cdot r^2 \).

To be compared with 854 and 9.6 respectively of ref./12/. In comparing the results one should keep in mind that whereas our formulas \(^{11,12}\) preserve the sum rule automatically: \( \sigma_- - \sigma_+ = \frac{4}{3} T r^2 \), in the calculation of ref./12/ 15% of the strength is missing in \( \sigma_- - \sigma_+ \).

Data on \( \sigma_+ / \sigma_- \) have been recently reported \(^{4b}\) for \(^{120}Sn\). We obtain \( \sigma_- / \sigma_+ = 5.3 \) to be compared with the measured value \( \sigma_- / \sigma_+ = 4 \).

IV - APPLICATIONS: OTHER EXCITATIONS

The quadrupole operator can be treated in a similar way, starting from the appropriate schematic quadrupole-quadrupole interaction. We use \( \vec{F} = \sum x_i y_i \), In this case one has

\[ O_1(V) = 0 \quad O_2(V) = 0 \quad O_2(T) = 0 \]
\[ \Delta V = \frac{1}{2} \frac{V_T}{A} \quad \frac{<x V>}{<x^2>} \quad \Delta T = \Delta V \frac{\Delta V}{2E_D} \] (16)
We report here some numerical results for Pb. Using \( V_T = 120 \text{ MeV} \),

\[
<r_4^2> = 1.3 \frac{<r_2^2>^2}{1252 \text{ fm}^4} \quad \text{and} \quad <r_v^4> = 1551 \text{ fm}^4
\]

\[
\sigma_+ = 1250 \text{ fm}^4 \quad \text{and} \quad \sigma_- = 10350 \text{ fm}^4
\]

to be compared with \( \sigma_+ = 1283 \) and \( \sigma_- = 8072 \) of ref. 12; in this last case, because of numerical limitations, 20% of the strength is missing in the difference \( \sigma_- - \sigma_+ \).

As a final example we work out the case of the dipole spin flip operator \( \slash \! \! \! \! T \frac{\hat{z}_1 \cdot \hat{u}_1}{\omega} \). One starts from the appropriate schematic Hamiltonian containing a Dipole-spin-flip

Dipole spin-flip separable interaction (we disregard small spin-orbit effects). Let us call \( V_{OT} \) the coupling constant relevant to this interaction: within this Hamiltonian the input parameters for our formulas 9,10,11,12 are:

\[
f_o = \frac{4}{3} T <r_v^2> \quad f_1 = \frac{A}{2m} \quad f_2 = \frac{\omega v}{f_o}
\]

\[
O_1 (V) = - f_o \left( V_T - V_{OT} \right)
\]

\[
O_2 (V) = - 2 f_o f_1 \left( V_T - V_{OT} \right)
\]

\[
O_2 (T) = f_o \left( V_T - V_{OT} \right)^2
\]

\[
E_o = \sqrt{\frac{2}{\omega^2 + \frac{V_{OT} f_1}{4A <r_v^2>}}}
\]

Note that for this case \( O_1 (V) \) and \( O_2 (V) \) are different from zero and proportional to \( (V_T - V_{OT}) \), (first order contribution) and \( O_2 (T) \) to \( (V_T - V_{OT})^2 \) (second order contribution). As a consequence

\[
\Delta_V = \frac{T}{A} \left( V_T - V_{OT} \right) \frac{r_v^2}{r^2} \quad \Delta_T = \frac{T}{2E_o} \left( V_{OT} \right)^2 \frac{<r_v^2>}{<r^2>}
\]

In this case the asymmetry term \( \Delta_T \) depends on \( V_{OT} \) and \( \Delta_V \) is the net result emerging from the effect of the symmetry energy \( T \frac{V_T}{A} \) and the collective (opposite) shift induced by the dipole-dipole interaction \( \frac{V_{OT}}{A} T \). Once more these results are the same as those deduced with a direct diagonalization of the schematic Hamiltonian within R.P.A. /13/.

V - CONCLUSIONS

We have illustrated with some examples the usefulness of formulas (9,10,11,12) using simple schematic Hamiltonians and excitation operators. To study the predictions given on this matter by more sophisticated Hamiltonians one has to work-out the commutators entering in (6). Skyrme forces or more conventional Hamiltonians of the type \( H = V(r_{ij}) (\mathbf{N} + \mathbf{M}^x + \mathbf{B}^T + \mathbf{B}^G) \) can be profitably used to this aim. For this last interaction and a dipole operator explicit expressions for the various commutators can be found in ref. /14/. Dynamical Coulomb effects can also be included
in (6) and the analysis can be extended to other interesting operators, as the monopole and the magnetic dipole operators. For the first, if one disregards Coulomb effects and exchanges, the formal results would be the same as those obtained for the quadrupole (formulas 16) where now \( E_o \) is the giant resonance energy of the monopole excitation. For the second the results would be similar to those expressed in formulas 17 with the factor \( r_z^2 / r^2 \) replaced by unity and \( E_o \) interpreted as the energy of the magnetic giant resonance.

This matter will be the subject of a more detailed work.

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