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RECENT DEVELOPMENTS IN FIELD AND FORCE COMPUTATION

J. Simkin

Rutherford Appleton Laboratory, Chilton Didcot, Oxon OX11 0QX, U.K.

Abstract - Recent developments in methods for electromagnetic field computation are reviewed, in particular work reported at the COMPUMAG Conference held in Genoa in June 1983. The accuracy of partial differential and integral equation solutions is compared and the evaluation of forces is examined.

1 - INTRODUCTION

The computation of electromagnetic fields is important for many scientific and industrial applications. Most of these applications result in computational problems that are unique to this discipline. One of the major problems is that results must be extremely accurate if they are to be of any practical use. This requirement would be impossible to satisfy if the materials used in the construction of electromagnetic devices were less easy to model. Commonly used materials are easily measured and characterised, and their properties are reasonably constant. However, there are exceptions, for example permanent magnet materials such as Alnico. There are many other specific problems, for example the unbounded nature of the field, the geometrical complexity of the devices and skin effects in eddy current solutions.

This paper reviews the progress in electromagnetic field computation, paying particular attention to work reported at the June 1983 COMPUMAG conference. Current research is dominated by the application of finite elements to the solution of partial differential equations, the reasons for favouring this approach are examined. The calculation of forces produced by electromagnetic fields is often subject to large errors, the techniques used for force calculations are compared and the cause of the errors is identified.

2 - ACCURACY

In many applications of electromagnetic devices the fields must be computed to an accuracy of the order of 1 part in 1000 or better. This can only be achieved when the materials involved are stable and their properties can be easily measured. Even then, guaranteeing the results to this accuracy is extremely expensive and for truly three dimensional fields may be impossible. Problems that involve linear magnetic materials can be solved to very high precision in two dimensions using boundary integral methods/1/. A simple problem has been solved with constant permeability in order to show rates of convergence of the solution for different methods. Although relatively simple the problem shown in Figure 1 does provide a non-trivial test. One quarter of a 'H' frame magnet with a sloping pole was solved...
using a scalar potential boundary integral method and a vector potential, partial differential equation, finite element method. The problem was assumed infinitely long out of the plane, with an iron relative permeability of 1000. A very large (500 degrees of freedom) boundary integral solution was used to obtain the 'correct' result. Figure 2 shows the flux distribution in the problem and from this figure the boundary conditions should be obvious. Figures 3a and 3b show the convergence of the integrated flux across the air gap for the two solution methods and also show the time required for the solutions. This problem was selected because the rapidly varying field gives a more realistic test of the finite element method, accuracy is much easier to obtain if the field is constant. In both methods quadratic variation of the potential was used in each discrete element. The partial differential equation solution employed smoothing on the field solution, direct differentiation of the element shape functions gives much larger errors.

Figure 1. Cross section of an H frame magnet with a sloping pole. Only one quarter of the magnet is displayed.

Figure 2. Flux distribution in the H frame magnet.

The results clearly show that for this problem the boundary integral method is by far the best. However, this cannot be used for non-linear problems except by introducing area discretisation, whereas the partial differential equation method can be used for non-linear solutions and will give similar precision. The figures do not tell the whole story, the increase in accuracy from 0.08% to 0.02% with partial differential equation solution was obtained by only increasing the discretisation within the aperture of the magnet. This strong correlation between accuracy and purely local subdivision is one of the strengths of the method, it is not a property of the integral method except in areas close to the discretised surface.
2.1 Cost Comparison

The results plotted in Figure 3 were obtained by programs that give nearly optimal performance for the types of method they use. Table 2 shows how the operation count varies as a function of accuracy for boundary integral, volume integral and partial differential equation methods in 2D solutions. Accuracy is assumed to depend on the element side length raised to a power that is independent of the method. Table 3 shows the same comparison for 3D solutions.

### Table 2

<table>
<thead>
<tr>
<th></th>
<th>Boundary integral</th>
<th>Volume integral</th>
<th>Partial Differential</th>
</tr>
</thead>
<tbody>
<tr>
<td>accuracy</td>
<td>$\alpha n^a$</td>
<td>$m^3$</td>
<td>$p^a$</td>
</tr>
<tr>
<td>discretisation</td>
<td>$\alpha n$</td>
<td>$m^2$</td>
<td>$p^2$</td>
</tr>
<tr>
<td>matrix evaluation</td>
<td>$\alpha n^2$</td>
<td>$m^2$</td>
<td></td>
</tr>
<tr>
<td>equation solution</td>
<td>$\alpha n^3$</td>
<td>$m^2$</td>
<td>$p^2\ln(p)$</td>
</tr>
<tr>
<td>field recovery</td>
<td>$\alpha n$</td>
<td>$m^2$</td>
<td>$1$</td>
</tr>
</tbody>
</table>
Table 3

A comparison of operation counts for different solution methods in 3D problems

<table>
<thead>
<tr>
<th></th>
<th>Boundary integral</th>
<th>Volume integral</th>
<th>Partial Differential</th>
</tr>
</thead>
<tbody>
<tr>
<td>accuracy</td>
<td>$a^n$</td>
<td>$m^a$</td>
<td>$p^a$</td>
</tr>
<tr>
<td>discretisation</td>
<td>$n^2$</td>
<td>$m^3$</td>
<td>$p^3$</td>
</tr>
<tr>
<td>matrix evaluation</td>
<td>$n^6$</td>
<td>$m^6$</td>
<td>$p^3$</td>
</tr>
<tr>
<td>equation solution</td>
<td>$n^6$</td>
<td>$m^2$</td>
<td>$p^3 \ln(p)$</td>
</tr>
<tr>
<td>field recovery</td>
<td>$n^2$</td>
<td>$m^3$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

The most obvious conclusion from these tables is that in the limit of very large discretisations all methods are dominated by equation solution. The next conclusion is that for solutions in three dimensions partial differential methods must become more effective than the other approaches; again, in the limit of large discretisations when the constants in the proportionality relationships become insignificant. It is only in two dimensions that boundary integral methods are particularly attractive, Figure 3 demonstrates this quite effectively. In solving practical problems on existing computing hardware the constants in the proportionality relationships become important. Experience with existing programs in three dimensions shows that once the number of unknowns rises above 1000 for integral methods, the partial differential methods are better. Accuracies of the order of .2% can be achieved with this level of discretisation.

2.2 Linear Algebra

In deriving the operation counts for Tables 2 and 3 it was assumed that direct equation solution methods were used for integral methods (Gaussian elimination) and that the most effective methods available were used for the partial differential methods. In the latter, there have been significant developments in the last 5 years and it is only as a result of these that the partial differential methods have been so successful. The majority of new, large, finite element programs now use preconditioned conjugate gradient methods/2,3/. The computer storage required by these methods increases linearly with the number of unknowns and is independent of ordering of the unknowns. Similarly the solution times are almost independent of the ordering of the unknowns, this is to be contrasted with other sparse matrix methods where the bandwidth or profile of the matrix was very important especially for problems in three dimensions. Table 4 gives an indication of the solution times required using an incomplete cholesky conjugate gradient method for a finite element solution of Laplace's equation in three dimensions.

Table 4

Solution times for incomplete cholesky conjugate gradient methods.
An IBM 3081D was used for these calculations.

<table>
<thead>
<tr>
<th>Number of unknowns</th>
<th>Solution time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3500</td>
<td>18</td>
</tr>
<tr>
<td>7000</td>
<td>70</td>
</tr>
<tr>
<td>19000</td>
<td>225</td>
</tr>
</tbody>
</table>

The Rutherford Appleton Laboratory implementations of these methods do not use any disk backing store, the solution of a 25000 unknown set of equations for the Laplacian problem requires 5MB of virtual storage.

2.3 3D Solutions

In order to show the convergence of solutions to three dimensional problems an
axisymmetric model with the cross-section as shown in Figure 1 was solved using a 2D axisymmetric model and a full 3D model in which one octant of the geometry was discretised. Figure 4 shows a computer generated display of the model with hidden lines removed. Figure 5 shows the convergence of the integrated field as a function of discretisation. Moving from Figure 3 to Figure 5 using Tables 2 and 3 it would seem reasonable to expect to require 600000 degrees of freedom to give 0.2% accuracy. Figure 5 shows that only 24000 unknowns were required, this was achieved by using high order small elements in the region of interest and low order elements in the outer areas. These results were obtained using the TOSCA program/1/.

Figure 4. A 3D model of an axisymmetric H frame magnet.

Figure 5. Convergence of the integrated flux crossing the gap of the 3D H frame magnet.
Integral methods can be formulated such that the field decays correctly to infinity. In solving partial differential equations using finite element procedures the field is in general artificially terminated at the boundary of the mesh with either a Dirichlet or Neumann boundary condition. This can result in large errors if the false termination of the field is strongly coupled to the regions of interest.

Several methods of overcoming this limitation have been developed. A boundary integral method/4/ or boundary Galerkin method/5/ can be applied to the false boundary or a ballooning technique/6/ could be used. The ballooning technique uses recursive generation of similar rings of elements around the problem space and the degrees of freedom so created are condensed out of the system of equations. This technique and the boundary integral method, both produce coupling between all degrees of freedom associated with the boundary and hence create a dense block in the final system matrix. Another alternative is to use an element in the exterior which extends from the boundary of the mesh to infinity/7/, these are usually called 'infinite elements'. Various decay functions may be selected in order to approximate the variation from the boundary to infinity, the most commonly used functions are either reciprocal, exponential or orthogonal polynomial/7,8/. The advantage of this approach over the others is that the system matrix does not contain a densely populated block connecting the nodes on the boundary. Its disadvantage is that the decay function must be closely related to the actual decay if large errors are not to be produced.

There have been exciting developments in the calculation of eddy currents. In two space dimensions boundary integral methods have been extended to include formulations capable of modelling eddy currents/9/. In three dimensions vector potential or electric field solutions have been directly coupled to scalar potentials for the exterior space/10/. Uniqueness has been demonstrated for the vector potentials/11/, with a definite implied gauge/12/, providing this formulation is only used inside conducting regions. Such methods minimise the number of unknowns required at each discretisation point and also avoid the cancellation problems inherent in other techniques/13/. The electric field and vector potential approaches are very similar/14/, it is not yet clear which of them will be best for transient solutions.

The most successful methods that are now available are based on coupled network formulations/15/. These methods have been greatly extended and now use networks based on tetrahedra, this makes the modelling of real devices much more straightforward. Two other developments are particularly interesting; eddy current solutions using coupled finite elements and boundary elements/16/, where special finite elements for vector fields are used; finite difference solutions, which have been developed and refined to give very good results/17/.

The forces associated with electromagnetic fields are in many cases the primary motive for the design of the device or they are a constraint on the design. This is unfortunate because it is particularly difficult to reliably compute the forces. In all methods any error in the fields is enhanced in computing the forces. The methods of computing forces are,

- virtual work/18/
- body force integration/19/
- Maxwell stress integration/18/
- change in stored energy.

It has been shown that virtual work calculations become, in the limit of small
Body force integration is subject to large errors because the external forces on components may be many times smaller than the internal forces that are balanced by stresses in the component. In a similar way a surface integration of the Maxwell stress tensor often results in nearly balancing positive and negative contributions. Virtual work has the same problem and calculations based on actual displacements of objects are very inconvenient.

Figure 6: The flux distribution for a pair of infinitely long parallel conductors carrying equal and opposite currents, with an iron bar slightly offset from the centre.

The errors that will occur are very dependent on the problem being solved, very accurate answers are easy to obtain if there is no cancellation in the integrals, for example, computing the force between a current carrying coil and an infinite half plane. Figure 6 shows a problem that is particularly difficult to solve. This is a two dimensional section through a pair of conductors carrying equal and opposite currents, slightly displaced from the centre is an iron rail, the figure also shows the flux distribution. The problem is to compute the force on the iron rail. This can be solved very easily by computing the change in stored energy or by computing the reaction on the coils using the body force. It is particularly difficult to solve by Maxwell stress integration. The resultant force on the rail is less than 10% of the integrated modulus of the stress tensor. For the rail there is a weak singularity in the field near the corners; if a partial differential finite element method is used this singularity is only approximated crudely and the answers on the left and right hand corners are very dependent on the local discretisation. Figure 7 shows the variation of the modulus of the field around a half circle centred on the rail, with a radius such that the circle passes close to, but does not touch, the surface of the rail. There is a slight left to right asymmetry due to the displacement of the rail, but the discontinuity on the left is purely due to the left to right asymmetry of the discretisation. The effects of the corner damp out quickly as the radius of the path is increased such that points on the path do not lie within the band of elements touching the surface of the rail. This effect has a tremendous influence on the computed forces, Figure 8 shows the force computed by Maxwell stress integration as a function of the radius of the circle used for the integration. The expected result was obtained using a very large number of unknowns, from the body force on the conductors. The figure shows results for two levels of discretisation, if very small elements are used close to the surface of the bar the poor modelling of the singularity does not affect the computed forces.
The major headache associated with finite element analysis is in the preparation of data. Electromagnetic fields are particularly hard to model because the external space must also be discretised. There are major developments in this area that should be available to the designer in the near future, these include automatic mesh generation on a geometric model, and adaptive mesh generation to satisfy some
specified accuracy criteria. Both developments are important particularly if they can be used in order to give reliable results without a highly experienced and able finite element expert. Also linked with the adaptive mesh generation is the idea of using duality principles to obtain upper and lower bounds on solution variables/20/, this must be important in future software developments. There are already some programs that use adaptive mesh generation in the solution of electromagnetic fields/21/.

7 - CONCLUSION

The review paper on field computation/22/ that was presented in 1972 at MT-4, included tables showing programs for field computation, at the 1983 Compumag conference a review of eddy current programs/23/ produced a list at least three times longer. Methods for electromagnetic field computation are still being actively developed and the accuracy of the results is only limited by the computer resources available. In practice this means that reliable results can only be obtained by experienced personnel. The trend to adaptive mesh generation should improve productivity and reduce the degree of expertise that is required, however, increased computing power will be needed for very high accuracy. Todays serial computers favour the use of partial differential equation methods, parallel computers on the other hand, would revive the interest in integral equations. It must be said though, that integral solutions are more prone to disastrous errors.

REFERENCES


