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EFFECTIVE INTERACTIONS AND THE EXCITATION OF GIANT RESONANCES

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Résumé - On étudie le rôle des interactions effectives entre projectiles et nucléons-cible dans l'excitation des résonances géantes. On se limite au cas des excitations induites par des protons d'énergies intermédiaires, où les caractéristiques principales de la sonde peuvent se comprendre à partir de la matrice t nucléon-nucléon libre. On étudie la fiabilité quantitative de l'approche microscopique des résonances géantes dans laquelle les interactions effectives sont basées sur des matrices G ou des matrices t libres, en mettant l'accent sur les incertitudes provenant des paramètres des potentiels optiques.

Abstract - The role of projectile-nucleon effective interactions in the excitation of giant resonances is studied. Explicit considerations are limited to proton-induced excitations at intermediate energies where the main features of the probe can be understood in terms of the free nucleon-nucleon t-matrix. The quantitative reliability of a microscopic approach to the study of giant resonances using effective interactions based on G-matrices and free t-matrices is studied and optical model uncertainties are stressed.

1. INTRODUCTION

The field of giant resonances in nuclei has been substantially enriched /1-5/ during the past several years. In addition to obtaining more systematics for the more "classical" modes of excitation such as the giant dipole and giant quadrupole resonances, new varieties of spin and isospin modes of various multipolarities have been identified. The newer developments have been made possible largely by the operation of accelerators at intermediate energies (100 ≤ E_{prot} (MeV) ≤ 1000) where the characteristics of the projectile-nucleus couplings are somewhat different from those at lower energies.

At any energy an important part of interpreting the excitation of giant resonances (or any other states) by hadron scattering is an understanding of the interaction between the projectile and the target nucleus. In a microscopic approach this means knowing the projectile-nucleon coupling at the energy of interest. Loosely speaking, two types of microscopic approaches have been pursued. The first /6/ is purely empirical and consists of calibrating some simple interaction by insisting that it describe the excitation of well-known low-lying states of the same symmetry (spin, isospin, etc.) as the giant resonance to be studied. With the proliferation of giant resonances of different multipolarity, spin and isospin, this purely empirical approach becomes awkward and to some extent impractical. In the second approach one attempts to derive the projectile-nucleon coupling from more "basic" principles or data. In the case of proton scattering, which is emphasized here, this coupling is based on nucleon-nucleon (N-N) data and is obtained from either the free N-N t-matrix /7,8/ or a density-dependent G-matrix /9,10/. In principle this second approach is much more satisfactory; in practice it is also monitored by comparison with the excitation of low-lying states having relatively well-known transition densities /5/. Only the second approach will be considered here.
In section 2 some of the primary elements for describing nucleon-nucleus scattering are discussed briefly. In section 3 effective interactions based on the free t-matrix and a density-dependent G-matrix /10/ are considered and some of their most important characteristics are examined. In section 4 mostly isoscalar spin-independent excitations are studied. In section 5 charge exchange reactions are considered and in section 6 spin excitations are discussed briefly.

2. SOME GENERAL CONSIDERATIONS

The interpretation of nuclear excitations requires the calculation of a projectile-nucleus t-matrix which in the single-scattering distorted-wave approximation considered here is given by

$$T_{\text{FI}}^{\text{DW}} = \int d\vec{r}_p \chi^{(+)\dagger}(\vec{k}',\vec{r}_p) \langle \Gamma' M' | \sum_{i=1}^{A} V_{ip} | IM, \uparrow \uparrow \rangle \chi^{(+)}(\vec{k},\vec{r}_p)$$

(1)

where the $\chi^{(+)}$ denote the distorted waves and $V_{ip}$ is the effective interaction between the projectile and each target nucleon. The initial and final target and projectile spin projections are denoted by $(M,M')$ and $(\uparrow \uparrow,\uparrow \uparrow)$; $\vec{k}(\vec{k}')$ is the initial (final) projectile-nucleus momentum and relativistic kinematics are used. Eq. (5) is oversimplified in that no explicit allowance has been made for exchange effects; in actual calculations of nucleon-nucleus scattering, the knock-on exchange terms are included by replacing $V_{ip}$ by $V_{ip}^{\text{IE}} = V_{ip}[1-P_{ip}]$ where $P_{ip}$ is the operator which exchanges nucleons $i$ and $p$. It has been demonstrated elsewhere /11/ that it is usually adequate for $E_p \geq 75$ MeV to regard $V_{ip}^{\text{IE}}$ as a local operator of the same form as $V_{ip}$ and this is assumed in the formalism presented here.

2.1. Transition potentials and transition densities

To illustrate more explicitly the rather general way in which nuclear structure enters the calculation of $T_{\text{FI}}^{\text{DW}}$, it is convenient to introduce a transition potential which is the target matrix element in Eq. (1). In particular,

$$U_{\text{FI}}(\vec{r}_p') \equiv \langle \Gamma' M' | \sum_{i=1}^{A} V(\vec{r}_{ip}') | IM \rangle$$

(2)

which may be rewritten as:

$$U_{\text{FI}}(\vec{r}_p') = \langle \Gamma' M' | \int d\vec{r} V(\vec{r}_p-\vec{r}) \sum_{i} \delta(\vec{r}-\vec{r}_i) | IM \rangle = \int d\vec{r} V(\vec{r}_p-\vec{r}) \rho_{\Gamma' I}(\vec{r})$$

(3)

where $\rho_{\Gamma' I}(\vec{r})$ is the nuclear transition density defined by

$$\rho_{\Gamma' I}(\vec{r}) \equiv \langle \Gamma' M' | \sum_{i=1}^{A} \delta(\vec{r}-\vec{r}_i) | IM \rangle$$

(4)

The spin and isospin dependence of $V_{ip}$ introduces other densities. As long as the probe acts as a one-body operator in the target's space the transition density contains all of the nuclear structure information which enters the transition $I\Gamma'$. Of course different probes sample the transition density differently and this makes them complementary. This same complementarity requires that care must be exercised when comparing strengths obtained from the use of different probes.

2.2. Momentum space considerations

Some of the most important features of the transition amplitude are better illustrated in momentum space. If $V(\vec{r}_p-\vec{r})$ in Eq. (3) is expressed in terms of its Fourier transform $\tilde{V}(q)$ Eq. (1) becomes

$$T_{\text{FI}}^{\text{DFW}} = \int dq D(\vec{q};\vec{k}',\vec{k}) \tilde{V}(q) \beta_{\Gamma' I}(q)$$

(5)
where \( D \) is a distortion function given formally by:

\[
D(q; k, k') = \frac{1}{(2\pi)^3} \int d\chi \left( \begin{array}{c} -1 \end{array} \right) \left( \begin{array}{c} k' \ r \\ k \ r \end{array} \right) e^{-i\mathbf{q}\cdot\mathbf{r}} \chi(k, r)
\]

(6)

and becomes \( \delta(q-k+k') \) in the plane wave limit; \( \hat{\rho} \) is the Fourier transform of \( \rho \),

\[
\hat{\rho}_{I'I}(q) = \int d\mathbf{r} e^{i\mathbf{q}\cdot\mathbf{r}} \rho_{I'I}(\mathbf{r}) = \langle I'I' | \sum_{i=1}^A i_1^\mathbf{q} \mathbf{r}_i | IM \rangle
\]

(7)

and \( \mathbf{q} \) is the local momentum transfer. In addition to important differences in the distortion function for different projectiles \( V(q) \) will also be different. In particular, within a folding model context /12/ the Fourier transform of the nucleon composite-particle interaction is related to that of the N-N interaction by

\[
\hat{V}_{NC}(q) = \hat{\rho}_C(q) \hat{V}_{NN}(q),
\]

(8)

where \( \hat{\rho}_C(q) \) is the Fourier transform of the composite projectile density. From Eq. (8), the finite size of \( \rho_C(r) \) is seen to suppress the higher momentum transfers in the single scattering of a composite projectile relative to those allowed in nucleon scattering. By inserting Eq. (7) into Eq. (8) we may identify the probe function \( P \) of ref. 2 defined by

\[
\tau^D_{FI} = \int d\mathbf{r} \rho_{I'I}(r) P(k', k; r)
\]

(9)

as

\[
P(k', k; r) = \int dq e^{i\mathbf{q}\cdot\mathbf{r}} D(q; k, k') \hat{V}(q).
\]

(10)

It is the extent to which \( P(k', k; r) \) resembles the multipole operator \( r^J_{IJ}(\mathbf{r}) \) near the peak value of the differential cross section that determines, for example, how reliably \((p, pf)\) and electromagnetic transition rates may be compared and related to multipole sum rules /12,13/. A few comparisons of the probe function \( P \) with the multipole operator \( r^J_{IJ} \) have already been reported /14/ which show significant differences for proton energies near 150 MeV.

3. TYPES OF EFFECTIVE N-N INTERACTIONS AND THEIR PROPERTIES

Effective N-N interactions for nucleon scattering are usually assumed and constructed to be of the form:

\[
\begin{align*}
V_{ip} &= V_C^C(r) + V_{LS}^C(r) + V_{T}^C(r) + V_{C}^C(r) + V_{LS}^C(r) + V_{T}^C(r) \\
&\quad + V_{LS}(r) L^i \cdot L^i_p + V_{LS}(r) L^i \cdot L^i_p + V_{LS}(r) L^i \cdot L^i_p \\
&\quad + V^T(r) S_{ip} + V^T(r) S_{ip} \tau^z_p
\end{align*}
\]

(11)

\[(S, T) = (0, 0), (1, 0), (0, 1), (1, 1)\]

\( V_C, V_{LS}, V^T \) and \( V_{C}^{Coul} \) stand for central, spin-orbit, tensor and Coulomb terms respectively, \( r = r_{ip} = |\mathbf{r}_i - \mathbf{r}_p| \) and \( S_{ip} \) is the usual tensor operator. \( V_C, V_{LS} \) and \( V^T \) typically depend on the incident projectile energy /8/ and, in the case of \( G \)-matrix
interactions may also depend on the local density /10/. When $V_{ip}$ of Eq. (11) is regarded as including the knock-on exchange terms, each column the contributing parts of $V_{ip}$ to excitations of specific S and T transfers to the target. Note that in general $V^{LS}$ contributes to both S=0 and S=1 excitations /15/; $V^{Coul}$ has been included for completeness. (It should be stressed that the interaction strengths given in refs. 8 and 10 assume that the knock-on exchange terms will be included explicitly.)

For future reference it is convenient to write $V_{ip}$ in terms of the linear (q), orbital (L), spin (S), and total (J) angular momentum which it can transfer to the target in a single step. To do this we introduce the elementary tensors

$$M_{LSJ}(q^T, 0_S) = j_L(qr) \ (i^L Y_L(q) \otimes 0_S)^J, \quad 0_0 = 1, \quad 0_1 = \vec{s}$$

(12)

and for simplicity restrict consideration to the central parts of $V_{ip}$; the spin-orbit and tensor parts of $V_{ip}$ are discussed in more detail in ref. 5. In terms of these tensors

$$V_{ip}^C = \frac{2}{\pi} \int_0^\infty q^2 dq \sum_{LSJ} (-)^{J+S} V_{LSJ}^C(q) M_{LSJ}(i) \cdot M_{LSJ}(p)$$

(13)

where isospin indices have been suppressed for brevity. For a given L, S, J and (local) momentum transfer q, $M_{LSJ}(i)$ is the operator to which the nuclear structure is sensitive, and for small q this operator is proportional to the multipole operator $r^T(Y_L \otimes 0_S)^J$ occurring in long-wavelength electromagnetic processes.

3.1. Free N-N t-matrix interactions

Where applicable an effective interaction based directly on the free N-N t-matrix provides one of the simplest approaches for obtaining the projectile-nucleus coupling. This approach is especially appealing for the study of giant resonances where the primary objective is to extract nuclear structure information. This approach bypasses in an approximate way many of the difficulties /9,10/ associated with constructing and applying a N-N potential. Unfortunately, the impulse approximation (IA) is not expected to be reliable for proton bombarding energies ($E_p$) below ~100 MeV and for certain classes of transitions, namely S=T=0, a number of corrections to the IA have been found necessary /10,16/ at even higher energies (see sect. 4). Despite their limited realm of quantitative validity, effective interactions based on the free N-N t-matrix provide useful insights into trends which may be expected in nucleon-nucleus scattering.

The steps in obtaining an effective interaction like that in Eq. (11) from free N-N scattering amplitudes are described in detail in ref. 8. The net result /8/ at each energy is a sum of Yukawa terms for representing $V^0$ and $V^{LS}$ and $r^2$ times a sum of Yukawa terms for $V^1$. Some of the most important characteristics of the t-matrix interaction can be illustrated by plotting the moduli of its anti-symmetrized matrix elements ($t_{NN}$) in momentum space in the nucleon-nucleus system /8/ against momentum transfer (q), projectile kinetic energy ($E_p$) and spin and isospin transfers (S and T).

Figure 1 shows a plot for the central parts of the force at q=0 for $100 \leq E_p$ (MeV) $\leq 800$. The subscripts $\sigma$ and $\tau$ refer to spin and isospin transfers of one unit. The most striking feature of these curves is the strong dominance of the
Fig. 1. Energy dependence of the magnitudes of the central (direct + exchange) parts of the N-N t-matrix. On the right is shown the energy dependence of the ratio \( |t_{CG}/t_{C}|^2 \) at zero momentum transfer.

Scalar-isoscalar part of \( t_{NN} \) at all energies considered. The very small (and poorly determined) \( t_{C} \) suggests that the central part of the force is ineffective for exciting isoscalar spin modes. In striking contrast, we see that for isovector excitations \( t_{GT} \) dominates over \( t_{T} \) at small \( q \) in this energy regime. This is emphasized in Fig. 1 where the ratio \( |t_{GT}/t_{T}|^2 \) is plotted versus \( E_p \) at \( q=0 \). It is now well established /4,5,8,17-13/ that this \( S=1 \) dominance of \( t_{NN} \) renders proton scattering an especially sensitive probe of isovector spin modes at intermediate energies. This dominance, together with the fact that \( |t_{GT}| \approx |t_{T}| \) below about 60 MeV has been especially important for the identification and interpretation of Gamow-Teller (GT) resonances using the \((p,n)\) reaction /4,5,17,18/.

Fig. 2. Energy dependence of the spin-orbit and tensor parts of the t-matrix interaction (t) at \( q = 1.5 \text{ fm}^{-1} \).
Figure 2 illustrates the relative importance of the spin-orbit and tensor parts of the interaction as a function of bombarding energy at a momentum transfer of $q=1.5 \text{ fm}^{-1}$. This value of $q$ was chosen for illustration since it represents a compromise of the peak positions of the spin-orbit and tensor terms which, from ref. 8, peak roughly near 2.0 and 1.0 $\text{fm}^{-1}$ respectively. More importantly, these non-central terms tend to dominate in this region of $q$. Most apparent from Fig. 2 is the near negligible size of the isovector spin-orbit term so that isovector $S=1$ modes at large $q$ are dominated by the tensor force at all energies. The smallness of $t_{LS\sigma}$ also implies a strong sensitivity of isovector $S=1$ analyzing powers to optical model parameters since there is typically little source of asymmetry in the inelastic reaction mechanism. Isoscalar $S=1$ excitations are expected to be excited competitively by the spin-orbit and tensor parts of $t_{NN}$ and this is borne out in detailed calculations /8,17,19,20/.

Figure 3 shows the $S=1$ and $S=0$ transfer parts of $t_{NN}$ at $E_p=140$ MeV as a function of momentum transfer $q$. As described in ref. 8, $t_{LS}$ and $t_{LS\sigma}$ should be multiplied by -2.0 in order to compare directly with $t_T$ and $t_T\sigma$ for nucleon-nucleus scattering. These latter quantities ($E^T$) have already been normalized to represent the strength of the tensor force in nucleon-nucleus collisions as described in the appendix of ref. 8. (The left-hand sides of equations A5a, A5b and A5c of ref. 8 should be $t^T_{LS}$, $t^T_{LS\sigma}$ and $t^T_{LS\sigma}$ respectively.) For both isoscalar and isovector transitions, it is seen to be important to include the tensor force in calculations of nucleon-nucleus scattering for essentially all values of momentum transfer. A significant exception is seen to occur for the excitation of $S=T=1$ modes for $q \leq 0.5 \text{ fm}^{-1}$ and this is especially important /4,8,17,18/ for interpreting GT excitations.

![Fig. 3. Moduli of $S=1$ and $S=0$ parts of $t$ as a function of $q$ at $E_p = 140$ MeV.](image)

![Figure 3. Moduli of $S=1$ and $S=0$ parts of $t$ as a function of $q$ at $E_p = 140$ MeV.](image)
3.2. Density dependent G-matrix interactions

The most sophisticated (and in principle the most reliable) effective interactions constructed to date are those derived from G-matrices based on realistic N-N potentials and calculated in the presence of nuclear matter where one of the two interacting nucleons has energy greater than the Fermi energy. These G-matrices are obtained by solving the Bethe-Goldstone equation at a number of different values of the nucleon density or Fermi momentum \( \rho = \frac{2k^2}{5\hbar^2} \). The density dependence arises from the limitation of scattering to unoccupied intermediate states and from the single particle energies which depend on the Fermi energy through the single particle potential /9,10/ in which the nucleons move. The density dependence gets weaker with increasing bombarding energy where the fraction of intermediate states excluded by the Pauli principle decreases and the kinetic energy dominates the single particle energies.

The calculated G-matrix is much too complicated for nucleon-nucleus calculations so that a local effective interaction \( V_{\text{eff}} \) is introduced which reproduces some of the most important matrix elements of G. What finally emerges from this procedure is an effective interaction like that in Eq. (11) which depends not only on \( \vec{r}_{16} \) but also on the local density at which the particles interact. This is called the local density approximation /9,10/ (LDA); scattering programs now exist which can treat density dependence in this way.

Figure 4 shows a comparison /16/ of effective interactions at 140 MeV derived from the free t-matrix of ref. 8 and from density-dependent G-matrices /10/ based on the Paris /21/ and Hamada-Johnston /21/ (HJ) potentials. This figure suggests that density dependent effects should be quite strong over a large range of momentum transfers for S=0, T=0 excitations especially when the HJ based effective interaction is used. Even at low density there are large differences between the HJ effective interaction and the free t-matrix at this energy; it has not been determined to what extent this can be attributed to changes in N-N phase shifts over the past twenty years. The tensor and spin-orbit parts of the effective interaction are calculated to be considerably less density dependent /16/.

![Fig. 4. Comparison of \( |t_{10}^C(q) = t_{10}^G(q)| \) at 140 MeV for the t-matrix interaction from ref. 8 (solid curve on both sides) with the effective interactions from ref. 10 based upon the Paris and HJ potentials. Long (short) dashed curves show low (high) density limits.](image-url)
4. NATURAL PARITY EXCITATIONS: \((S=0, T=0)\)

With a few notable exceptions (such as the isovector \(Z^+\) excitation in \(^{12}\text{C}\) at 16.1 MeV) natural parity states \((\Delta\pi = (-)^J)\) of spin \(J\) in nuclei with \(0^+\) ground states are excited by the central \((S=O)\) and spin-orbit \((S_p)\) terms of \(V_{\text{Y}}\) through the spin-independent and current-independent transition density of Eq. (4). There are, however, both proton and neutron transition densities \(\rho_p\) and \(\rho_n\). For \(N=Z\) nuclei these are usually assumed to be related by \(\rho_n = (-)^T\rho_p\); for collective modes in heavier nuclei one often assumes \(\rho_n = (N/Z)\rho_p\) so that if \(\rho_p\) is known, say from electron scattering, \(\rho_n\) can be fixed.

To implement the above ideas it is convenient to rewrite the transition densities in Eqs. (4,7) in terms of their radial parts. Using the multipole expansion of \(\delta(\mathbf{r}-\mathbf{r}')\) we find:

\[
\rho_{I'\lambda}(\mathbf{r}) = \frac{1}{\sqrt{2I'+1}} \sum_{J,M} <I'\lambda M'|I'M>|Y_{J\mu}(\mathbf{r})\rho_J(\mathbf{r})
\]

(14)

where

\[
\rho_J(\mathbf{r}) \equiv \langle I'|\sum_i \frac{\delta(\mathbf{r}-\mathbf{r}_i)}{r^2} \hat{Y}_J(\mathbf{r}_i)|I\rangle
\]

(15)

is the radial transition density and the reduced matrix elements are in the convention of ref. 22. The corresponding "radial" part of \(\tilde{\rho}_{I'\lambda}(\mathbf{q})\) in Eq. (7) is

\[
\rho_J(q) = \int_0^\infty r^2 dr \, j_J(qr)\rho_J(r).
\]

(16)

For purposes of testing or calibrating an effective interaction it is useful (if not essential) to compare hadron scattering with results using electromagnetic probes for the same transition. For natural parity transitions the common ingredient is the proton transition density. For inelastic electron scattering the square of the longitudinal form factor is to a good approximation given by

\[
|F_L(q)|^2 = \frac{\left|\tilde{\rho}_{Jp}(q)[f_p(q) + \frac{N}{Z} f_n(q)]\right|^2}{(2I+1)}
\]

(17)

where \(\tilde{\rho}_{Jp}(q)\) is the proton point density of Eq. (16) and \(f_p(f_n)\) is the intrinsic charge form factor of the proton (neutron) /12/. Similarly, the \(B(EJ)\) is given by

\[
\frac{B(EJ; I'\rightarrow I)}{e^2} = \frac{\left|\int_0^\infty r^2 dr \, r^J \rho_J(\mathbf{r})\right|^2}{(2I+1)} \left(\frac{(2J+1)!!}{q^J} \rho_{J,p}(q)\right)^2.
\]

(18)

Eqs. (17,18) provide important constraints on \(\rho_{J,p}\); when supplemented by isospin considerations such as discussed earlier \(\rho_{J,n}\) is also determined.

Two different models /2/ which require explicit knowledge of the effective interaction are often used. The first is a fully microscopic treatment in which \(\rho_J(\mathbf{r})\) is obtained by a shell model diagonalization or an RPA calculation and is given by

\[
\rho_J(\mathbf{r}) = \frac{1}{\sqrt{2I'+1}} \sum_{\lambda J \lambda' \lambda''} R_{\lambda}(\mathbf{r}) \, R_{\lambda'}(\mathbf{r}) \, |j_2| |Y_{J\lambda'}| |j_1| \langle I'| |A^+_{J\lambda J \lambda'}(J)\rangle |I\rangle
\]

(19)

where \(R\) is the full radial part of the single particle (or hole) wave function and \(A^+\) creates a particle\((j_2)\)-hole\((j_1)\) pair coupled to angular momentum \(J\). For very
The transition density $\rho_{j,i}(r)$ is taken to be

$$\rho_{j,i}(r) = \delta_{j,i} \rho_{gS}(r)$$

(20)

where $\rho_{gS}$ is the ground state density and $\delta_{j,i}$ (the deformation length) is obtained from the B(E2) or from the longitudinal electron scattering form factor via Eqs. (17,18). The proton point transition density may be determined even more directly when the charge transition density is available by unfolding the nucleon's charge distribution as implied in Eq. (17).

In the next two sections we apply the above ideas to the excitation of low-lying natural parity states in $^{12}$C and $^{208}$Pb where relatively reliable transition densities are available from inelastic electron scattering. This is done with both t-matrix and G-matrix effective interactions at a number of energies in an attempt to assess their reliability for studying the excitation of less well-known excitations such as giant resonances.

4.1. The 4.44 MeV, $2^+$ T=0 excitation in $^{12}$C

Although $^{12}$C is quite removed from infinite nuclear matter this transition was chosen because its proton transition density and ground state density are well established over a wide range of momentum transfer and $\rho_p = \rho_n$ for $N=Z$ nuclei. In addition, $(p,p')$ data /19,23/ for this transition are available at several energies between 100 and 800 MeV. The unpublished data of K. Jones was used at 398 MeV. The transition density was constructed from the $1p$ shell wave functions /24/ of Cohen and Kurath and renormalized (upward) to agree with the longitudinal $(e,e')$ form factor out to $q^{-2}-2.5$ fm$^{-1}$. This calibrated transition density was then used throughout. Comparisons between calculated and measured cross sections are shown in Fig. 5 for proton beam energies of 120, 200, 398 and 800 MeV. The phenomenological (PH) optical model parameters at 120 and 200 MeV are those used in ref. 19; those at 398 and 800 MeV are unpublished. The label IA denotes that an updated version of the free t-matrix interaction described in ref. 8 has been used; the label GV denotes the density-dependent G-matrix of ref. 10 of the appropriate energy. The label FC (folded-consistent) denotes that the optical potential has been calculated in a folding model from the same effective interaction as that mediating the transition. The calculated differential cross sections have been renormalized as shown in Fig. 5. Renormalization of the calculated peak cross sections near 1 fm$^{-1}$ is necessary at each beam energy other than 800 MeV for both t- and G-matrices. At 120 and 200 MeV the G-matrix calculations using phenomenological distorted waves which describe elastic scattering generate differential cross sections whose shapes are clearly superior to those calculated with the free t-matrix interaction. The broader peak predicted by the G-matrix calculations may be traced to the enhancement (suppression) of large (small) momentum transfers relative to the t-matrix calculations noted by Kelly /16/. (See Fig. 4.) This is illustrated explicitly in Fig. 5 at 200 MeV where the central contribution alone (GVPH) is shown. This same curve also illustrates the importance of the N-N spin-orbit interaction at this energy; even at the peak cross section inclusion of VLS roughly doubles the calculated cross section and this has important implications for the study of giant resonances. The N-N spin-orbit terms in the t-matrix and G-matrix interactions are quite similar. As expected, these shape differences between the IAPH and GVPH calculations are relatively small near 400 MeV where the shape and magnitude of the IAPH cross section are in slightly better agreement with the data.

It has been noted /16/ that better agreement between calculated and measured observables are often obtained when consistent (FC) distorted waves are used in the inelastic calculation. The results of using FC distorted waves are shown in Fig. 5 for the case of the G-matrix at $E_p = 120$, 200 and 398 MeV. The shapes of the calculated cross sections are uniformly poorer than with phenomenological waves; the required renormalizations are slightly closer to unity. In particular, if the calcu-
Fig. 5. Cross sections for the $^{12}\text{C}(p,p')$ reaction to the 4.44 MeV $2^+$ state at proton energies of 120, 200, 398 and 800 MeV. The renormalization factors $N(N')$ are discussed in the text as are the different types of calculation IAPH etc. $N'$ is only used at 398 MeV. The curves (---) at 120 and 200 MeV correspond to a fixed-density ($k_F = 0.5 \text{ fm}^{-1}$) G-matrix calculation and to a central-only density-dependent G-matrix calculation (GVPHC) respectively.
lated cross sections are scaled to match the peak experimental cross sections, the renormalization factor for the GVPH and GVFC calculations may be summarized approximately by (see Table 1):

\[
N(\text{GVPH}) = 0.65 \pm 0.05, \quad N(\text{GVFC}) = 0.70 \pm 0.07
\]

Over this same energy range \(N(IAPM) = 0.72 \pm 0.12\). It is reasonable to ask how well these FC distorted waves describe elastic scattering. This has been examined for \(q \leq 2.5 \text{ fm}^{-1}\) at 120 and 200 MeV where the G-matrix folded potentials overestimate the differential cross sections in a very similar way at each energy, especially for \(q \leq 0.8 \text{ fm}^{-1}\) and \(q \geq 1.5 \text{ fm}^{-1}\). At each energy the calculated elastic analyzing powers are significantly improved relative to those obtained with the (searched) phenomenological optical parameters, especially for \(q \leq 0.8 \text{ fm}^{-1}\).

The considerations of this section suggest that either the t-matrix or G-matrix interaction may be used to calculate peak cross sections in light nuclei after renormalization; when shape considerations are important for multipolarity identification say, the G-matrix, is more reliable between 100 and 200 MeV. The necessity to renormalize the G-matrix is not understood. Also, from Fig. 5, the low-density \((k_F = 0.5 \text{ fm}^{-1})\) G-matrix calculation for the \(^{12}\text{C}\) excitation at 120 MeV differs from the free t-matrix calculation at large \(q\) by as much as it differs from the full density-dependent G-matrix calculation. This result indicates that the noted improvement obtained using the G-matrix is only partially due to its density dependence.

4.2. The 2.61 MeV, \(3^-\) excitation in \(^{208}\text{Pb}\)

This classic collective excitation was chosen for study because of what we know about it as a result of the vast amount of attention it has received, both theoretically and experimentally. In particular, the proton ground state and transition /25/ densities have been extracted from electron scattering measurements; in addition, calculated (RPA) proton and neutron transition densities are available which provide a good description of the longitudinal charge form factor. Inelastic proton data are also available for this transition at a number of energies. Here we consider \((p,p')\) data at 135 /26/, 200 (ORNL - Orege. collaboration and ref. 27), 334 (F. Bertrand et al., unpublished) and 800 /28/ MeV.

Two different transition densities were used. One is from an RPA calculation by the Jülich group. The other is a phenomenological density in which the proton part was obtained directly from \((e,e')\) measurements /25/ after unfolding the charge distributions of the proton and neutron; the neutron point density was assumed to satisfy

\[
p_{J,p,n}(r) = \sigma_x^N \rho_{J,p}(r).
\]

The constant \(\sigma\) was adjusted /28/ until the \((p,p')\) cross section at 800 MeV was described in the distorted wave impulse approximation (DWIA). The value of \(\sigma\) is 1.1 reflecting a nearly pure collective excitation in the hydrodynamical sense. Calculations using these two densities at 800 MeV are shown in Fig. 6. The unadjusted Jülich transition density leads to cross sections (denoted by \(B\) in Fig. 6) which fall off somewhat too rapidly with angle and which are slightly shifted forward relative to the data. The phenomenological transition density \((A)\) yields an excellent description of the shape and magnitude of the experimental data. This shape difference arises from the shape and magnitude differences between the neutron transition densities in the Jülich and phenomenological densities. The Jülich neutron transition density peaks at a larger radius than the proton transition density; in terms of the \(r^3\) moments, \(\alpha\) in Eq. (21) is 1.02 for the Jülich transition density. The phenomenological transition density was used below 800 MeV.
At 135 MeV neither the G-matrix nor t-matrix calculation provides a good description of the data when the phenomenological (Woods-Saxon) optical potential is used. (The squares are actually data /26/ for the L=3 doublet in 207Pb.) From Fig. 6 each of these calculated cross sections requires a renormalization of 0.7 in rough agreement with the results for 12C at 120 MeV. The consistent G-matrix calculation provides an excellent description of the cross section between 20° and 40° without any renormalization. This is in sharp contrast to the situation for the 21 state in 12C where the agreement in shape between the calculated and experimental cross sections deteriorated when the consistent optical potential was used. The calculated elastic scattering cross section (not shown) is in only modest agreement with the experimental data when the G-matrix folded optical potential is used and the neutron ground state density is taken to be N/Z times the proton density. It is unfortunate that data for the first maximum are unavailable at this energy since that is the most relevant peak for giant resonance studies and because the different microscopic models predict significantly different cross sections at the first peak. The standard collective model, which provides an excellent description of the shape of the cross section for this transition at each energy considered, yields a peak cross section of 27 mb/sr near 12.5° when scaled to match the available data for θ = 15°. Somewhat older data /29/ at 155 MeV suggest a peak cross section near 35 mb/sr.

At 200 MeV and 334 MeV data is available only for the first maximum. At 200 (334) MeV both t-matrix and G-matrix calculations require a renormalization factor of 0.5 (-0.38) when phenomenological optical model parameters are used. The consistent G-matrix calculations (GVFC) require renormalization factors of -0.8 and 0.5 at 200 and at 334 MeV respectively. The available data do not differentiate between the shapes derived from different calculations; larger angle data would be most welcome.

A summary of renormalization factors defined by σ(expt) = Nσ(theory) near the first peak for the different types of calculations is shown in Table 1 for both 12C and 208Pb. The relatively large spread in the renormalization factors for the 31 transition in 208Pb is confusing. The experimental situation is equally confusing, especially between 100 and 200 MeV where the peak experimental cross section for the 31 transition varies erratically; the largest /29/ and smallest /28/ peak cross sections differ by a factor of ~2.5 in this energy interval! Although the
Fig. 6. Cross sections for the $^{208}$Pb (p,p') reaction to the 2.61 MeV $^3-$ state at proton energies of 135, 200, 334 and 800 MeV. The 200 MeV data is that of the ORNL-Oregon collaboration. The legend is explained in the text (see sect. 4.1.).

Data sets containing these extreme values of the cross section were not used in the present analysis, these considerations help place the renormalization factors in Table 1 in perspective.

Table 1. Renormalization Factors for S=0, T=0 Transitions

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$E_p$ (MeV)</th>
<th>N(IAPH)$^a$</th>
<th>N(GVPH)$^b$</th>
<th>N(GVFC)$^c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}$C($^2_1^+$)</td>
<td>120</td>
<td>0.60</td>
<td>0.67</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.55</td>
<td>0.64</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>398</td>
<td>0.85</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{208}$Pb($^3_1^-$)</td>
<td>135</td>
<td>0.70</td>
<td>0.70</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.50</td>
<td>0.50</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>334</td>
<td>0.38</td>
<td>0.38</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

$^a$) T-matrix interaction, phenomenological optical potential.
$^b$) G-matrix interaction, phenomenological optical potential.
$^c$) G-matrix interaction, folded G-matrix optical potential.
The data for the $3^-$ transition in $^{208}$Pb may also be described by the collective model which yields deformation lengths $\delta \equiv \beta R$ of 0.73, 0.67, 0.61 and 0.82 fm at 135, 200, 334 and 800 MeV respectively, with an average value of 0.71. (This average value is unchanged when data at 61/26/ and 95/30/ MeV are included.) The value of $\beta R$ at 334 (800) MeV is lower (higher) than this average by 14 (17)%. Values of $\beta R$ at the other energies are within -5% of the average.

Assuming that the data considered are properly normalized, the $^{208}$Pb results in Table 1 imply that neither effective interaction provides a satisfactory description of the experimental data as a function of energy and this limits their predictive power for giant resonance studies. There does appear to be a preference for a folded model type of optical potential near 135 MeV for $^{208}$Pb. For the $3^-$ excitation in $^{208}$Pb the choice of optical potential between 100 and 200 MeV introduces uncertainties as large or larger than those associated with the choice of effective interaction. The two optical potentials considered are not, however, equivalent, for elastic scattering; the resolution of this optical potential problem clearly warrants high priority.

5. ISOVECTOR EXCITATIONS: NUCLEON CHARGE EXCHANGE REACTIONS

The nucleon charge exchange reactions $(p,n)$ and $(n,p)$ are especially selective because only isovector nuclear excitations are allowed. This renders the $S=0$, $T=0$ part of the N-N interaction, which dominates the $(p,p')$ spectrum at small momentum transfers inoperative (see Fig. 1). Moreover, since most of the $(p,n)$ data /31/ at intermediate energies have been taken at forward angles where the momentum transfer is small and therefore the non-central parts of $V_{\text{HE}}$ are relatively unimportant, the ratio of $|t_{pp}/t_{np}|^2$ shown in Fig. 1 should, apart from nuclear matrix elements, be a reasonable measure of the relative cross sections for $S=1$ and $S=0$ excitations and this is borne out by experiment /32/.

From Eqs. (11-13) the nucleon couples to the nucleus through the tensors

$$\sum_{i=0}^{1} M_{ij}(q, O_{i}(i)) \tau_{\mu}(i), \quad \begin{cases} \mu = -1, (p,n) \\ \mu = 0, (p,p') \\ \mu = 1, (n,p) \end{cases}$$

for isovector excitations in the convention $T_z = (N-Z)/2$; $\tau_{\mu}$ is the $\mu$th component of the Pauli isospin operator. At small (local) momentum transfer $q$ these tensors may be related to the usual isovector multipole operators /32,33/:

$$M_{000} \rightarrow \frac{1}{\sqrt{4\pi}} \left[1 - \frac{q^2 r_1^2}{6}\right] - (P) - \frac{1}{6} q^2$$

$$M_{011} \rightarrow \frac{1}{\sqrt{4\pi}} \left[1 - \frac{q^2 r_1^2}{6}\right] - (GT) - \frac{1}{6} q^2$$

$$M_{101} \rightarrow \frac{4q}{3} r_1 Y_1(\hat{r}_1) \sim (IVDR)$$

$$M_{11J} \rightarrow \frac{4q}{3} r_1 Y_1(\hat{r}_1) \otimes \sigma(i)^J \sim (IDSR); \quad (J = 0, 1, 2)$$

$$M_{202} \rightarrow - \frac{2q^2 r_1^2}{15} Y_2(\hat{r}_1) \sim (IVQR)$$
where the abbreviations on the right denote the multipole strength operators. For example, IVMR (IMSR) stands for isovector monopole resonance (isovector monopole spin resonance) etc. The actual response probed by the projectile is more complicated because of the coherent superposition of different q values due to distortion effects; see Eq. (5). The correspondence between charge exchange cross sections and multipole strengths should be closest for light nuclei where the relevant values of r are smaller.

It is by now well established /8,19,31,33/ that the free t-matrix interaction (t part) describes isovector spin excitation strengths at small momentum to within 10-20% in light nuclei at proton beam energies between 100 and 800 MeV. The isovector spin-independent part of the effective interaction is not nearly as well established because of the limited amount of data at intermediate energy for transitions where it may be tested. There does exist published data for the (p,n) reaction to the isobaric analogue state (IAS) of the ground state of Zr at Ep = 120 MeV and we consider this transition here. The study of such well-known excitations is helpful if not essential for studying other similar excitations such as the IVMR etc. The results of both t-matrix and G-matrix calculations are shown in Fig. 7. As before, both phenomenological and G-matrix folded model distorted waves were considered. Using phenomenological (PH) distorted waves which were fit to the elastic scattering cross sections of Scott et al. (unpublished) both t-matrix (SM82) and G-matrix calculations provide reasonable descriptions of the existing cross section data. The consistent G-matrix calculation underestimates the differential cross section, which in this case (L=0) is especially sensitive to the distorted waves in the interior of the nucleus, by a factor of 2.5. This result is in sharp contrast to the results for 12C and 208Pb discussed earlier where the GVFC calculation improved in magnitude with experiment suggesting that the isovector spin-independent part of the G-matrix interaction is too weak or that the improvement obtained using GVFC calculations for 12C and 208Pb was fortuitous. The results of Kelly /16/ indirectly suggest that the problem may be the isovector spin-independent part of the G-matrix interaction. The folded G-matrix optical potential provides a qualitative to semi-quantitative description of the Zr + p elastic scattering data; the fit (unsearched) is comparable to that obtained for Pb + p elastic scattering at 135 MeV.

Fig. 7. Differential cross sections for the Zr(p,n) reaction at 120 MeV to the IAS in Nb. None of the calculated curves have been renormalized; the legend is explained in the text (see sect. 4.1.).
The $\nu = 1$ component of the IVMR has recently been reported /34/ using the $(n^-,\pi^0)$ reaction on $^{120}\text{Sn}$ and $^{90}\text{Zr}$. Estimates of cross sections for the nucleon charge exchange reaction exciting this mode have also been made recently /35/ using the $t$-matrix interaction and phenomenological distorted waves. Calculations for the isovector spin monopole mode which may be even stronger are underway. As yet there is no experimental $(p,n)$ or $(n,p)$ data for these modes.

6. SPIN EXCITATIONS AT SMALL MOMENTUM TRANSFER: $1^+$ STATES

As noted in sect. 5, the strengths of isovector spin excitations at small $q$ appear /8/ to be quantitatively described by the free $t$-matrix interaction; isoscalar spin excitations are found to be poorly described at small $q$, at least below 300 MeV, by the $t$-matrix or $G$-matrix. This is primarily due to the large tensor exchange terms which tend to overestimate the small $q$ cross section. However, reliable transition densities for isoscalar spin modes are not readily available for testing the effective interaction. $\beta$-decay is inoperative and isospin mixing tends to mask the isoscalar transition density in electromagnetic processes. Therefore, the calibration of transition densities or wave functions depends on the less reliable $(\pi,\pi')$ reaction and the $(p,p')$ reaction at higher energies /8/. We mention this because a description of the recently reported "M1" resonances /36,37/ in heavy nuclei near 200 MeV often involve primarily neutron excitations and thus a superposition of isoscalar and isovector $S=1$ amplitudes. In the case of the $1^+$ (M1) excitation in $^{90}\text{Zr}$ at 8.9 MeV we have estimated that the isoscalar contribution increases the small-angle cross section by 10–30% depending on the $N-N$ interaction, optical potential etc. when a $(1g_{9/2} 1s_{1/2}^{7/2})$ neutron configuration is assumed.

Since there has been considerable disagreement between $(e,e')$ and $(p,p')$ groups as to what fraction of the (very model-dependent) shell-model sum rule strength is observed in these two reactions and how it is distributed, we have examined some of the uncertainties attributable to differences between different effective interactions and different optical potentials in extracting a value of the strength from $(p,p')$ measurements. This is illustrated in Fig. 8 for the $^{90}\text{Zr}$ $(p,p')$ reaction at 201 MeV assuming a pure neutron particle-hole excitation as described above. Harmonic oscillator bound state wave functions were used with an oscillator length of 2.12 fm; the phenomenological optical model parameters are set II of ref. 37; the data are from ref. 36. If all of the "M1" strength were concentrated in the state...
at 8.9 MeV and the neutron ground state were (1g9/2)10 (with no protons in the 89/2 state), no renormalization of the calculated cross section would be expected for the correct effective interaction. Although the effective interaction is known to be uncertain largely because of uncertainties in its isoscalar part, we expect the transition to be dominated by the much better understood and larger isovector part. When phenomenological distorted waves are used both t-matrix and G-matrix interactions require comparable renormalization factors near 0.23. However, when consistent distorted waves are used, the calculated cross section is significantly reduced leading to a larger renormalization factor of ∼0.40 corresponding to 40% of the simple shell-model strength. To the extent that it is very difficult to reconcile this larger value (40%) with (e,e') data, this result suggests that the distorted waves generated from the G-matrix folded model may be inappropriate even though they provide a reasonable description of the elastic scattering forward of ∼-20°. The use of these calculated distorted waves should have similar effects on the extraction of Gamow-Teller strengths in heavy nuclei. Satchler /38/ has studied similar consequences amongst phenomenological potentials and finds quenching due to wine-bottle shaped potentials. Calculated cross sections (not shown) using phenomenological and folded consistent distorted waves for excitation of the T=1, I+ state in the 12C (p,p') reaction at 200 MeV differ by only ∼-20% at small q.

7. SUMMARY

This work may be characterized as a very limited survey of the present status of our ability to calculate proton induced inelastic scattering and charge exchange reactions at intermediate energies using realistic N-N effective interactions. Particular emphasis has been placed on the excitation of low-lying S=0 collective excitations in 12C and 208Pb at several beam energies between 100 and 800 MeV where calculated (p,p') differential cross sections have been compared with available experimental data. These transitions were selected because the transition densities are relatively well determined from (e,e') measurements and this enables us to test and/or calibrate the effective interaction for use in other transitions.

At 800 MeV the DWIA provides a good description of the observed peak cross sections in both 12C and 208Pb (see sect. 4) without renormalization. Below 400 MeV three different types of theoretical calculations have been considered. The simplest type (TAPH) uses phenomenological distorted waves chosen to fit elastic scattering data together with a free t-matrix interaction for the inelastic scattering amplitude; another type (GVPH) uses these same distorted waves but employs a density-dependent G-matrix for the transition operator; the third type (GVFC) uses the G-matrix to generate both the distorted waves and the transition potential. The renormalization factors required by the theoretical cross sections to match the observed peak cross sections have been given in Table 1. For the 2+ transition in 12C the GVPH type of calculation appears best overall in that the renormalization factors are roughly energy dependent and the angular distributions are well described, especially at 120 and 200 MeV. For 208Pb there is a considerable spread in the peak cross sections reported by different groups so that only tentative conclusions can be drawn. Namely, the GVFC type of calculation requires the least renormalization at each energy and at 135 MeV predicts a differential cross section in much better agreement with experiment than those obtained with the two other types of calculation. It has been shown in the case of 12C that it is inappropriate to ascribe the improvement obtained by using the G-matrix relative to the t-matrix entirely to density-dependence; much of the improvement arises from somewhat different low-density limits of the t and G-matrices especially at large momentum transfers.

Due in part to the scarcity of relevant data, S=0 charge exchange reactions have been considered only briefly. For this type of transition, both t- and G-matrix interactions give reasonable results for the TAS transition in 90Zr with less than a 20% renormalization of the calculated differential cross sections when phenomenological distorted waves are used. The GVFC type of calculation for this monopole transition yields cross sections which are much too small (N = 2.5) suggesting that the isovector part of the G-matrix may be too weak.
The sensitivity of the extracted "Ml" strength in $^{90}$Zr to each type of theoretical calculation has also been explored. As for the isobaric analogue state this transition was found to be most sensitive to the choice of distorted waves; the GVFC calculated cross section is only half that of the GVPH calculation.

In summary, the single largest uncertainty in a microscopic description of the excitation of the states considered here is the choice of distorted waves, especially below 300 MeV. This same uncertainty limits the quantitative validity of a microscopic study of other states including giant resonances. There does appear to be some preference for a folding-model type of optical potential, especially when analyzing power data are included. It should be stressed that no variation of the G-matrix folded optical potentials has been made here; perhaps that is too restrictive at this time.

ACKNOWLEDGMENTS

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4. Goodman C. D., ibid, p. 117.


Addendum: After this manuscript was submitted the unpublished data for the $^{208}\text{Pb}(3^+)$ excitation at 334 MeV was reexamined and now needs to be multiplied by an overall factor of 1.6. Those parts of section 4.2 (including Table 1) which refer to this data need to be considered accordingly.