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AN ESTIMATION OF THE RATES OF (TWO-NEUTRINO) DOUBLE BETA DECAY
AND RELATED PROCESSES

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Résumé - La désintégration double-bêta, la capture électronique avec émission de positrons et la capture électronique double (toujours avec émission de deux neutrinos) ont été calculées en supposant que l'état $1^{+}$ le plus bas du noyau intermédiaire est dominant. En utilisant les valeurs expérimentales pour les éléments de matrice de désintégration bêta simple et les dénominateurs d'énergie, on obtient des résultats qui ne dépendent pas des calculs de physique nucléaire. Les résultats pour la désintégration double bêta sont en accord avec les données géochimiques récentes. D'autres prédicitions sont à la portée des possibilités actuelles de détection.

Abstract – The (two-neutrino) double beta decay, the electron capture with positron emission and the double electron capture decay rates are estimated for several nuclides by assuming the dominance of the lowest $1^+$ state of the intermediate isobar. By using single-$\beta$ decay matrix elements and energy denominators as experimental inputs we get values independently of nuclear physics calculations. The results for double $\beta^-$ decay are in agreement with the recent geochemical values. Other predictions are within the present experimental sensitivity.

The recent years have witnessed a great deal of work /1/ in the field of double beta decay because of its relevance for the question of the, (Majorana) neutrino mass which might be inferred if the lepton number violating neutrinoless double beta decay would take place. From the theoretical point of view there exists alarming discrepancies /2/ between the conventional ($2\beta$)-decay theory and the geochemical measurements /3,4,5/. Although it is commonly accepted that the uncertainties in the computations arise mainly from the nuclear matrix elements it is troubling to note that recent improved calculations with the state-of-the-art shell model techniques have exacerbated these discrepancies /2,6/. Moreover, the proper treatment /2,7/ of the Coulomb correction amounts also to an increase of the absolute rates, i.e., each new refinement seems to go in the wrong direction. In the direct experiments there exists, so far, one "positive" result /8/, in the $^{82}$Se case, which is in contradiction with the corresponding geochemical result /6/. On the other hand, there are two geochemical experiments /3,4/ measuring the famous Tellurium ratio which give incompatible results of far reaching consequences /2,4,7/. Should the geochemical results /4,5/ be correct one is faced to the uneasy situation of having theoretical absolute rates considerably larger than the experimental ones and, which is worse, all the theoretical improvements make the rates even larger. One might give up some of the approximations usually made in computing the rates, like for instance, the closure approximation employed when performing the sum over the intermediate nuclear states /6,9/ or to look for alternative mechanisms in order to reduce the theoretical rates /10/. In Ref 6, devoted to the Te isotopes, the sum over the intermediate $1^+$ states is performed for excitation energies up to about 20 MeV, inserting theoretically computed single $\beta^-$ and $\beta^+$ decay amplitudes from the $0^+$ ground states to the various $1^+$ intermediate states, instead of using the ($2\beta$)-decay amplitudes conventionally summed up by closure. With the nuclear states described in a BCS pairing model they obtain even higher rates than those previously computed.
In Ref. 9 the sum over the intermediate states is saturated by keeping only the lowest-lying 1+ state, inserting the experimental matrix elements of the single \( \beta^- \) and \( \beta^+ \) decay amplitudes from the 1+ to the 0+ ground states. This very naive and drastic approach, as noted also in Ref. 6, already yields the observed 2\( \beta^- \)-decay rates in the 128Te, 130Te and 82Se cases, as given by the geochemical data. These results may cast some doubts about the reliability of the geochemical measurements or the closure approximation, or both.

In the present paper we briefly review our results /9/ and extend them to other two-neutrino processes related to the double beta decay, i.e. double positron emission, \( e^- \rightarrow e^+ \) conversion and double electron capture /11/.

In the conventional (2\( \beta \))^± decay theory the (two-neutrino) decay rate in 0+ → 0+ transitions is given by

\[
\Gamma(2\beta)^{\pm}_{2\nu} = (96\pi)^{-1} (G_F^2 \tan \theta)^4 \lambda^4 m^9 |M_{GT}|^2 |\tau|^2 \text{ years}^{-1}
\]

where

\[
\frac{1}{\tau} = m^{-3} \int m \ F^\pm(Z \pm 2, E_1) \left| \langle 0 | e | 1 \rangle \right|^2 \left| \langle 0 | e | 2 \rangle \right|^2 \sum_{m=0}^{T_0+2m-E_0} E^{2(E_0-E_2)4} \left| \langle 0 | e | 2 \rangle \right|^2
\]

\[
\times \left[ K_1 \left( E_{1^-} + E_{1^+} \right) \right]_1 \left( E_{1^-} + E_{1^+} \right)_1
\]

\[
K_1 = \left( E_{2^-} + E_{2^+} \right)_1 \left( E_{2^-} + E_{2^+} \right)_1
\]

\[
\lambda = 1.25 \text{ and } \Delta E = E_0 - E_1 \text{ is the difference between the energy of the initial state and some averaged energy of the intermediate states. The Gamow-Teller matrix element is given as usual by}
\]

\[
|\langle 0 | e | 1 \rangle|^2 = (61 \pi^2)^2 / (\lambda^2 G^2 \cos^2 \theta \tan^2 \theta \text{fm}^2) \text{ i.e. } |\langle 0 | e | 1 \rangle|^2 = (1.44 \times 10^8) / (10^{10} \text{fm})(4\pi \text{fm})^2
\]

where the indexes 1 and 2 refer to the two branches of decay from the intermediate state 1+ to the initial and final 0+ ground states. The resulting half-life is

\[
T_{1/2}(2\beta)^{\pm}_{2\nu} = (4.26 \times 10^{22}) / (|\langle 0 | e | 1 \rangle|^2 \text{ years})
\]

The phase space integrals \( T_0^{\pm} \) have been computed numerically, with the Coulomb correction factors \( F^\pm \) as suggested by Haxton et al /2/. Table 1 shows the half-life pre-

| Nuclide | (log ft)_1 | (log ft)_2 | \( \nu \) / sec | \( |\langle 0 | e | 1 \rangle|^2 \) | \( T_{1/2} \) (years) |
|---------|-------------|-------------|-----------|-----------------|----------------|
| Zn      | 4.7         | 5.1         | 0.30      | 2.28 × 10^2     | 2.77           | 6.74 × 10^23 |
| Se      | 4.7         | 5.5         | 2.68      | 9.08 × 10^3     | 5.30 × 10^3    | 8.85 × 10^30 |
| Mo      | 4.5         | 4.7         | -0.6      | 9.08 × 10^2     | 4.43 × 10^1    | 1.05 × 10^19 |
| Ru      | 4.4         | 4.5         | 1.29      | 1.81 × 10^1     | 3.45 × 10^1    | 6.82 × 10^21 |
| Pd      | 4.1         | 4.7         | 0.77      | 2.28 × 10^1     | 1.22 × 10^3    | 1.53 × 10^20 |
| Cd      | 4.0         | 4.5         | 1.88      | 4.55 × 10^1     | 6.20 × 10^2    | 1.51 × 10^24 |
| Cd      | 4.0         | 4.7         | -0.40     | 2.87 × 10^1     | 2.87 × 10^4    | 5.16 × 10^18 |
| Te      | 5.1         | 6.1         | 1.53      | 9.08 × 10^4     | 3.34           | 1.40 × 10^25 |
| Se      | 7.3         | 9.08 × 10^3 | 2.36 × 10^6 | 1.99 × 10^20 |
| Te      | 130         | 9.08 × 10^4 | 1.98 × 10^4 | 2.37 × 10^21 |
| Nuclide | $\log(\text{ft})_1$ | $\log(\text{ft})_2$ | $|\vec{\beta}_{\text{GT}}|^2$ | $\nu$/m | $T_0$/m | $I_1$ | $\tau_{1/2}$/yr | $T_0$/m | $I_2$ | $\tau_{1/2}$/yr | $T_0$/m | $I_3$ | $\tau_{1/2}$/yr |
|---------|-------------------|-------------------|-----------------|---------|--------|--------|----------------|--------|--------|----------------|--------|--------|----------------|
| $^{64}_{30}$Zn | 5.0 | 5.3 | 7.22x10^{-3} | 2.13 | - | - | - | 0.13 | 0.70x10^{-9} | 6.4x10^{34} | 2.11 | 0.315 | 7.3x10^{26} |
| $^{78}_{36}$Kr | 4.8 | 5.5 | 7.22x10^{-3} | 2.35 | 1.64 | 0.835x10^{-3} | 7.07x10^{27} | 3.61 | 12.0 | 2.2x10^{24} | 5.58 | 12.0 | 6.4x10^{24} |
| $^{106}_{48}$Cd | 4.9 | 4.4 | 7.22x10^{-3} | 1.39 | 1.44 | 0.206x10^{-3} | 2.86x10^{27} | 3.39 | 16.0 | 6.8x10^{22} | 5.34 | 21.0 | 6.5x10^{22} |
| $^{108}_{48}$Cd | 4.8 | 4.4 | 9.08x10^{-2} | 4.23 | - | - | - | - | - | - | 0.43 | 4.4x10^{-5} | 2.5x10^{28} |
| $^{112}_{50}$Sn | 4.7 | 4.1 | 2.28x10^{-1} | 2.29 | - | - | - | 1.70 | 0.59x10^{-1} | 5.2x10^{24} | 3.64 | 2.42 | 1.4x10^{23} |
| $^{120}_{52}$Te | 4.5 | 5.1 | 3.62x10^{-2} | 2.92 | - | - | - | 1.26 | 0.40x10^{-2} | 4.4x10^{26} | 3.19 | 0.93 | 1.8x10^{24} |
| $^{130}_{56}$Ba | 5.1 | 5.4 | 4.55x10^{-3} | 1.86 | 1.04 | 0.358x10^{-5} | 2.61x10^{30} | 2.97 | 3.7 | 3.0x10^{24} | 4.90 | 10.35 | 8.3x10^{23} |
| $^{132}_{56}$Ba | - | - | 4.55x10^{-3} | 3.50 | - | - | - | - | - | - | 1.48 | 2.2x10^{-2} | 3.9x10^{26} |
| $^{136}_{58}$Ce | 4.6 | 5.5 | 1.14x10^{-2} | 1.92 | 0.70 | 0.39x10^{-7} | 9.55x10^{31} | 2.62 | 1.4 | 2.8x10^{24} | 4.54 | 7.4 | 3.7x10^{23} |
| $^{138}_{58}$Ce | - | - | 1.14x10^{-2} | 3.04 | - | - | - | - | - | - | 1.23 | 1.3x10^{-2} | 2.1x10^{26} |
| $^{162}_{68}$Er | 5.0 | 5.6 | 3.61x10^{-3} | 1.56 | - | - | - | 1.50 | 0.44x10^{-1} | 1.7x10^{26} | 3.39 | 3.3 | 1.1x10^{24} |
dictions for (2β)\textsubscript{2}\textnu decay in some nuclides where the 1\textsuperscript{+} intermediate-state-dominance-hypothesis is applicable. As it was said above the results are in fair agreement with the geochemical data for the \textsuperscript{128}Te, \textsuperscript{130}Te and \textsuperscript{82}Se (2β)\textsuperscript{-} decay cases.

In much the same way the 1\textsuperscript{+} dominance approach can be applied to the (two neutrino) e\textsuperscript{-}\textrightarrow e\textsuperscript{+} conversion process (Kβ)\textsubscript{2}\textnu whose rate is given conventionally \cite{12}, by

\[ \Gamma_{(K\beta)_{2}\textnu} = (24\pi^5)^{-1}(G\cos\theta_c)^4|\Psi(0)|^2|\mathcal{M}_{GT}|^2 m^6 I_2 \text{ (years)}^{-1} \]

where

\[ I_2 = m^{-6} \int F^4(Z-2,F_{e^\pm}) dP_{e^\pm} \int T_{0+m-E_{e^\pm}}^0 E_{v1}^2 E_{v2}^2 dE_{v1}^2 (K_2^2 + L_2^2 + K_2 L_2) \]

\[ K_2 = (S+B_K-m+E_{v1})^{-1} + (S+E_{e^+}+E_{v1})^{-1}, \quad L_2 = (S+B_K-m+E_{v2})^{-1} + (S+E_{e^+}+E_{v2})^{-1} \]

\[ B_K \text{ is the binding energy of the K electron and S has the same meaning as above. By inserting the experimental N and S instead of MGT and S, one gets the e\textsuperscript{-}\textrightarrow e\textsuperscript{+} half-life from the expression} \]

\[ T_{1/2}^{(K\beta^+)}_{2\nu} = (3.53 \times 10^{-2})/(a^3 Z^3)^{1/2} \text{ years} \]

where the atomic wave function has been approximated, as usual, by |\Psi(0)|^2 = (aZ)^3 m^3 \pi^{-1}.

As far as the (two-neutrino) double electron capture (KK)\textsubscript{2}\textnu is concerned, the 1\textsuperscript{+} dominance hypothesis amounts, as in previous cases, to replace the experimental N and S instead of MGT and S in the conventional rate \cite{13}/

\[ \Gamma_{(KK)_{2}\textnu} = (4\pi^3)^{-1}(G\cos\theta_c)^8 |\Psi(0)|^4 |\mathcal{M}_{GT}|^2 I_3 \text{ (years)}^{-1} \]

where

\[ I_3 = m^{-6} \int T_{0}^0 dE_{v1}^2 E_{v1}^2 E_{v2}^2 (K_3^2 + L_3^2 + K_3 L_3)/3, \quad K_3 = (S-m+B_K+E_{v1})^{-1}, \quad L_3 = (S-m+B_K+E_{v2})^{-1} \]

The half-life is given by

\[ T_{1/2}^{(KK)_{2}\textnu} = (1.82 \times 10^{-20})/(a^6 Z^6)^{1/2} \text{ years} \]

The parameters entering the above formulae and the predicted half-lives for processes(88)\textsubscript{2}\textnu, (K\beta^+\textsubscript{2})\textnu and (KK)\textsubscript{2}\textnu are given in table 2.

REFERENCES