A CONTRIBUTION TO THE UNDERSTANDING OF A.C. LOSSES IN MULTIFILAMENTARY WIRES

P. Rem, F. Van Beckum, D. Dijkstra, L.J.M. Van de Klundert

To cite this version:


HAL Id: jpa-00223752
https://hal.archives-ouvertes.fr/jpa-00223752
Submitted on 1 Jan 1984

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
A CONTRIBUTION TO THE UNDERSTANDING OF A.C. LOSSES IN MULTIFILAMENTARY WIRES

F.C. Rem, F.P.H. van Beckum*, D. Dijkstra* and L.J.M. van de Klundert

Twente University of Technology, Department of Applied Physics, *Department of Applied Mathematics, P.O.B. 217, 7500 AE Enschede, The Netherlands

Résumé - Nous présentons un modèle numérique, basé sur les équations de Maxwell, pour calculer la dissipation d'un fil multifilamentaire, en champs magnétiques transverses.

Abstract - We present a numerical model, based on Maxwell's equations, for calculating transverse field losses in multifilamentary wires.

Introduction

In the past few years, the problem of finding the electromagnetic field in a multifilamentary wire, when placed in a changing perpendicular magnetic field, has been extensively studied. Although various authors [1,2,3] have contributed approximate solutions to the problem, no one, to our best knowledge, has shown a way to solve it rigorously.

In this paper, we describe a method, which has yielded results in simple cases, but may also be applied to more general problems.

The method

In general the method is iterative in that the magnetic field components are assumed to be known functions of space and time. We first solve \( E^0 \) from the simple equations:

\[
\begin{align*}
\frac{\partial}{\partial \phi} E^0_z &= -\frac{1}{\mu} \frac{\partial B_z}{\partial r} ; & \phi &= 0 : E^0_z = 0, \\
\frac{\partial}{\partial \phi} E^0_r &= \frac{1}{\mu} B_z - \frac{P}{2\pi} E^0 \phi ; & \phi &= \frac{\pi}{2} : E^0_r = 0, \\
E^0_\phi &= -\frac{P}{2\pi} E^0_z,
\end{align*}
\]

which are essentially Maxwell's equations and the condition that \( E^0 \) be perpendicular to the direction of the filaments. To find the real electric field \( E \), we divide the cross-section of the wire into three regions, as shown in figure 1 and write \( E \) as

\[
E = E^0 + Vq^i ; \quad i = 0,1,2,
\]

where superscript \( i \) distinguishes between the regions in figure 1.

figure 1. the geometry.
It is easy to see that $\Phi^0 = 0$ and $\Phi^1$ depends only on $r$ since both regions are non-saturated and so

\[(5) \quad V\Phi^0,1,\mathbf{p} = 0 ; \mathbf{p} = (0,2\pi r,\mathbf{I}_p).\]

Conservation of current in region 1 relates $\Phi^1(r)$ to the shape of the boundary $\mathbf{r} = r_O(\Phi)$ between 1 and 2:

\[(6) \quad \frac{2\pi}{|\mathbf{p}|} j_c = \Phi^1 \frac{1}{r} \frac{\partial}{\partial r} \left[ \sigma_r r \left( \frac{\partial}{\partial r} \Phi^1 \right) \right] \bigg|_{r=r_O(\Phi)} + \frac{\Phi}{r} \Phi^0 + \int_O \frac{\partial}{\partial r} (\sigma_r E_r) d\Phi.\]

On the boundary $r = r_O(\Phi)$ we have

\[(7) \quad \left. \Phi^1 \right|_{r=r_O(\Phi)} = \left. \Phi^2 \right|_{r=r_O(\Phi)} ; \left. V\Phi^1 \right|_{r=r_O(\Phi)} = \left. V\Phi^2 \right|_{r=r_O(\Phi)} ,\]

while on the outer edge, $r = R$:

\[(8) \quad \frac{\partial}{\partial r} \Phi^2 = -E^O_r.\]

In region 2, the condition of zero divergence of total current density, (normal and superconducting),

\[(9) \quad \text{div } \mathbf{j}_n = -\text{div } \mathbf{j}_s ,\]

leads to a Laplace problem for $\Phi^2$.

If, in addition, we knew the component $E_n$ of $E$, normal to $r = r_O(\Phi)$, we would have another differential equation for $\Phi^1$ and $r_O(\Phi)$.

\[(10) \quad E_n = (E^O + \partial \Phi^1) n_r + E^n_\Phi n_\Phi , \quad (n_r,n_\Phi) = \frac{(1,\frac{1}{r_O} \frac{\partial r_O}{\partial \Phi})}{\sqrt{1+\frac{1}{r_O^2} \left( \frac{\partial r_O}{\partial \Phi} \right)^2}}.\]

The solution of this set of equations is found by the following iteration scheme:

(a) initial guess for $E_n$
(b) solve for $r_O(\Phi)$, $\Phi^1(r)$ using (6) and (10)
(c) solve the Laplace problem
   ($\Phi^2$ is known on $r_O(\Phi)$ by (7); $\Phi^2 = 0$ if $\Phi = \pi/2$
   by symmetry; $\partial_r \Phi^2 = -E^O_r$ on $r = R$)
(d) compute $V\Phi^2$ from the calculated $\Phi$-values in region 2
   $r=r_O(\Phi)$
(e) Since $V\Phi$ is continuous on the boundary by (7) we can recalculate $E_n$ from
   
   $E_n = (E^O + V\Phi^1) n_r$  \quad $r=r_O(\Phi)$

(f) Check on convergence, (return to b)
(g) print solution and calculated values of the power and the induced central magnetic field.

At step (f) $E_n$ is compared with previously obtained values for $E_n$, and the iteration is stopped when a convergence criterion is satisfied.
The results

The results shown are calculated for the case that the magnetic field is uniform and \( a \) is isotropic and independent of \( r \). In this case, the problem is depending only on the two parameters

\[
\alpha = \frac{\mu_0 B L}{2\pi j_c}; \quad \beta = \left(\frac{P}{2\pi R}\right)^2
\]

and so are the scaled values of the induced central magnetic field \( B^i_\ast \) and the power \( P^\ast \):

\[
B^i_\ast = \frac{B^i}{2\pi \mu_0 B_0 R^2}; \quad P^\ast = \frac{P}{\mu_0 B_0^2 R}.
\]

If the external field is dominating, the approximation of the magnetic field by a uniform one with magnitude

\[
B^\text{ext} + B^i
\]

is convenient. However, if \( B^i \) gets larger with respect to \( B^\text{ext} \), fluctuations of e.g. \( j_c(B) \) become increasingly important and the problem should be iterated for the true magnetic field.

The figures

For several values of the parameters \( a \) and \( \beta \), fig. 2-4 show the shape of the saturated region. For large values of \( B \), these differ markedly from ellipses. Fig. 6, 7 show that the reduced values of \( p^\ast \) and \( B^i_\ast \) do not depend heavily on the parameters. Fig. 5 shows the function \( \Psi, \) where \( \Psi \) is defined by \( E = E_x e_x + \Psi \). Note that \( \delta_x \Psi = 0 \) at the boundary. Earlier solutions, obtained by other authors, failed to satisfy this boundary condition.

The authors are indebted to W. van Halteren for programming the numerical solution of Laplace's problem.

References

Fig. 2: Saturated region when \( B = 1 \).
\( \alpha/\beta \) is indicated in the figure.

Fig. 3: Saturated region when \( B = 4 \).
\( \alpha/\beta \) is indicated in the figure.

Fig. 4: Saturated region when \( B = 25 \).
\( \alpha/\beta \) is indicated in the figure.

Fig. 5: \( \psi(x,y) \);

Fig. 6: \( P \) as a function of \( \alpha \) and \( B \).

Fig. 7: \( B^* \) as a function of \( \alpha \) and \( B \).