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To cite this version:

HAL Id: jpa-00223427
https://hal.archives-ouvertes.fr/jpa-00223427
Submitted on 1 Jan 1983

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PREDICTION AND MEASUREMENT OF THE NATURAL FREQUENCIES AND DAMPING CAPACITY OF CARBON FIBRE-REINFORCED PLASTICS PLATES

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Abstract - The objective of this investigation is to predict the natural frequency and specific damping capacity of laminated composite plates in various modes of vibration by using the finite element method. The simple definition of specific damping capacity is \( \psi = \Delta U/U \), where \( U \) is the maximum strain energy stored per cycle and \( U \) is the energy dissipated per cycle. In this work, \( U \) is calculated by a finite element method which includes transverse shear deformation; \( \Delta U \) is determined from a damped element model. The specific damping capacities, mode shapes, and natural frequencies of various free-free carbon and glass fibre-reinforced plastics plates have been predicted and measured.

I. INTRODUCTION

Flat panels are used in many structural applications in land and ocean-going vehicles, aircraft and spacecraft. Both sandwich construction and filamentary composite materials are used in plate type structures because the structural stiffness can be increased without adding excess weight. For the optimal design of panels made from composite materials, structural designers require more useful and practical methods for obtaining the correct numerical results of the stiffness and damping of laminated composites. During the last decade, several authors have tried to predict the stiffness and damping of plates, but most of them have been restricted to the natural frequencies of anisotropic laminated composite plates, and there have been few on damping.

In this work we have investigated the prediction of the natural modes and specific damping capacity of anisotropic laminated plates using, and extending, the finite element method described by Cawley and Adams /1/ and the damped element model proposed by Adams and Bacon /2/.

To investigate the accuracy of the theoretical prediction, a comparison will be made with the experimental results from various laminated CFRP and GFRP rectangular plates in the free-free condition for the first six modes of vibration. Some graphical methods for simplifying the prediction are presented.

All the plates used in this investigation were mid-plane symmetric so as to eliminate bending-stretching coupling. It is, however, possible to include this effect in the analysis if asymmetrical laminates were to be used.
II. THEORY

The specific damping capacity is defined as

$$\psi = \frac{\Delta U}{U}$$

(1)

where \(\Delta U\) is the energy dissipated during a stress cycle and \(U\) is the maximum strain energy.

\(U\) is obtained as for an undamped system as follows:

$$U = \frac{1}{2} \int_{V} \{\varepsilon_{ij}\}^T \{\sigma_{ij}\} \, dV$$

(2)

where \(\varepsilon_{ij}\) and \(\sigma_{ij}\) are the strains and stresses related to the fibre direction.

Equation (2) may be reduced to a standard form as

$$U = \frac{1}{2} \{\delta\}^T [K] \{\delta\}$$

(3)

where \(\{\delta\}\) is the nodal point displacement matrix. Here, five degrees of freedom for each nodal point and 8 nodal points for each element are used, and \([K]\) is the stiffness matrix. In the evaluation of the maximum strain energy \(U\), the Young's modulus of \(0^\circ\), \(90^\circ\) unidirectional fibre reinforced beams, \(E_L\), \(E_T\), and the shear modulus of a \(0^\circ\) unidirectional rod \(G_{LT}\) are used.

As given in Ref. 2

$$\Delta U = \int_{V} \delta(\Delta U) \, dV$$

(4)

where \(\delta(\Delta U)\) is the energy dissipated in each element, and is defined as

$$\delta(\Delta U) = \delta(\Delta U_1) + \delta(\Delta U_2) + \delta(\Delta U_3) + \delta(\Delta U_4) + \delta(\Delta U_5)$$

and

$$\delta(\Delta U_1) = \frac{1}{2} \psi_L \varepsilon_{11} \sigma_{11}, \delta(\Delta U_2) = \frac{1}{2} \psi_T \varepsilon_{22} \sigma_{22}$$

$$\delta(\Delta U_3) = \frac{1}{2} \psi_{TT} \varepsilon_{23} \sigma_{23}, \delta(\Delta U_4) = \frac{1}{2} \psi_{LT} \varepsilon_{13} \sigma_{13}$$

$$\delta(\Delta U_5) = \frac{1}{2} \psi_{LT} \varepsilon_{12} \sigma_{12}$$

Suffix 1 denotes the fibre direction, while 2 and 3 are the two directions transverse to the direction of the fibres. \(\psi_L\), \(\psi_T\), etc. are the associated damping capacities in each direction, and they are obtained from tests on unidirectional beams.

Equation (4) may be reduced to matrix form as:

$$\Delta U = \frac{1}{2} \int_{V} \{\varepsilon_{ij}\}^T \{\psi\} \{\sigma_{ij}\} \, dV$$

(5)

where

$$[\psi] = \begin{bmatrix}
\psi_L & 0 & 0 & 0 & 0 \\
0 & \psi_T & 0 & 0 & 0 \\
0 & 0 & \psi_{TT} & 0 & 0 \\
0 & 0 & 0 & \psi_{LT} & 0 \\
0 & 0 & 0 & 0 & \psi_{LT}
\end{bmatrix}$$

Using the same method as with Eqn. (2), Eqn. (5) may be reduced to

$$\Delta U = \frac{1}{2} \{\delta\}^T [K_d] \{\delta\}$$

(6)
where \( \{6\} \) is the same matrix as in Eqn. (2) and was obtained from the finite element results. \([K_d]\) is the stiffness matrix of the damped system, and it may be evaluated separately.

III. RESULTS AND DISCUSSION

The composite plates used in this investigation consisted of either glass or HM-S carbon fibre in DX-210 epoxy resin. The plates were made of 8 or 12 layers of pre-impregnated fibre in a hot press, such that different laminate orientations could be obtained; details of the plates used are given in Table 1. The material properties used in the theoretical prediction are given in Table 2. All the values in

Table 1  Plate Data

<table>
<thead>
<tr>
<th>Plate number</th>
<th>Material</th>
<th>No. of layers</th>
<th>Density kg m(^{-3})</th>
<th>(v_f)</th>
<th>Ply orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CFRP</td>
<td>8</td>
<td>1.446.2</td>
<td>0.342</td>
<td>(0(^\circ), 90(^\circ), 0(^\circ), 90(^\circ))</td>
</tr>
<tr>
<td>2</td>
<td>CFRP</td>
<td>12</td>
<td>1.636.4</td>
<td>0.618</td>
<td>(0(^\circ), -60(^\circ), 60(^\circ), 0(^\circ), -60(^\circ), 60(^\circ))</td>
</tr>
<tr>
<td>3</td>
<td>GFRP</td>
<td>8</td>
<td>1.813.9</td>
<td>0.451</td>
<td>(0(^\circ), 90(^\circ), 0(^\circ), 90(^\circ))</td>
</tr>
<tr>
<td>4</td>
<td>GFRP</td>
<td>12</td>
<td>2.003.5</td>
<td>0.592</td>
<td>(0(^\circ), -60(^\circ), 60(^\circ), 0(^\circ), -60(^\circ), 60(^\circ))</td>
</tr>
</tbody>
</table>

N.B. Suffix s means mid-plane symmetric

Table 2  Moduli and damping values for materials used in the plates

<table>
<thead>
<tr>
<th>Material</th>
<th>(E_1) (GPa)</th>
<th>(E_2) (GPa)</th>
<th>(G_{12}) (GPa)</th>
<th>(\psi_1)</th>
<th>(\psi_2)</th>
<th>(\psi_{12})</th>
<th>(v_{1,2})</th>
<th>(v_f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HMS/DX-210</td>
<td>172.1</td>
<td>7.20</td>
<td>3.76</td>
<td>0.45</td>
<td>4.22</td>
<td>7.05</td>
<td>0.3</td>
<td>0.50</td>
</tr>
<tr>
<td>Glass/DX-210</td>
<td>37.87</td>
<td>10.90</td>
<td>4.91</td>
<td>0.87</td>
<td>5.05</td>
<td>6.91</td>
<td>0.3</td>
<td>0.50</td>
</tr>
<tr>
<td>DX-210/BF3400</td>
<td>3.21</td>
<td>3.21</td>
<td>1.20</td>
<td>6.54</td>
<td>6.54</td>
<td>6.68</td>
<td>0.34</td>
<td>0</td>
</tr>
</tbody>
</table>

this table were established either by using beam specimens cut from a unidirectional plate (longitudinal and transverse damping and Young's moduli) or cylindrical specimens (for measuring the shear modulus and damping in torsion). It should be noted that the value of the torsional damping of a 90\(^\circ\) fibre orientation rod, \(\psi_{23}\), is not important in the prediction, since changing it from 6% to 15% gave no difference to the theoretical results. In the prediction, \(\psi_{23}\) is taken as the same value as \(\psi_{12}\) which is the value of torsional damping of a 0\(^\circ\) fibre orientation rod (in longitudinal shear). Because of variations in the fibre volume fraction of the plates, the material properties used in the theoretical prediction were corrected from a standard set given for a 50% fibre volume fraction.

3.2 Comparison of theoretical and experimental results

Tables 3 and 4 show for the first six modes the theoretical prediction and the experimental results of CFRP plates for various fibre orientations. On the whole, there is good agreement between the predicted and measured values. The discrepancies in natural frequencies are less than 10%, and the values of specific damping capacity are very close. Mode 6 in plate 3 could not be obtained experimentally because the input energy from the transient technique was insufficient. Tables 5
<table>
<thead>
<tr>
<th>No.</th>
<th>Freq. (Hz)</th>
<th>Mode shape</th>
<th>SDC(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>58.10 (68.88)</td>
<td><img src="image" alt="Mode shape" /></td>
<td>7.80 (6.65)</td>
</tr>
<tr>
<td>2</td>
<td>213.31 (218.9)</td>
<td><img src="image" alt="Mode shape" /></td>
<td>0.91 (1.05)</td>
</tr>
<tr>
<td>3</td>
<td>243.47 (251.2)</td>
<td><img src="image" alt="Mode shape" /></td>
<td>2.50 (2.6)</td>
</tr>
<tr>
<td>4</td>
<td>302.51 (305.4)</td>
<td><img src="image" alt="Mode shape" /></td>
<td>0.60 (0.92)</td>
</tr>
<tr>
<td>5</td>
<td>324.16 (323.5)</td>
<td><img src="image" alt="Mode shape" /></td>
<td>1.51 (1.7)</td>
</tr>
<tr>
<td>6</td>
<td>441.62 (452.5)</td>
<td><img src="image" alt="Mode shape" /></td>
<td>2.74 (3.0)</td>
</tr>
</tbody>
</table>

Table 3 Natural frequencies and damping of various modes of an 8-layer plate (0°, 90°, 0°, 90°, 0°, 90°, 0°, 90°, 0°) carbon FRP plate (Plate no. 1). Experimental values in brackets.

and 6 give the results for GFRP plates. All show good agreement between prediction and measurement.

The effect of air damping and the additional energy dissipation associated with the supports and the small piece of metal required (which is connected to earth) for the transducer affect the results of the very low damping modes such as the 4th mode of plate 1, the 4th mode of plate 3 and so on. These are essentially beam modes in which the large majority of the strain energy is stored in tension/compression in the...
fibres and not in matrix tension or shear. However, the results for all the plates used are satisfactory, even when the specimens have imperfections such as slight variations in thickness and the nominal angle of the fibres.

It can be said that the more the twisting, the higher the damping. For instance, for an 8 layer cross-ply ($0^\circ/90^\circ$) GFRP plate (see Table 5) the two beam-type modes, numbers 2 and 3, appear similar, but the relationship of the nodal lines to the outer fibre direction means that the higher mode has much less damping than the lower one. The other modes of vibration of this plate all involve much more plate
twisting and hence matrix shear than do modes 2 and 3, and so the damping is higher. It is important for designers to realise the significance of these results, which show that for all the plates the damping values are different for each mode.

V. CONCLUSIONS

In this paper a method for predicting the natural frequencies, mode shapes and vibration damping parameters of laminated composite plates has been described. The method is based on the finite element technique using the damped element model in which the effects of transverse shear deformation and rotary inertia were considered. The significance of the difference of damping values from different mode shapes and fibre orientations must be emphasized. The damping values for mode shapes in which there is a lot of twisting are greater than for those in which the large majority of the strain energy is stored in tension/compression in the fibre and not in matrix tension or shear.

REFERENCES
