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DEEP HOLE AND HIGH-LYING PARTICLE STRENGTH FUNCTIONS

V.G. Soloviev

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Abstract - The basic assumptions of the quasiparticle-phonon nuclear model are expounded. The results of calculation of the fragmentation of one- and two-quasiparticle states are compared with the experimental data for spherical nuclei. The change of the fragmentation of the $1h_{11/2}$ subshell as one moves from a spherical to a deformed nucleus is studied. The strength distribution of the charge-exchange resonances in spherical and deformed nuclei is calculated.

The strength distribution (fragmentation) of single-particle and collective states in atomic nuclei is intensively studied experimentally and theoretically. A great progress is associated with the study of the fragmentation of deep hole and proton particle states in spherical nuclei and of collective states of the giant resonance-type in light, medium and heavy nuclei.

With increasing excitation energy the level density of atomic nuclei grows and their structure becomes more complicated. Thus, a transition proceeds from simple low-lying to very complicated states at intermediate and high excitation energies. It is almost impossible to measure and describe the characteristics of each of many thousand levels, the wave functions of which contain a large number of components. Therefore, the problem is to investigate the laws of complication of the structure with increasing excitation energy. The study of the fragmentation of one- and two-quasiparticle states is a first step in solving this problem. An increasing interest in the study of the fragmentation of deep hole neutron and proton states stems from that the spreading widths $\Gamma_1$ are considerably larger than the escape widths $\Gamma_1$. An experimental discovery of many multipole, spin-multipole and charge-exchange resonances raised a problem about the strength distribution of these collective states. This problem turned out to be more important because in some cases the strength of the collective mode was apparently less than half of the predicted value.

An effective method for describing the fragmentation of one-quasiparticle and one-phonon states has been developed within the quasiparticle-phonon nuclear model /1-3/. In this report we expound the basic assumptions of the model, compare it with other models and demonstrate...
the advantages of the quasiparticle-phonon nuclear model. The fragment-
tation of one- and two-quasiparticle states in spherical nuclei are
described in the framework of the model. The results are compared with
the experimental data. Specific properties of the fragmentation of
charge-exchange collective states in spherical and deformed nuclei are
discussed.

1 - BASIC ASSUMPTIONS OF THE QUASIPARTICLE-PHONON NUCLEAR MODEL

The fragmentation of one-quasiparticle, one-phonon and quasiparticle
phonon states is calculated within the quasiparticle-phonon nuclear
model. The characteristics of nuclear states at low, intermediate and
high excitation energies, determined by this fragmentation, are calcu-
lated. The model is based on the following assumptions:

i) The RPA one-phonon states are used as a basis. All the model pa-
rameters are fixed while constructing the phonon basis.

ii) The quasiparticle-phonon interaction affects the fragmentation
of quasiparticle and collective motion and thus the complication of
the structure of nuclear states with increasing excitation energy.

iii) The excited state wave functions are given as a series in the
number of phonon operators (in odd-A nuclei they are added by the qua-
siparticle operator). In the above calculations the expansion is limi-
ted by two phonons.

iv) The strength function method is used to calculate the reduced
transition probabilities, transition densities and other nuclear cha-
acteristics without solving the relevant secular equations.

The Hamiltonian of the quasiparticle-phonon nuclear model includes the
average field as the Saxon-Woods potential and the pairing, multipole-
multipole, spin-multipole - spin-multipole isoscalar and isovector
including charge-exchange interactions. A general description of the
model Hamiltonian for the deformed nuclei is given in refs./1,2/ and
for the spherical nuclei in refs./3,4/.

In transforming the model Hamiltonian by the canonical Bogolubov tran-
sformation, one moves from the nucleon operators to the quasiparticle
operators $\alpha_{jm}^+$. The pairs of operators $\alpha_{jm}^+, \alpha_{jm}$ are expres-
ssed through the phonon operators, whereas the quasiparticle
operators remain in the form $\alpha_{jm}^+ \alpha_{jm}$. Such an inclusion of the phonon
creation operator overcomes the difficulties with the double counting of some
diagrams, which take place in the nuclear field theory/5/.

The secular equation determining the energies $\omega_{\lambda i}$ of the one-phonon
states is

$$\mathcal{F}(\omega) = (\omega + \omega)(X^i(n) + X^i(p)) - 4 \lambda \omega, X^i(n) X^i(p) - 1 = 0.$$  (2)

Then by $\lambda \omega$, $\lambda \omega$ and $\lambda \omega$, $\lambda \omega$ we denote the isoscalar and isovector
constants of the multipole and spin-multipole forces,

$$X^\lambda M(\tau) = \sum_{jlm} \left( \frac{f^\lambda_{jldz}}{E^\lambda_{jlm} - \omega_{\lambda i}} \right)^2 E^\lambda_{jldz}$$

for the multipole forces, (3)
\[
\chi_{S}^{Li}(\tau) = \sum_{i}^{T} \frac{(f_{i}^{TE}(i_{i}, i_{z}) U_{i_{i}, i_{z}}(\tau))^{2}}{\varepsilon_{i_{i}, i_{z}} - \omega_{Li}^{2}} \quad \text{for the spin-multipole forces (3')}
\]

\[f_{i_{i}, i_{z}}(i_{i}, i_{z}) \] are the single-particle reduced matrix elements of the multipole and spin-multipole operators, \( T = n, p \) implies the summation over neutron (\( n \)) and proton (\( p \)) single-particle states: \( U_{i_{i}, i_{z}}^{(} = U_{i_{j}, i_{z}}^{(} U_{i_{j}, i_{z}}^{(} ; U_{i_{i}, i_{z}}^{(} = U_{i_{j}, i_{z}}^{(} U_{i_{j}, i_{z}}^{(} \), where \( U_{i_{j}} \) and \( U_{i_{j}}^{(} \) are the Bogolubov transformation coefficients, \( E_{j} \) and \( E_{j}^{(} \) are the one- and two-quasiparticle energies;

\[
y_{i}^{(T)} = y_{i}^{(T)} + y_{i}^{(-T)} \left\{ \frac{1 - (x_{i}^{*} x_{i})}{(x_{i}^{*} x_{i})^{2}} \right\}^{2}, \quad y_{i}^{(T)} = \frac{1}{2} \frac{\partial}{\partial \omega} X_{i}^{(T)} \bigg|_{\omega = \omega_{i}}.
\]

To describe charge-exchange resonances we introduce charge-exchange multipole and spin-multipole phonons in the form /6/

\[
\Omega_{\lambda_{i}}^{\mu} = \sum_{i_{p} i_{n}} \left\{ U_{i_{p} i_{n}}^{\lambda_{i}} A_{(p, i_{n}; \lambda_{i} \mu)}^{(\lambda_{i})} V_{i_{p} i_{n}}^{\lambda_{i}} \right\}
\]

The secular equation for finding the charge-exchange one-phonon energies \( \Omega_{\lambda_{i}}^{\mu} \) is

\[
G_{CM}(\Omega_{\lambda_{i}}^{\mu}) = (1 - X_{M}^{\lambda_{i}^{(+)}})(1 - X_{M}^{\lambda_{i}^{(-)}}) - (X_{M}^{\lambda_{i}^{(++)}})^{2} = 0,
\]

where

\[
X_{M}^{\lambda_{i}^{(+)}} = \sum_{i_{p} i_{n}} \frac{(f_{i}^{TE}(i_{p}, i_{n}) U_{i_{p} i_{n}}^{(})^{2} \varepsilon_{i_{p} i_{n}}}{\varepsilon_{i_{p} i_{n}} - \Omega_{\lambda_{i}}^{2}}
\]

(7)

\[
X_{M}^{\lambda_{i}^{(-)}} = \sum_{i_{p} i_{n}} \frac{(f_{i}^{TE}(i_{p}, i_{n}) U_{i_{p} i_{n}}^{(})^{2} \varepsilon_{i_{p} i_{n}}}{\varepsilon_{i_{p} i_{n}} - \Omega_{\lambda_{i}}^{2}}
\]

(7')

\[
y_{\lambda_{i}}^{i} = \frac{X_{M}^{\lambda_{i}^{(+)}}}{1 - X_{M}^{\lambda_{i}^{(-)}}}, \quad y_{\lambda_{i}}^{-2} = \left\{ (1 - X_{M}^{\lambda_{i}^{(-)}}) / \left( -\frac{\partial G_{CM}(\Omega)}{\partial \Omega} \right) \Omega = \Omega_{\lambda_{i}} \right\}^{2} \]

(8)

Taking into account the secular equations for one-phonon states, the model Hamiltonian can be transformed into

\[
H_{M} = \sum_{i} \varepsilon_{i} \chi_{i}^{(+) \lambda_{i}} \chi_{i}^{(-) \lambda_{i}} + H_{MV} + H_{M\sigma} + H_{S\sigma} + H_{CM\sigma} + H_{CMS\sigma} + H_{Cp\sigma} + H_{CS\sigma} + H_{CSP\sigma},
\]

where

\[
H_{MV} = \frac{1}{4} \sum_{\lambda \mu i \tau} \frac{X_{M}^{\lambda \lambda_{i}^{(+) / \lambda_{i}^{(-)}}(\tau)} \varepsilon_{\lambda \mu_{i \tau}}}{\sqrt{y_{\lambda_{i}}^{\lambda_{i}^{(+) / \lambda_{i}^{(-)}}}} Q_{\lambda \mu_{i \tau}}^{+} Q_{\lambda \mu_{i \tau}}^{+},
\]

(10)

\[
H_{M\sigma} = -\frac{1}{2 \sqrt{2}} \sum_{\lambda \mu_{i \tau}} \left\{ \left(-\frac{\lambda^{+} \mu_{i \tau}}{\varepsilon_{\lambda \mu_{i \tau}}} + Q_{\lambda \mu_{i \tau}}^{+} Q_{\lambda \mu_{i \tau}}^{+} \right) \sum_{i_{i}, i_{z}} \frac{f_{i_{i}, i_{z}}^{(i, i_{i}, i_{z})} B_{i_{i}, i_{z}}^{(i, i_{i}, i_{z})} + h.c.}{\sqrt{y_{\lambda_{i}^{(+) / \lambda_{i}^{(-)}}}}} \right\},
\]

(11)

\[
H_{S\sigma} = -\frac{1}{4} \sum_{\lambda \mu i \tau} \frac{X_{S}^{\lambda \lambda_{i}^{(+) / \lambda_{i}^{(-)}}(\tau)} \varepsilon_{\lambda \mu_{i \tau}}}{\sqrt{y_{\lambda_{i}}^{\lambda_{i}^{(+) / \lambda_{i}^{(-)}}}} Q_{\lambda \mu_{i \tau}^{+} Q_{\lambda \mu_{i \tau}}^{+}},
\]

(12)
Formulae for $H_{csv}$ and $f_{csv}$ differ from (14) and (15) as instead of the reduced matrix elements of the multipole operators they contain those of the spin-multipole operators.

We present a general scheme of calculations within the quasiparticle-phonon nuclear model. The wave function of an odd-A spherical nucleus is

$$\psi_0 = \psi_{0g.s.} \otimes \psi_{e.m.},$$

where $\psi_{0g.s.}$ is the ground state wave function of a doubly even nucleus.

Then, we calculate an average value of $H_{csv}$ over (16), and using the variational principle. As a result, we find the secular equation for the energies $E_i$ of the states $\psi_i$

$$\mathcal{F}(\gamma_i) = \mathcal{F}(\gamma_0) - \sum_{ij} \Gamma(Jj \lambda i) D_j^{2i}(J^0) = 0,$$

where

$$\Gamma(Jj \lambda i) = \langle \psi_0 | H_{csv}^{(*)} | \psi_{0g.s.} \rangle |a_{j,i}^{+} Q_{\lambda \mu i}^{+} \rangle |a_{j,i}^{+} Q_{\lambda \mu i}^{+} \rangle | \psi_0 \rangle.$$

and the equation for the $D_j^{2i}$ functions, which is given, for instance, in refs. /7, 8/.

The excited state wave function of a doubly even spherical nucleus is written as

$$\psi_i = \left\{ \sum_i R_i(J^0) Q_{\lambda \mu i}^{+} + \sum_{\lambda_1 \lambda_2} D_{\lambda_1 \lambda_2}^{2i}(J^0) [Q_{\lambda_1 \lambda_2}^{+} Q_{\lambda_1 \lambda_2}^{+}] | \psi_0 \rangle \right\}_{i}.$$
2 - FRAGMENTATION OF ONE-QUASIPARTICLE STATES

Many papers /14-18/ have been devoted to the experimental and theoretical study of the fragmentation of one-quasiparticle states. Earlier, the fragmentation of one-quasiparticle states has been calculated in studying the low-lying states (see, for instance, ref./1/). The fragmentation of one-quasiparticle states far from the Fermi energy has been calculated in ref./19/ in deformed nuclei within the quasiparticle-phonon nuclear model. The first microscopic calculations of the fragmentation of particle states in spherical nuclei have been aimed at the study of $s$ - and $p$ -wave neutron strength functions. These calculations were performed with the excited state wave function in the form of (16). The fragmentation of deep hole states has been calculated in ref./21/, where the coupling with the first quadrupole and octupole phonons has been taken into account explicitly whereas with other configurations by introducing an effective width, and in ref. /11/ within the nuclear field theory. The fragmentation of deep hole states has been calculated in ref./22/ with the wave function (16), in which $F=O$, and a large number of phonons. The fragmentation of one-quasiparticle states in many spherical nuclei has been calculated in refs./23-26/ with the wave function (16) taking into account the terms $\sim x^+Q^+Q^+$ with a large one-phonon basis.

The fragmentation of low-lying hole states has first been measured experimentally /27/ in 1960. The one-nucleon transfer reactions turned out to be effective for obtaining the spectroscopic factors which characterise the strength distribution of high-lying particle and deep hole states in spherical nuclei (see refs./14,23-39/). The polarized particle beams turned out to be very useful for the assignment to the spins of one-quasiparticle fragmented states /40,41/.

Now we give the formulae for calculating the fragmentation of hole and particle, neutron and proton states in odd-A spherical nuclei. The strength function describing the fragmentation of the subshell $j$, has the form

$$C_j^2(\hbar) = \sum_j C_{j0}^2 \int \frac{\Delta}{2\pi} \left( \hbar - \epsilon_{j0} \right)^2 + \Delta^2 / 2 = \pi^{-1} \text{Im} \mathcal{F}^{-1}(\hbar + i\Delta / 2),$$

(19)

where $C_{j0}$ enters into (16), $\mathcal{F}(\hbar)$ is defined by (17), and $\Delta$ is the averaging parameter. The spectroscopic factor, summed over the energy interval $\Delta \bar{E}$, for the excitation of the hole state $j$ in the one-nucleon transfer reaction is

$$C^2S = (2j+1) \mathcal{A}_j^2 \int_{\Delta \bar{E}} C_j^2(\hbar) d\hbar.$$

(20)

In calculating the spectroscopic factor for a particle state $U^2_p$ is substituted by $U^2_p$. The fragmentation of one-quasiparticle states is determined by the spectroscopic factors $C^2S$, centroid energies and widths

$$\bar{E}_x = \frac{\int_{0}^{E_m} C_j^2(\hbar) d\hbar}{\int_{0}^{E_m} C^2(\hbar) d\hbar} - \hbar_{gs},$$

$$\bar{E}_x = \frac{\int_{0}^{E_m} C_j^2(\hbar) d\hbar}{\int_{0}^{E_m} C^2(\hbar) d\hbar} - \hbar_{gs},$$

(21)

$$\Gamma = \frac{\int_{\Delta \bar{E}} C^2(\hbar) d\hbar}{\int_{0}^{E_m} C^2(\hbar) d\hbar}$$

(22)

where $\hbar_{gs}$ is the ground state energy of an odd-A nucleus. These values are compared with the corresponding experimental data. The calculations within the quasiparticle-phonon nuclear model, which are presented below, have been performed with the wave function (16). The $(2-5) \cdot 10^2$ terms of the type $\alpha^+Q^+$ and the $(1-5) \cdot 10^4$ terms of the type $\alpha^+Q^+Q^+$ were included into the wave function (16). In comparison with ref./11/ and other papers we take into account the terms $\sim \alpha^+Q^+Q^+$,
playing an important role in some cases in spherical nuclei.

Previously, the distribution of the single-particle strength for the states far from the Fermi energy has been thought of to be of the Breit-Wigner form with the center corresponding to the quasiparticle energy. The first calculations/19,20/ of the fragmentation of one-quasiparticle states have shown that the distribution of their strength has a complex form, which differs from the Breit-Wigner one. Typical examples of the fragmentation of neutron hole states in $^{114}$Sn, calculated in ref./23/, and of proton particle states in $^{144}$Eu, calculated in ref./25/, are given in fig. 1. More than 90% of strength of these states is exhausted up to an energy of 14 MeV. It is seen from fig. 1 that the strength distribution has several peaks. They are due to large matrix elements corresponding to the poles of the secular equation. Consider the energy dependence of the width $\gamma'$. It has been assumed in some papers (see, for instance ref./19/) that $\gamma' \sim E_x^{2}$. The available experimental and theoretical data on the fragmentation of one-quasiparticle states did not reveal any regularities of the dependence of $\gamma'$ on $E_x$. It is obvious that with increasing $E_x$ the region of location of the most part of the subshell strength increases. The width $\gamma'$, determined by formula (22), specifies the strength distribution if it is of the Breit-Wigner form or has one maximum. If the strength distribution has several maxima, as in fig. 1, the $\gamma'$-values turn out to be less informative. For instance, in $^{89}$Zr more than 90% of the $^{1g_{9/2}}$ subshell strength is concentrated in the ground state. Taking into account the rest part of the $^{1g_{9/2}}$ strength lying above and using formula (22), one gets the value of $\gamma'$ close to that of $\gamma'$ in $^{119}$Sn, in which $E_x$ = 6.5 MeV and the strength is distributed over many levels in the interval 4–5 MeV.

The calculations within the quasiparticle-phonon nuclear model describe well the fragmentation of one-quasiparticle states in spherical nuclei. The calculations have no free parameters. Since there is no fit of the experimental data, one cannot expect a complete agreement

Fig. 1. Fragmentation of neutron hole states in $^{115}$Sn and of proton particle states in $^{145}$Eu.
of theory with experiment. A typical agreement of the calculations with experiment is shown in fig. 2. Almost such an agreement of the calculations/24/ with the experimental data/42/ is observed for the fragmentation of the proton subshell $1g_{9/2}$ in $^{207}$Tl. The comparison of the spectroscopic factors $C^2S$, summed over $\Delta E$, is shown in table 1. The experimental and calculated values of $C^2S$ for $^{57}$Co are obtained by Pugach, Vagner, Vdovin et al. For $^{89}$Zr the experimental data are taken from ref./33/ and the calculations from ref./26/ for $^{115}$Sn from refs./40/ and /23/, respectively. It is seen from the table that the calculated values of $C^2S$ for $1d_{5/2}$ in $^{57}$Co and for $2p$ in $^{115}$Sn agree fairly well with the experimental data. The $1f_{7/2}$ strength distribution in $^{89}$Zr is an example of a discrepancy between theory and experiment. The description can be somewhat improved by fitting the parameters of the Saxon-Woods potential.

The available experimental information on the fragmentation of one-quasiparticle states in spherical nuclei is scarce. The calculations are restricted to several states in a few nuclei. Therefore, one cannot make conclusions about the regularities of the fragmentation. We can only make the following remarks about the fragmentation of one-quasiparticle states in spherical nuclei.

i) About (70-90)\% of the single-particle strength is concentrated in each ground or low-lying state of an odd-A nucleus. The fragmentation depends on the one-quasiparticle energy. With increasing energy of the one-quasiparticle state the fragmentation grows, and the single-particle strength is distributed over several levels.

ii) The form of the one-quasiparticle strength distribution differs from the Breit-Wigner one, and the distribution has several peaks.

iii) The fragmentation depends on quantum numbers of the single-particle state and on the properties of collective vibrational states of the relevant doubly even nucleus. Under other equal conditions, the subshells with small $j$ are fragmented stronger than the subshell with large $j$. This can be seen from figs. 1 and 2. The fragmentation of one-quasiparticle states in the nuclei with closed shells is weaker.

It is interesting to study how the fragmentation of the subshell strength changes as one passes from spherical to deformed nuclei. Due to the deformation, each subshell is splitted, and thus the obtained fragmentation is large. A further fragmentation is caused by the quasiparticle-phonon interaction. Now, we consider the fragmentation of the neutron $1h_{11/2}$ subshell in the spherical $^{143}$Sm and deformed $^{151}$Sm nucleus. The calculations have been performed by Vdovin, Maiev, Nguyen Din Vin and Soloviev with the parameters of the Saxon-Woods potential given in ref./2/. in $^{151}$Sm with the phonons belonging to $^{152}$Sm at the deformation $\beta_2 = 0.3$, $\beta_4 = 0.04$, and in $^{145}$Sm with the phonons belonging to $^{146}$Sm, $\beta_2 = \beta_4 = 0$. Due to the deformation the $1h_{11/2}$ strength is distributed as follows: 90% is concentrated in the single-particle states $\pi_2 [505]$, $\pi_2 [514]$, $\pi_2 [523]$, $\delta_2 [532]$, $\delta_2 [541]$, $\pi_2 [550]$, 6% in the particle states near the Fermi energy, and 4% in the particle states in...
Table 1. Sums of spectroscopic factors in the energy intervals $\Delta E$

<table>
<thead>
<tr>
<th>Nuclei $n l_j$</th>
<th>$\Delta E$, MeV</th>
<th>$c^2 S_{\text{exp.}}$</th>
<th>$c^2 S_{\text{calc.}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$57^{\text{Co}}$</td>
<td>4.11-5.35</td>
<td>0.46</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>5.35-7.06</td>
<td>0.54</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>7.06-9.16</td>
<td>1.12</td>
<td>1.65</td>
</tr>
<tr>
<td></td>
<td>9.16-11.18</td>
<td>1.72</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>11.18-13.24</td>
<td>0.83</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>13.24-15.30</td>
<td>0.81</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>15.30-19.42</td>
<td>0.37</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>4.11-19.42</td>
<td>5.85</td>
<td>5.42</td>
</tr>
<tr>
<td>$89^{\text{Zr}}$</td>
<td>3.5-7.7</td>
<td>3.6</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>7.0-12.3</td>
<td>1.3</td>
<td>5.8</td>
</tr>
<tr>
<td>$115^{\text{Sm}}$</td>
<td>3.5-12.3</td>
<td>4.9</td>
<td>6.8</td>
</tr>
</tbody>
</table>

The range of the neutron binding energy. Due to the deformation 90% of the $\frac{1}{2}^{+}$ strength is fragmented in the energy interval 7 MeV. The fragmentation of $\frac{1}{2}^{+}$ (see formula (19)), calculated with the wave function (16) with $\frac{\hbar}{2\pi} = 0.5$ MeV taking into account the quasiparticle-phonon interaction, is exhibited in fig. 3. For $151^{\text{Sm}}$ the energies of hole states are plotted in the right scale and of particle states in the left scale. The single-particle states $\frac{1}{2}^{+}$ [505] and $\frac{3}{2}^{+}$ [514] (peaks are at an energy of 1 and 2.5 MeV) are not practically fragmented. The main part of other hole state strength is distributed in the interval 1-3 MeV. Note, that a very small portion of the $\frac{1}{2}^{+}$ strength is in the states $\frac{3}{2}^{+}$ [523] and $\frac{5}{2}^{+}$ [521] observed experimentally in $151,153^{\text{Sm}}$. It is seen from fig. 3 that the main part of the $\frac{1}{2}^{+}$ strength in $145^{\text{Sm}}$ is fragmented in the energy interval 2-3 MeV and the rest part (38%) at higher energies. These calculations are aimed at demonstrating the general regularities of the fragmentation of subshells in spherical and deformed nuclei rather than at elucidating the experimental data/38,39/ on the $\frac{1}{2}^{+}$ fragmentation in the Sm isotopes. Based on the investigation one may conclude that: first, the single-particle strength in a spherical nucleus is fragmented stronger than in the deformed nucleus (due to a large collectivity of vibrational states), and second, the strength of each subshell in a deformed nucleus is fragmented stronger than in a spherical nucleus (due to a stable deformation).
The experimental and theoretical investigations of the fragmentation of one-quasiparticle states show that the strength is concentrated in a certain energy interval and the strength distribution contains peaks up to an energy of 15-20 MeV. This fact is nontrivial, since previously it was assumed that above the neutron binding energy the statistical regularities predominate, whereas nonstatistical effects are very small. A further study of the fragmentation of one-quasiparticle states requires an extension of the region of experimental investigation by increasing the number of nuclei studied and by going towards higher excitation energies. It would be interesting to study the fragmentation of the deepest hole states via high resolution \((e,e'p)\) and \((p,2p)\) experiments.

3 - FRAGMENTATION OF TWO-QUASIPARTICLE STATES

The experimental information on the fragmentation of two-quasiparticle states in spherical nuclei is gained from the spectroscopic factors of the one-nucleon transfer reactions and from the two-nucleon transfer reactions /14,43,44/. In recent years the experimental data from the \((p,t)\) and \((\alpha,^6\text{He})\) reactions have been enriched /45-47/.

Now we give the formulae for describing the fragmentation of two-quasiparticle states within the quasiparticle-phonon nuclear model, which have been obtained in ref./48/. In the RPA the two-quasiparticle component \(d_1d_2\) in the state \(\alpha_{\text{sh}}\) with spin \(J=\lambda\) is determined by the phonon amplitude \(\gamma^\lambda_{d_1d_2}\). In the calculations of the fragmentation of one-phonon states with the wave function \(18\) and model Hamiltonian \(9\), we get

\[
\Phi_{d_1d_2}^\lambda (J_1, J_2) = \frac{1}{2} \left| \sum_i R_i^\lambda (J) \gamma_{d_1d_2}^\lambda \right|^2,
\]

an explicit form of the strength function is given in ref./48/. The wave function of an odd-\(A\) target-nucleus is taken in the form

\[
U_{\nu_0} (d_0 m_0) = C_{d_0\nu_0} \alpha_{d_0 m_0}^* U_{\nu_0},
\]

and the wave function of the final state with spin \(J_f\) is taken in the form \(18\). The strength function for the spectroscopic factor of the nucleon pickup reaction on the subshell \(j\) is

\[
S_{d_0}(J_f; \lambda) = C_{d_0\nu_0}^2 \Phi_{d_0}^\lambda (J_f; \lambda).
\]

Summing over \(J_f\), which form the states \(j\) and \(j_0\), we get

\[
S_{d_0}(\lambda) = \frac{1}{J_f+1} \sum_{J_f} S_{d_0}(J_f; \lambda), \quad \Phi_{d_0}(\lambda) = \sum_{J_f} \frac{2J_f+1}{J_f} \Phi_{d_0}(J_f, \lambda).
\]

The results of calculation of the fragmentation of two-quasiparticle states have been compared with the experimental data for some spherical nuclei in ref./48/. In this paper we shall discuss the new results. The fragmentation of the "valence particle-hole" states is studied via the pickup reaction. We shall consider the spectroscopic factors of the "valence particle-hole" states in \(^{206}\text{Pb}\), the experimental data for which have been obtained in ref./43/ from the \(^{207}\text{Pb} (\alpha,^6\text{He})^{206}\text{Pb}\) reaction. The experimental and calculated spectroscopic factors summed in the energy intervals \(\Delta E\) are given in table 2. It is seen from the table that the calculations describe fairly well the fragmentation of two-quasiparticle states, though provide a somewhat larger concentration of strength at low energies. According to ref./43/ 35% of the \([3p_{1/2}, 1h_{11/2}]\) strength is concentrated in the interval \((7.4-10.6)\) MeV, the calculations give a value two times larger /49/. It is of interest to find out where the rest part of this strength is. The calculated
Table 2. Distribution of the two-quasiparticle strength in 206Pb

<table>
<thead>
<tr>
<th>{i\d_0}</th>
<th>\Delta E, MeV</th>
<th>\Phi_{i\d_0}(E) dB</th>
<th>\exp/43/</th>
<th>calc/49/</th>
</tr>
</thead>
<tbody>
<tr>
<td>{2f_7/2,3p_1/2}</td>
<td>0 -4.33</td>
<td>6.27</td>
<td>6.2</td>
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<td>1.3</td>
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<td>{1f_5/2,3p_1/2}</td>
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<td>6.82</td>
<td>7.5</td>
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<td>8.05</td>
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<tr>
<td>{9.29-10.59}</td>
<td>1.35</td>
<td>1.0</td>
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<tr>
<td>{7.4 -10.6}</td>
<td>4.04</td>
<td>8.3</td>
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structures at (7-9) MeV. According to the calculations made by Voronov, these structures are caused by the fragmentation of two-quasiparticle states. Thus, a wide structure with \(E_X = 8.3\) MeV is observed in \(^{94}\text{Mo}\). The results of calculation of four two-quasiparticle states of the type of "valence particle-hole" and "hole-hole" are presented in fig. 5. It is seen from this figure that the structure around 8.3 MeV may be due to the excitation of the configurations \(\{2d_{5/2}, 1f_{5/2}\}\) and \(\{1g_{9/2}, 1g_{9/2}\}\). The nature of this structure and the strength distribution of the \(\{2d_{5/2}, 1f_{5/2}\}\) state can be explained by measuring the \(^{95}\text{Mo}(pd)^{94}\text{Mo}\) reaction cross section.

In the two-nucleon transfer reactions one can observe resonance-like structures, which can be compared with the calculated fragmentation of two-quasiparticle states. In ref./45/ a structure at the energy 8.0 MeV with the width \(\Gamma = 2.20 \pm 0.15\) MeV has been observed in the \(^{116}\text{Sn}(\alpha, \alpha')^{114}\text{Sn}\) reaction. The analysis of the angular distribution indicates an excitation of the states with \(J^\pi = 6^+\) with a possible admixture of states with \(J^\pi = 8^+\). The fragmentation of the two-quasiparticle strength with \(J^\pi = 6^+\) and \(8^+\) in \(^{114}\text{Sn}\), calculated by Voronov/50/, is shown in fig. 4. It follows from the comparison of fig. 4 with the experimental data that the observed at 8 MeV structure in \(^{114}\text{Sn}\) may be due to the fragmentation of the two-quasiparticle states \(\{1g_{9/2}, 1g_{9/2}\}\) with \(J^\pi = 8^+\) and \(6^+\) and \(\{1g_{9/2}, 1d_{5/2}\}\) with \(J^\pi = 6^+\). In the \((p, t)\) reaction cross sections measured in ref./46/ many nuclei contain wide

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Fig. 4 - Strength functions of two-quasiparticle neutron states in \(^{114}\text{Sn}\)

Fig. 5 - Strength functions of two-quasiparticle neutron states in \(^{94}\text{Mo}\)
In recent years the first attempts have been made to study the fragmentation of two-quasiparticle states in spherical nuclei. Many investigations of the one- and two-nucleon transfer reactions should be made for a consistent study of the fragmentation of two-quasiparticle states in spherical and deformed nuclei.

4 - FRAGMENTATION OF CHARGE-EXCHANGE COLLECTIVE STATES

The fragmentation of single-particle and collective vibrational motions is mainly due to the quasiparticle-phonon interaction and is described in the framework of the same model /2,7-11,18/. We shall demonstrate how the fragmentation of charge-exchange collective states, which is intensively studied in recent years/51,52/, is described within the quasiparticle-phonon nuclear model.

In studying the giant multipole resonances in deformed nuclei, it has been shown /2/ that the RPA-calculations give correct values of their widths. They are slightly influenced by the $2p-2h$ configurations/53/. The reason is that the fragmentation of the single-particle spherical states due to the deformation is stronger than that due to the quasiparticle-phonon interaction. Therefore, we calculate in the RPA the $(\rho,n)$ and $(n,\rho)$ strength functions. The secular equations and strength functions of the $(\rho,n)$ and $(n,\rho)$ reactions for the excitation of charge-exchange states on doubly even deformed nuclei are given in ref./54/. The strength functions for the excited $1^+$ states including the Gamow-Teller (GT) resonance on $^{238}$U calculated with $\alpha_{E2}^{GT} = 17/\Lambda$ MeV, $\Delta = 0.5$ MeV and other constants of the model from ref. /2/ are shown in fig. 6. In the deformed nuclei the GT resonance consists of the components with $I^=K^=1^{+0}$ and $1^{+1}$, which are splitted by 0.6 MeV in $^{166}$Er and by 0.3 MeV in $^{238}$U. In the GT strength distribution one can separate the low-energy part, the region of maximum around $E_X = 20$ MeV and the high-energy part with contributions 27%, 56% and 16%, respectively, about 1% of strength is at 50 MeV. Almost the same distribution of strength is observed in all calculated nuclei. The centroid energy of the low-energy part in $^{238}$U is 14 MeV, it is below the maximum of the GT resonance by 6 MeV. The ratio $B(\rho,n)/B(n,\rho)$ is equal to 20 for the rare-earth nuclei and to 57 for $^{238}$U. The model independent sum rule is exhausted by (98-99)%.

The strength functions for the spin-dipole charge-exchange resonances with $\lambda^=0^-, 1^-, 2^-$ on deformed nuclei have been calculated by Solo- viev and Sushkov. The strength function of the $\langle \rho,n \rangle$ reaction on $^{166}$Er, calculated according to ref./51/ with $\alpha_{E2}^{GT} = 0.75 4\pi:\epsilon^2 > \alpha_{E2}^{GT}$, $\alpha_{E2}^{GT} = 17/\Lambda$ MeV, and $\Delta = 0.5$ MeV, is shown in fig. 7. The centroid energy is $E_X = 24$ MeV, $E_x$ for different components is: 28 MeV for $0^-$, 25 MeV for $1^-$ and 21 MeV for $2^-$. The spin-dipole strength is distributed over the energy intervals as follows: 6% up to 11 MeV, 20% within 11-20 MeV, 30% within 20-25 MeV, 41% within 25-33 MeV and 3% above 33 MeV. According to the calculations 70% of the spin-dipole strength is concentrated in the interval of 15 MeV. The energy region of location of the spin-dipole charge-exchange strength in deformed nuclei is quite large. It exceeds considerably the region of concentration of the GT strength.

As is known /9,18/, the widths of the giant resonances in spherical nuclei are caused by the fragmentation of one-phonon states due to the coupling with the $2p-2h$ configurations. While studying the charge-exchange resonances in spherical nuclei, one should calculate the fragmentation of $np$ phonons. The method of including $np$ phonons into the quasiparticle-phonon nuclear model has been developed in ref./55/. The equations for describing the fragmentation of $np$ phonons in spherical nuclei were obtained in a general form. The calculations were performed with the wave function in the form

$$\psi_{JM}^{(\lambda)} = \{ \sum_i R_i^{(\lambda)} Q^{\lambda_i}_{\lambda M} + \sum_{\lambda_1,\lambda_2} P_{\lambda_1,\lambda_2}^{(\lambda)} (\sigma) [\lambda_1,\lambda_2,\lambda M, J M] \} \psi_0.$$ (26)
In ref./55/ a transition has been made to the system of approximate equations, which is now used for the description of the \((p,n)\) and \((n,p)\) strength functions for excitation of the charge-exchange states. As is known, less than 50\% of the GT resonance strength is found experimentally in spherical nuclei. It is assumed in some papers (see refs./51,52,56,57/) that the missing of total GT strength is caused by an admixture of excitations of the \(\Delta\) - isobar - nucleon hole to the proton-particle - neutron-hole GT states. However, one did not succeed in explaining such a large missing of the GT strength. In ref./57/ the influence of the \(2p-2h\) configurations on the GT resonance in \(208\text{Pb}\) has been calculated by taking into account the first vibrational \(3^-\) and \(2^+\) states. However, a very small number of the \(2p-2h\) configurations was used in these calculations. Therefore, it is reasonable to calculate the fragmentation of \(np\) phonons with a large number of two-phonon states in the wave function (16) and thus to find out to what extent the fragmentation is responsible for the missing of GT strength. Such calculations have been performed by Kuzmin and Soloviev. The results of calculations of the \((p,n)\) reaction on \(^{124}\text{Te}\) with excitation of the GT resonance are shown in fig. 8. A rather large phonon basis with the wave function (26) has been used in the calculations. In the RPA calculations the GT strength is distributed as follows: 27\% up to 11 MeV, 58\% within 13-16 MeV, 9\% within 16-20 MeV, and 6\% above 20 MeV. If the quasiparticle-phonon interaction is taken into account the maximum almost does not shift. The GT strength is distributed as follows: 30\% up to 11 MeV, 30\% within 13-16 MeV, 20\% within 16-20 MeV and 20\% above 20 MeV. Thus, the GT strength decreases from 60\% to 30\% around maximum. Up to an energy of 20 MeV 80\% of the GT strength is exhausted.
These calculations are continued, a large space of one-phonon states is taken into account, and the GT and spin-dipole resonance is calculated for several nuclei including the nuclei with unfilled shells. A further investigation of the fragmentation of charge-exchange resonances in spherical and deformed nuclei is of great interest.

CONCLUSION

The study of the fragmentation of one-, two-quasiparticle and one-phonon states is an important stage in investigating the complication of the nuclear state structure with increasing excitation energy. The fragmentation of isobar-analog states is of great interest, the experimental information on it is very scarce/14/. One can easily calculate the fragmentation of isobar-analog states under the assumption that isospin is a good quantum number. The fragmentation of the quasiparticle plus phonon states is calculated within the quasiparticle-phonon nuclear model. The fragmentation of these states influences, for instance, the partial strength functions for \( \gamma \)-transitions to the low-lying states in spherical nuclei studied in ref. /8/. The distribution of the \( ^{19}g_{\frac{9}{2}} \otimes 2^+_1 \) strength with \( \mathcal{J}^\pi = \frac{9}{2}^+ \) in \(^{119}\text{Sn}\), calculated in ref. /16/, is shown in fig. 9. It is seen from the figure that a larger part of this strength is localized in the energy interval 5 MeV. The study of the fragmentation of the quasiparticle plus phonon states is of great interest. How are many-quasiparticle and many-phonon states fragmented at intermediate and high excitation energies? This question has been raised several years ago/58,59/, but still there is no experimental information on the fragmentation of many-quasiparticle states.

In conclusion it is worth mentioning that one should not expect a complete agreement of theory with experiment while describing nuclear states at intermediate and high excitation energies. The fit by changing the parameters is not very valuable. The nuclear many-body problem is stated so that one should take into account the difference of the properties of the system of 238 nucleons from the system of 240 or 235 or 190 nucleons. This problem is highly difficult. There are large difficulties in solving the three- and four-nucleon problem. Various nuclear models provide a fairly good description of the fragmentation of one-quasiparticle, two-quasiparticle and one-phonon states, and these models are to be further improved.

**Fig. 9** - Strength distribution of the \( ^{19}g_{\frac{9}{2}} \otimes 2^+_1 \) state in \(^{119}\text{Sn}\).
References

28. STOYANOV Ch., VDOVIN A.I., Preprint JINR E4-83-106, Dubna, 1983.
42. LANGEVIN-JOLIOT H., et al., Contribution to this symposium, 2-7.
47. GUILLOT J. et al., Contribution to this symposium, 2-7.
49. VORONOV V.V., ZHURAVLEV I.P., JINR E4-82-512, Dubna, 1982.
50. VORONOV V.V., Contribution to this symposium, 2-16.
52. GAARDE C., LARSEN J.S., RAPPOPORT J., Int. Conf. on Spin Excitations in Nuclei, 1982.
55. SOLOVIEV V.G., Preprint JINR E4-83-272, Dubna, 1983.
60. SoIoviev V.G. Particles and Nuclei, 3 (1972) 770.