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FLAVOR MIXING AND THE MASSES OF LEPTONS AND QUARKS

H. Fritzsch

Universität München, MPI für Physik, Poehringer Ring 6, D-8000 München, F.R.G.

All phenomena in particle physics presently known can be described within the so-called standard model, based on QCD - the gauge theory of strong interaction physics - and on the SU(2) x U(1) theory - the gauge theory of the weak and electromagnetic interactions. No doubt, the introduction of the standard model of leptons and quarks is a great achievement; all interactions observed in nature with the exception of gravity are described successfully. Yet it is surprising that the theory gives very little insight into the problem of masses. In the standard model the lepton and quark masses are introduced by hand. No explanation of these masses or of mass differences is given. The masses of the weak bosons W and Z are introduced as functions of the vacuum expectation value of a scalar Higgs field \( \phi \):

\[ M_W, Z \sim e \cdot \langle \phi \rangle \cdot (e = \sqrt{4 \pi \alpha}). \]

Phenomenologically this vacuum expectation value is determined by the Fermi constant, i.e. by the strength of the weak interaction amplitudes at low energies.

During the last ten years an important progress has been made in understanding the mass scales of strong interaction physics. The typical mass scale of phenomena in strong interaction physics is of the order of 1 GeV. This mass scale is a reflection of the confinement phenomenon in QCD. The latter implies that the quarks inside a hadron are confined in a finite region of space, whose extension is described by a confinement parameter \( \Lambda_c \). All hadronic masses can be calculated, at least in principle, in terms of \( \Lambda_c \). Mass ratios like \( M_\pi / M_\rho \) are independent of \( \Lambda_c \) and can be computed. Thus the theory of quantum chromodynamics allows us for the first time to calculate the masses of particles.

Of course, QCD does not give any information about the quark masses. The latter are introduced as free parameters\(^1\), i.e. they play a rôle similar to the one played by the electron - or muon mass in atomic physics or in quantum electrodynamics.

The origin of the electron - or quark masses is still unknown. At the present time it is not possible to reach a deep understanding of the properties of the lepton-quark spectrum. However time has come to speculate about certain properties, for example about possible connections between the weak mixing angles and the quark masses.

The leptons and quarks can be classified in three families. Each family has the charge structure

\[
\begin{pmatrix}
0 & 2/3 & 2/3 \\
-1 & -1/3 & -1/3
\end{pmatrix}
\]

i.e. it includes a neutrino, a lepton of charge -1 (in units of e), a quark of charge 2/3 and a quark of charge (-1/3) (the three colors are denoted explicitly). The masses of the charged fermions are as follows (in MeV):

- Neutrino: ~0 MeV
- Electron: 511 MeV
- Muon: 105 MeV
- Down quark: 4.2 MeV
- Up quark: 3.3 MeV
- Charm quark: 1.5 GeV

These mass differences are as yet unexplained. However, it is interesting to note that the down and charm quark masses are of the same order of magnitude as the radiative corrections of QCD, which is another indication that the theory of quantum chromodynamics is on the right track.
We shall suppose that the t-quark exists. Its mass must be greater than about 22 GeV otherwise the tt ground state would have been observed in the e^+e^- annihilation experiments. The masses of the quarks are, of course, not measurable directly, but must be inferred from observed properties of the hadronic spectra. The mass values depend on the energy scale used for the determination of the quark masses. We follow ref. (1) and denote the quark masses at an energy scale of 1 GeV.

The eigenstates of the quark mass matrix do not coincide with the weak interaction eigenstates. It is customary to denote the weak interaction mixing by introducing mixtures of mass eigenstates for the charge -1/3 - quarks d', s', b':

\[
\begin{pmatrix}
  d' \\
s' \\
b'
\end{pmatrix} =
\begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
d \\
s \\
b
\end{pmatrix}
\]

where \( V_{ij} \) is a 3 x 3 unitary matrix.

In the formal limit \( m_d = m_s = m_b = 0 \) an arbitrary unitary transformation of the fields \((d,s,b)\) can be made without changing the physics. In particular the matrix \( V_{ij} \) can be diagonalized:

\[
(V_{ij}) \rightarrow \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}
\]

Thus the matrix \( V_{ij} \) has nontrivial matrix elements only if at least one of the masses \( m_d, m_s, m_b \) is different from zero (or analogously one of the masses \( m_u, m_c, m_t \)). Therefore one may conclude: the quark masses and the weak mixing angles are related. Any theory of these masses must at the same time give information about the weak mixing angles, and vice versa. One may even discuss the hypothesis that the weak mixing angles are functions of the quark masses. This situation arises provided the quark mass matrix obeys certain constraints, which are fulfilled as a consequence of underlying discrete or continuous symmetries (see e.g. ref. (3)).

Below I shall discuss some possible connections between the weak mixing angles and the quark masses. The observed pattern of the quark masses exhibits clearly a well-defined hierarchy. The lightest quarks \((u,d)\) are much lighter than the quarks \((c,s)\), which in turn are substantially lighter than \((t,b)\). Before employing this hierarchy for a specific purpose, let me mention a few facts about the weak mixing matrix \( V_{ij} \).
The mixing matrix $V_{ij}$ is usually written as a function of three mixing angles $\theta_i$ ($i = 1,2,3$) and a phase parameter $\delta$:

$$
(V_{ij}) = \begin{pmatrix}
    c_1 & s_1 c_3 & s_1 s_3 \\
    -s_1 c_2 & c_1 c_2 c_3 + s_2 s_3 e^{i \delta} & c_1 c_2 s_3 - s_2 c_3 e^{i \delta} \\
    -s_1 s_2 & c_1 s_2 c_3 - c_2 s_3 e^{i \delta} & c_1 s_2 s_3 + c_2 c_3 e^{i \delta}
\end{pmatrix}
$$

$(c_i = \cos \theta_i, \ s_i = \sin \theta_i)$.

In the limit $\theta_2 = \theta_3 = \delta = 0$ the mixing matrix reduces to the Cabibbo mixing matrix

$$
\begin{pmatrix}
    V_{ud} & V_{us} \\
    V_{cd} & V_{cs}
\end{pmatrix} = \begin{pmatrix}
    \cos \theta_1 & \sin \theta_1 \\
    -\sin \theta_1 & \cos \theta_1
\end{pmatrix}
$$

($\theta_1$: Cabibbo angle).

For small mixing angles the matrix $V$ can be written approximately as follows:

$$
V = \begin{pmatrix}
    1 & \theta_1 & 0 \\
    -\theta_1 & 1 & n \\
    0 & -n & 1
\end{pmatrix} = n = \theta_3 + \theta_2 e^{i \delta}.
$$

Since $m_u \gg m_b$, the b-quark decay can proceed only via the matrix element $V_{cb}$ and $V_{ub}$. In the limit $\theta_2 = \theta_3 = 0$ the b-quark would be stable.

Recently the lifetime of the b-flavored mesons (quark composition $\bar{u}_b$, $\bar{d}_b$) has been determined:

$$
\tau(b) = (12.0 \pm 4.5 \pm 3.0) \cdot 10^{-13} \text{s} \quad (5)
$$

$$
\tau(b) = (18 \pm 0.6 \pm 0.4) \cdot 10^{-12} \text{s} \quad (6)
$$

This lifetime is surprisingly long. If the matrix elements $V_{cb}$ ($V_{ub}$) were of the same order as the matrix element $V_{us}$ ($\approx 0.22$), one expects $\tau(b) < 10^{-13} \text{s}$. Thus $V_{ub}$ and $V_{cb}$ must be much less than $V_{us}$.

Furthermore it is known that the matrix element $V_{ub}$ is much less than $V_{cb}$:

$$
|V_{ub} / V_{cb}| < 0.20 \quad (7)
$$

Gathering all the informations about the weak matrix elements (see e.g. ref. (7)), one finds a mixing matrix as follows:

$$
(V_{ij}) = \begin{pmatrix}
    0.9723 & 0.9737 & 0.228 & 0.234 & 0.000 & 0.008 \\
    0.228 & 0.234 & 0.9704 & 0.9726 & 0.042 & 0.067 \\
    0.003 & 0.016 & 0.041 & 0.066 & 0.9979 & 0.9991
\end{pmatrix}
$$

[the complex phase parameter $\delta$ has been omitted].
It is worthwhile to remark that the submatrix
\[
\begin{pmatrix}
V_{cs} & V_{cb} \\
V_{ts} & V_{tb}
\end{pmatrix}
\]
is rather close to the unit matrix, i.e. nature seems to be close to the limit where the heavy quark doublet \((t,b)\) decouples from the other quarks. One is invited to speculate about the reason for this surprising phenomenon. Below we shall discuss two hypotheses about the quark mass matrix, which are related to this "decoupling phenomenon".

I. Decoupling Hypothesis
Suppose only the first two families would be present. We consider the limit \(m_u = m_d = 0\). In this case the masses of the quarks \(c\) and \(s\) are "infinitely" heavy compared to the \((u,d)\) masses, and one might expect that in this limit the physics of the \((u,d)\) system decouples entirely from the physics of the \((c,s)\) system. In other words: the Cabibbo angle should vanish as \(m_u, m_d \to 0\):
\[
\tan \theta_c = \frac{m_u}{m_d} \to 0. \quad (12)
\]

In ref. (3) a special mass matrix for the quarks was discussed, which led to the relation:
\[
\theta_c = \sqrt{\frac{m_d}{m_s}} - e^{i\beta} \sqrt{\frac{m_u}{m_c} |}
\]
where \(\beta\) is an (unknown) phase parameter. This relation fulfills the decoupling condition.

In the case of six quarks the decoupling hypothesis implies that all weak mixing angles vanish as \(m_u, m_d \to 0\) and \(m_c, m_s \to 0\):
\[
\tan \theta_i = \frac{m_{u,d}}{m_{c,s}} \to 0 \quad (14)
\]
i.e. the heavy quarks \((t,b)\) decouple, and the \(b\)-quark is stable.

II. Scaling Hypothesis
Suppose we start from a certain set of weak eigenstates denoted by the doublets
\[
\begin{pmatrix}
u_o & c_o & t_o \\
d_o & s_o & b_o
\end{pmatrix}
\]
In this basis the mass matrices will not be diagonal. Let \(\mathcal{M}(2/3)\) the mass matrix for \((u,c,t)\), and \(\mathcal{M}(-1/3)\) the mass matrix for \((d,s,b)\). We consider the special case where \(\mathcal{M}(2/3)\) and \(\mathcal{M}(-1/3)\) are proportional to each other:
\[
\mathcal{M}(2/3) = \lambda \mathcal{M}(-1/3). \quad (16)
\]
In that case a diagonalization of \(\mathcal{M}(2/3)\) leads automatically to a diagonalization of \(\mathcal{M}(-1/3)\). In the basis where the quarks of charge \((2/3)\) are mass eigenstates, the quarks of charge \((-1/3)\) will also be mass eigenstates. No weak interaction mixing results, and the quark masses fulfill the relations:
Suppose it is possible to express the weak mixing angles as functions of the quark masses. In this case we find it natural to adopt the following scaling hypothesis A:

If the masses of the quarks of charge \((2/3)\) are proportional to the masses of \((-1/3)\)-charged quarks, all weak mixing angles vanish.

The mixing angles can only be functions of the dimensionless ratios of quark masses \(m_i / m_j\) \((i,j = u,d,s,...)\). Whatever the dynamics underlying the weak mixing phenomenon may be, it will presumably be a dynamics involving only properties of the quarks of the same electric charge. In other words: the matrix elements of the quark mass matrix \(M(2/3)\) will depend only on properties of the quarks of charge \(2/3\), and will not depend on properties of the quarks of charge \((-1/3)\). We arrive at the second part B of the scaling hypothesis:

The weak mixing angles depend only on ratios of quark masses of the same electric charge:

\[
\theta_i = \theta_i \left( \frac{m_u}{m_c}, \frac{m_u}{m_t}, \frac{m_c}{m_t}, \frac{m_d}{m_s}, \frac{m_d}{m_b}, \frac{m_s}{m_b} \right) \tag{18}
\]

Below we discuss a few useful applications of both the decoupling and scaling hypothesis.

If we apply the scaling hypothesis to the special two-family-case discussed above (see ref. (3)), one finds:

\[
\theta_c \approx \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}} \tag{19}
\]

As required, the mixing angle depends only on the ratios of masses of quarks with the same electric charge. In the limit where \((u,c)\) and \((d,s)\) are proportional, the angle vanishes.

In reality one has \(m_d / m_s \gg m_u / m_c\), i.e. one is rather far from a proportionality of the quark masses. Taking the quark masses as indicated above one finds \(\theta_c \approx 0.17\). However, the values of the \(u,d,s\)-quark masses are rather uncertain. The measured value of \(\theta_c (\theta_c \approx 0.22)\) is reproduced by taking e.g. \(m_u = 12\) MeV, \(m_d = 15\) MeV, \(m_s = 150\) MeV. We interpret the relatively large value of the Cabibbo angle as a consequence of the large deviation from the proportionality case \(m_u / m_c = m_d / m_s\). If this were true, the \(u\)-quark mass would be about eight times larger than \(m_d\). In reality \(m_u\) is less than \(m_d\), implying that the neutron is heavier than the proton. Thus the neutron-proton mass difference and the relatively large value of \(\theta_c\) are intimately related to each other. Therefore the relatively large value of the Cabibbo angle has important consequences for low energy physics, especially for physics at the ILL. If \(\theta_c\) were much smaller than observed, the neutron would be lighter than the proton, and not the neutron, but the proton would decay via the \(\beta\)-decay process.

Let us consider the second and third generation. The \(u\)- and \(d\)- quark masses are very small compared to all other quark masses. The limit \(m_u = m_d = 0\) is expected to be a good approximation. Applying the decoupling condition, one expects the \((u,d)\) system to be unmixed in this limit, i.e. \(\theta_c = 0\). The only mixing which can be present is a mixing between \(s\) and \(b\), i.e. the six quark mixing matrix takes the form:

\[
m_u = \lambda m_d \\
m_c = \lambda m_s \\
m_t = \lambda m_b.
\]

\(\lambda\) is a dimensionless parameter. If \(\lambda \approx 1\), the mixing is large, and the quarks of different electric charge are strongly mixed. If \(\lambda \approx 0\), the mixing is small, and the quarks of different electric charge are weakly mixed.
The (2 × 2) submatrix can be written in terms of one angle \( \eta \):

\[
\begin{pmatrix}
V_{cs} & V_{cb} \\
V_{ts} & V_{tb}
\end{pmatrix} = \begin{pmatrix}
\cos \eta & \sin \eta \\
-\sin \eta & \cos \eta
\end{pmatrix}.
\] (21)

The experimental data on the b-quark life time imply that the angle \( \eta \) is very small: \( \eta \approx 0.05 \). Thus one must be rather close to the proportionality case \( m_t = m_c = m_b \), i.e. \( m_t = \frac{m_b \cdot m_c}{m_t} \approx 39 \text{ GeV} \). (Of course, this value is rather uncertain, mostly due to the uncertainty of \( m_s \), which may vary between 150 and 200 MeV - \( m_t \) varies correspondingly between 35 and 47 GeV.)

Applying the special mass matrix discussed in ref. (3), one finds

\[ n = \sqrt{\frac{m_s}{m_b}} - \sqrt{\frac{m_c}{m_t}} \] (22)

The constraint \( n \approx 0.05 \) given two possible solutions for \( m_t \). The actual value of \( m_t \) depends rather sensitively on \( m_s \). In the following table we give the solutions for various values of \( m_s \). For \( m_c \) and \( m_b \) (normalized at \( E = 1 \text{ GeV} \)) we adopt the values \( m_c(1 \text{ GeV}) = 1.35 \text{ GeV}, m_b(1 \text{ GeV}) = 5.3 \text{ GeV} \) (see ref. (1)).

<table>
<thead>
<tr>
<th>( m_s [\text{GeV}] )</th>
<th>0.12</th>
<th>0.15</th>
<th>0.18</th>
<th>0.20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_t(1 \text{ GeV}) ) - 1. Solution</td>
<td>34</td>
<td>28</td>
<td>24</td>
<td>22</td>
</tr>
<tr>
<td>( m_t(m_t) )</td>
<td>22</td>
<td>18</td>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>( m_t(1 \text{ GeV}) ) - 2. solution</td>
<td>135</td>
<td>96</td>
<td>75</td>
<td>65</td>
</tr>
</tbody>
</table>
| \( m_t(m_t) \)         | 71  | 52  | 42  | 37  | (23)

Besides the values of \( m_t(\mu = 1 \text{ GeV}) \) we have also denoted the value of \( m_t \) renormalized at \( E = m_t \). (This is the mass parameter to be used in estimations of the spectrum of \( \ell \)-particles). The extrapolation of \( m_t \) (1 GeV) to \( m_t(m_t) \) depends rather sensitively on the magnitude of \( \alpha_s \) \( (\alpha_s: \text{strong interaction coupling constant}) \) or respectively, on the scale parameter \( \Lambda \). We have used \( \Lambda = 100 \text{ MeV} \).

The first solution for \( m_t \) is compatible with the experimental constraints for \( m_s < 120 \text{ MeV} \), while the second solution gives relatively large values for \( m_t \) (above 40 GeV). We should like to stress the large deviation between \( m_t \) (1 GeV), obtained from our considerations about mixing angles, and \( m_t(m_t) \), which is supposed to be the quark mass value to be used in estimations of the spectrum of \( \ell \ell \)- and \( \ell \)-flavored particles.

Independent of whether the special mixing scheme discussed in ref. (3) is correct or not, our hypotheses imply:
If the determinant
\[
\begin{vmatrix}
m_c & m_t \\
m_s & m_b
\end{vmatrix}
\]
vanishes, the s-b-mixing vanishes as well. Nature seems to be not far from this case (see also ref. (10)). Nevertheless the t-quark mass depends still rather sensitively on the actual value of \(m_b\) and on the s-b-mixing angle, and, of course, on the special properties of the mixing mechanism. The t-quark mass can vary over a large range (see the table above).

Unfortunately these observations about the structure of the quark mass matrix discussed here give very little information about the lepton mass matrix. Of course, the leptonic weak mixing angle can be set to zero, if all neutrino masses vanish, i.e. we are dealing with three unmixed weak doublets:
\[
\left( \begin{array}{cccc}
\nu_e & \nu_\mu & \nu_\tau \\
e^- & \mu^- & \tau^-
\end{array} \right)
\]
However as soon as at least one of the neutrinos acquires a non-zero rest mass, weak mixing angles are expected to be present.

We could discuss both the decoupling and scaling hypotheses in the lepton case as well. However it seems that the situation here is different. The third charged lepton \(\tau\) is very heavy, but the associated neutrino \(\nu_\tau\) need not be much heavier than the other neutrinos \(\nu_\mu\) and \(\nu_\tau\). Thus one needs not be close to the limit
\[
m_{\nu_{\mu,\tau}} / m_\tau \rightarrow 0, \quad m_{e,\mu} / m_\tau \rightarrow 0,
\]
where the third doublet \((\nu_\tau, \tau)\) decouples from the others. It seems that the mechanism responsible for the generation of neutrino masses (if there is any), must differ qualitatively from the mechanism which generates the masses for the charged leptons and the quarks. The neutrino masses may well show no hierarchical pattern and be of similar order of magnitude, i.e.
\[
m_{\nu_e} \sim m_{\nu_\mu} \sim m_{\nu_\tau},
\]
in which case the mixing among the neutrinos is expected to be large. The search for non-zero neutrino masses (for example by searching for neutrino oscillations) remains an important task for the future.

In this talk I have discussed some ideas about the structure of the fermion mass matrix, which have been developed in close contact with experimental observations. I have not mentioned the yet unknown underlying physics, which leads to observed pattern of masses and mixing angles. Despite the regularities in this pattern discussed above, the quark and lepton mass spectrum is rather complex. Probably a solution to the problem of masses can only be found with a theory, which takes the idea of a substructure of leptons and quarks seriously\(^{11}\). The most interesting possibility is to relate the finite range of the weak interaction (of the order of \(10^{-16}\) cm) to the radii of leptons, quarks and weak bosons. In this case the lepton and quarks masses are (perhaps) just electromagnetic self energies; the observed pattern of masses and mixing angles can be interpreted that way\(^{12}\). In this case the physics phenomena at very low energy, e.g. neutrino oscillations, the neutron-proton mass difference etc. are intimately related to the substructure of the leptons and quarks, i.e. to the physics at energies of the order of 1 TeV or larger.

References and Footnotes

1. See e.g.: J. Gasser and H. Leutwyler, Physics Reports 87 C (1982) 77.
2. The present lower limit on \(m_t\) is about 22 GeV (G. Wolf, private communication).


7. See e.g.: G. Altarelli, Proceedings of the Int. NATO Summer Institute, Munich (1983).


10. For a different approach which fulfils both the decoupling and scaling hypotheses see: B. Stech, CERN - preprint TH 3644 (1983).

11. See e.g.: H. Fritzsch, Composite W-bosons and Their Dynamics, MPI Munich preprint (Nov. 1983).