STUDIES FOR "MIXED-Mu" MAGNETIC LEVITATION

J. Coupland

To cite this version:

J. Coupland. STUDIES FOR "MIXED-Mu" MAGNETIC LEVITATION. Journal de Physique Colloques, 1984, 45 (C1), pp.C1-923-C1-926. <10.1051/jphyscol:19841188>. <jpa-00223664>

HAL Id: jpa-00223664

https://hal.archives-ouvertes.fr/jpa-00223664

Submitted on 1 Jan 1984

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
STUDIES FOR "MIXED-Mu" MAGNETIC LEVITATION

J.H. Coupland

Rutherford Appleton Laboratory, Chilton, Oxon OX11 OQX, U.K.

Résumé - La méthode "orthogonal analogue" est utilisée dans l'étude des forces sur les rails en fer en configuration "mixed μ" pour un véhicule à levitation magnétique. Les exemples illustrent les difficultés d'obtenir assez de contrôle latéral quand la distance entre les rails et l'écran supraconducteur est choisie de façon réaliste.

Abstract - The orthogonal analogue method is used to investigate forces on iron rails in a 'mixed μ' configuration for a magnetically levitated vehicle. The examples given illustrate some of the difficulties in obtaining sufficient guidance at realistic clearance between rail and superconducting screen.

INTRODUCTION

The solution of two-dimensional conductor dominated magnetic field problems by the orthogonal analogue was suggested by Peierls in 1946. It became simpler with the availability of 'Teledeltos' conducting paper and then, with the advent of sensitive voltmeters, Asner et al. used a thin stainless steel sheet as the conducting medium in studies of magnetic field as affected by conductor positioning. It is suggested here that it is also a useful way to determine forces on pieces of iron, and may be extended further to include flux excluding regions, i.e. superconducting screens. Electromagnetic forces on iron are not always easy to determine by computer program and an alternative method with useful accuracy can be helpful. The problem is to study iron rail shapes in configurations that might be expected to show favourable force characteristics leading to stable lift and guidance of a vehicle. It has already been shown that it is possible to circumvent Earnshaw's theorem in theory and in practice by inclusion of suitably placed diamagnetic material. The problem becomes one of proximity and hence difficulty in gaining sufficient clearance between stabilising screen and iron rail. Early experimental work has been reported by Joyce.

METHOD

The magnetic fields in the xy plane arise from currents entirely in the z direction. In terms of vector potential \( A_z \), the governing equation of the problem is \( V^2 A_z - \mu_0 j_z = 0 \).

For the analogue a scaled pattern of currents, density \( j' \), is fed into the stainless steel sheet (thickness \( t \approx 0.1 \text{ mm} \)) corresponding to the conductors of the problem.

Continuity of current leads to

\[
\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \frac{-j'}{\partial t}
\]

as the governing equation in the analogue with V corresponding to \( A_z \), assuming conductivity \( \sigma \) and \( t \) are each constant. Since flux lines enter normally or thereabouts in unsaturated iron, the boundary condition \( H_z = 0 \) means \( \partial A_z / \partial n = 0 = \partial V / \partial n \) i.e. zero normal current or an insulating surface. A hole representing the shape of the iron is therefore cut into the sheet. Lines of constant \( A_z \) or \( V \) represent field lines, and differences of potential between points indicate flux crossing between them.

A two pin probe measures potential gradient, or B, and calibration is easily seen from Fig 1, where a uniform field \( B_0 \) is generated by two opposing current lines of strength \( i \text{ Am}^{-1} \) to give \( B_0 = \mu_0 i \), which is indicated by \( \partial V = i_1 / \partial x \) or \( B_0 = \mu_0 \partial V / \partial x \). Alternatively if \( \sigma \) and \( t \) are unknown, it may be easier to calibrate the probe directly in terms of such a field which may be readily
COPPER BARS SOLDERED TO SHEET

CLEARANCE \( t_{cm} \),

**Fig 1** - Analogue representing a uniform field \( B_0 \). Dotted outline shows cut-out for triangular rail.

Calculated, whence \( B_{\text{calc}} \approx \frac{\mu_0 dt(3V)}{\partial \chi} \). Any other field is then given by

\[
B = \frac{D}{C_{\text{calc}}} \frac{B_{\text{calc}}}{V_{\text{cal}}}, \quad \text{where} \quad V_{\text{cal}} \quad \text{refers to probe voltage, and there is no need to know the pin spacing either. Since} \quad B_{\text{calc}} \quad \text{can be measured easily, the outward force at the surface of the iron is obtained from} \quad F_n = B_0^2 \frac{V_0}{2} \quad \text{since} \quad B_0 = 0 \quad \text{for} \quad d > 1\]

If the probe is inadvertently inclined at angle \( \theta \) to the surface a cos \( \Theta \) error arises, \( AB/BO^2 \) or \( 10^{-2} \) for \( \Theta = 8 \). Surface forces can be obtained to a few percent accuracy except at corners where chamfers should be used, in effect a realistic modification as iron saturation is likely at sharp corners.

**IRON SHAPING**

Although there is no translational force on a piece of iron placed in a uniform field of infinite extent, there may be forces if the field is of limited extent, for example, a parallel planar iron gap excited by current sheets bridging it. A force arises if the iron is not central, or effectively so for shapes lacking perfect symmetry. Such forces when estimated from the images, are required to maintain the boundary conditions, are found to be rather small unless the iron is very close. The question arises, if as a shaped piece of iron approaches a screen, or current sheet, there is an enhanced repulsive force as magnetic flux is caused to shift from an adjacent face to a remote one, there being a significant change in magnetic moment \( M \) of the iron.

The elemental force \( F \) is given by \( F = B_0 \frac{\partial M}{\partial \chi} \) where \( B_0 \) is the field before inserting the iron.

The results for \( F \), in Fig 2, for roughly equal volumes of iron in the same \( B_0 (= 1T) \) in a Fig 1 analogue arrangement show that there is little to be gained from this 'flux shifting' argument. The apparent advantage shown by the rectangular block may not be realisable if the available \( B_0 \) is sufficient to saturate it. The demagnetisation factor \( D \) becomes important since \( B_{\text{iron}} = B_0/D \) for \( d > 1 \), valid at the onset of saturation. From voltage differences one deduces \( D = 0.30 \) for this block, whereas an ellipse of the same aspect ratio would have \( D = b/(a+b) = 0.32 \). The case of the uniformly magnetised ellipse is analysed by image theory in the appendix. The calculated values, indicated in Fig 2, suggest that the assumption of constant magnetisation is not a bad approximation for practical rail to screen clearances. As a check on analogue accuracy, measurements for a circular section at a negative clearance equal to the radius, i.e. a semicircle in contact with the screen, gave \( F = 4.19 \) tonne \( m^{-1} \) to be compared with 4.03 \( m^{-1} \) by analytical calculation for this case with \( a = 3.8 \) cm and \( B_0 = 1T \).

For an ellipse which is not saturated by \( B_0 \), \( m = B_0/V_0 = B_0/\mu_0 D = B_0(a+b)/\mu_0 b \), and the force/unit volume, or \( F/a^2 \) see appendix, does not show a maximum; the limit to ever more eccentric ellipses will be when \( B_0 = (a+b) \). \( B_0/b \) reaches \( B_{\text{sat}} \). However, if \( \mu_0 m \) is fixed, say \( B_{\text{sat}} \), then the maximum of \( F/a^2 \) is given by the minimum of \( (4d^2 + 3b^2 + 8bd + a^2) \),
where \( y_i \) is replaced by \( 2(d + b) \) and \( d \) is the clearance. Table I gives these optimum eccentricities, \( b/a \), for different relative clearances \( d/\sqrt{ab} \).

### QUADRUPOLE ARRANGEMENT

The system of alternate currents, lower half shown in Fig 3, is a convenient way to use flat race track coils in conjunction with a single or double rail. Good lift and stiffness characteristics may be obtained in the vertical direction but at the expense of strong instability in the guidance direction. From previous experience one might expect a screen to offset this and perhaps yield stability. The analogue results indicated that screens as in Fig 3 make it worse, a little less so for longer screens. A more effective screen is a thicker copper bar made it slightly worse still. A F2ZD RAL computer calculation incorporating diamagnetic screens confirmed this destabilising effect. Measurements at each side and around the end region X, indicate current \( \sim 10^5 \) A within a few cm of the edge. This overall unsatisfactory behaviour causing a flux concentration in the first quadrant suggests that such screens should be quenched to allow field penetration as the windings are excited, with the rail in the central position, and thereafter activated to provide guidance.

An estimate of the stabilising force can be made from image theory by considering first the images of the currents in the iron rail to represent its magnetisation, and then the differential images of these in the screens to simulate a lateral displacement \( \Delta \). Considering only the primary images, on account of the rapid force attenuation with distance, the repulsive force is estimated as \( 6p_M M^2 \Delta/\pi D^4 \), where \( D \) is the distance from the screen of the dipole \( M(=I\times L) \) representing the iron. Such forces are rather weak and call for a scaling up of the geometry to suit a minimal iron to screen clearance of 6 cm. Alternatively a better solution may be to reduce the flux concentration in the first quadrant by separating the windings in the lift direction.

### EFFECT OF A GAP

The analogue may be used to assess the effect of a gap in a screen. Even though the flux concentrates in the region of the gap to give a local field maximum there, the total flux passing through the gap increases linearly with width for small gaps. The relative amount is simply given by the ratio, potential difference across the gap/p.d. conductor to conductor, which is also plotted in Fig 4. It suggests that for a strictly two-dimensional situation small gaps would not be too harmful.

---

Table I

<table>
<thead>
<tr>
<th>( d/\sqrt{ab} )</th>
<th>( b/a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.475</td>
</tr>
<tr>
<td>0.5</td>
<td>0.405</td>
</tr>
<tr>
<td>1.0</td>
<td>0.315</td>
</tr>
<tr>
<td>2.0</td>
<td>0.225</td>
</tr>
</tbody>
</table>

---

Fig 3 - Flux contours for lower half of quadrupole arrangement.

Fig 4 - Flux pattern for 2 mm gap in screen. Inset shows flux plot along PQ and fraction of total flux passing through gap for different gap widths.
CONCLUSION
Although it is possible to obtain complementary results by computer if available, the examples given illustrate how this simple analogue has proved useful in investigating geometries for 'mixed μ' levitation. They show that despite early enthusiasm difficulties remain in obtaining adequate force and stiffness when one combines necessary operational clearance with the thickness of the cryostat wall. Notional ideas of gain from 'flux shifting' on suitably shaped iron are seen to be illusory.

ACKNOWLEDGEMENT
The author wishes to thank Prof R J Paul and colleagues for helpful discussions and staff at RAL for assistance with the work.

APPENDIX - Force on elliptical rail.
The iron rail, section \(x^2/a^2 + y^2/b^2 = 1\), with \(D = b/(a+b)\) will be uniformly magnetised by the applied field \(B_0\). This may be replaced by an equivalent pattern of surface currents which may be regarded as arising from two superimposed ellipses of opposite current density \(\pm j\) which have been slightly displaced by \(\Delta y\), where \(j\Delta y = m\), the magnetic moment/unit vol of the iron. Such a system of currents will induce a flux excluding current pattern in the screen which may be simulated by a corresponding image (Fig 5). The calculation of the repulsive force between iron and image assumes \(m\) is unchanged.

If \(A\) is the vector potential at \(Q\) due to + ellipse of current centred at \(P\), or the flux linked \(P\) to \(Q\), then the next flux linked due to + and - displaced ellipses at \(P\) will be \(3A\Delta y = B_\alpha\Delta y\), where \(B_\alpha\) is the field due to a single ellipse. Hence

\[
\frac{\partial}{\partial y} \text{force on + ellipse at } Q, \quad F_y = -3B = -3 \left( B_\alpha \Delta y \right) \text{ab}. \quad \text{By the complex number analysis of Beth [8] for an elliptical conductor, the external field } B \text{ is given by } B_y + iB_x = \mu_0 j \Delta y \left( x-i\frac{\sqrt{x^2 - c^2}}{c^2} \right), \text{ whence } B_x = \mu_0 j \Delta y \left( x-i\frac{\sqrt{x^2 - c^2}}{c^2} \right) \left( y-i\frac{\sqrt{y^2 + c^2}}{c^2} \right). \text{ Substituting for } B_x, \quad F_y = \mu_0 j \Delta y \left( y-i\frac{\sqrt{y^2 + c^2}}{c^2} \right) \left( x-i\frac{\sqrt{x^2 - c^2}}{c^2} \right) \left( 1-i\frac{\sqrt{y^2 + c^2}}{c^2} \right). \text{ The differential force on an image composed of + ellipses displaced by } \Delta y \text{ is } F = -\frac{\partial}{\partial y} \left( F_y \right) \Delta y, \text{ which reduces to } -\mu_0 \pi \left( j \Delta y \right)^2 \left( y_1^2 + c^2 \right)^{3/2}, \text{ or with } j\Delta y = m, \quad = \mu_0 m a_2 b^2 m^2 / \left( y_1^2 + c^2 \right)^{3/2}, \text{ and } y_1 \text{ = dist. of image from iron centre. This gives } F = 2.6 \text{mT}^{-1}, \text{ if } a = 5 \text{ cm}, \quad b = 2.35 \text{ cm}, \quad d = 2.65 \text{ cm and } \mu_0 m = B_1 = B_0 / 3.1 \text{ IT for } B_0 = 1 \text{IT.}

REFERENCES
7. Smythe W. R. Static and Dynamic Electricity, 3rd edn (1968) 344.