ENERGY DEPOSITION DISTRIBUTIONS FOR MULTIPLE ION-BEAM IRRADIATION OF ICF TARGETS

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JOURNAL DE PHYSIQUE
Colloque C8, supplément au n°11, Tome 44, novembre 1983

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Résumé

Un formalisme est présenté afin d'obtenir la fonction de distribution de l'énergie de déposition, pour une cible ICF irradiée par des faisceaux d'ions multiples. Des résultats pour une irradiation symétrique à 6 faisceaux, montrent que des fluctuations larges pourraient avoir lieu à cette fonction de distribution et qu'au moins 20 faisceaux sont nécessaires pour adoucir ces effets.

Abstract

A formalism is presented for obtaining the energy deposition distribution function for an ICF target irradiated with multiple ion beams. Results for symmetrical 6-beam irradiation show that large fluctuations would occur in this distribution function, and that at least 20 beams are required to smooth these effects.

1. Introduction

Hydrodynamic studies of ion beam - driven targets for inertial confinement fusion (ICF) require a detailed treatment of the geometry of the multiple beam (ie.'beamlet') configuration, in addition to the physics description of the slowing down process. The spatial distribution of the energy deposition arising from beamlet irradiation may well determine the subsequent stability of the compression of the target, particularly in the case of ion beam irradiation where the largest part of the ion energy is deposited near the end of the ion's range. Thus, regions of beamlet overlap in the target, even when each beamlet has a uniform intensity, can give rise to local 'hot spots' which could drive instabilities. Directly - driven targets might require a relatively large number of beamlets not just to maximise the irradiance symmetry but to minimize the effects of radial space charge spreading and self-field effects.

In this paper a detailed formalism is presented which allows a total energy deposition distribution function to be computed prior to its use in a suitable hydrodynamics code. Although the formalism is general, specific applications are limited in this paper to spherically symmetric targets.

2. Theory

The vector flux of test particles, $\Phi(\underset{\sim}{r},E,\underset{\sim}{\Omega})$, at energy E, position vector $\underset{\sim}{r}$ and transport unit vector $\underset{\sim}{\Omega}$ in a medium where only rectilinear Coulomb slowing down can occur, is given by the solution of the Boltzmann equation

$$\underset{\sim}{\Omega}\cdot\nabla\Phi\,(\underset{\sim}{r},E,\underset{\sim}{\Omega}) + \Sigma_a\,(\underset{\sim}{r},E)\,\Phi\,(\underset{\sim}{r},E,\underset{\sim}{\Omega}) - \frac{d}{dE}\,(S(\underset{\sim}{r},E)\,\Phi\,(\underset{\sim}{r},E,\underset{\sim}{\Omega}))$$

$$= Q\,(\underset{\sim}{r},E,\underset{\sim}{\Omega}) \quad \ldots\ldots(1)$$

Equation (1) is simply a balance equation in phase-space for the steady-state 6-vector particle density $n(\underset{\sim}{r},\underset{\sim}{v})$ where $\underset{\sim}{v} = v\underset{\sim}{\Omega}$ and $E = \gamma mc^2$ for a test particle of rest mass m. $Q(\underset{\sim}{r},E,\underset{\sim}{\Omega})$ is the source of such particles, $S(\underset{\sim}{r},E)$ is the slowing down power of the medimum and Σ_a is the macroscopic absorption cross section.

The solution to (1) for a monodirectional and monoenergetic plane source situated at $x = 0$, $Q = 2(\pi)^{-1}\,\delta(x)\,\delta(E-E_o)\,\delta(\mu-\mu_o)$, which produces an ion beam, is

$$\Phi(x,E,\mu) = \frac{\delta(\mu-\mu_o)\ \delta(x-x_o)}{2\pi S(E)}\ \exp - \int_E^{E_o} \frac{\Sigma_a(E')}{S(E')}\, dE' \quad \text{.......(2)}$$

for a homogeneous medium with the geometry of fig. 1, where

$$x_o = \mu_o \int_E^{E_o} \frac{dE'}{S(E')} \quad \text{...... (3)}$$

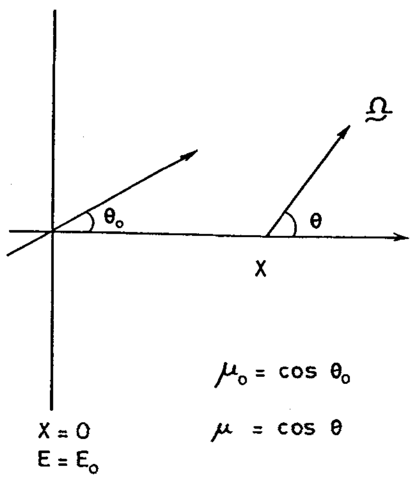

Fig. 1 Plane geometry for eqn (2).

The energy current associated with the particle source of unit strength is defined as

$$J_\epsilon\ (x) = 2\pi \int_o^\infty E\ dE \int_{-1}^{+1} \mu\Phi\ (x,E,\mu)\ d\mu \quad \text{..... (3)}$$

and is simply, from (2),

$$J_\epsilon(x) = \bar{E}(x) \exp - \int_{\bar{E}(x)}^{E_o} \frac{\Sigma_a(E)}{S(E)} \, dE \qquad \ldots\ldots(4)$$

where we have used the relationship

$$\delta(x(E) - x_o(E)) = \delta(E-\bar{E}) \Big/ \left| \frac{dx}{dE} \right|_{\bar{E}}$$

and $\bar{E}(x)$ is defined, for a given x-value, by the equation

$$x = \mu_o \int_{\bar{E}(x)}^{E_o} \frac{dE'}{S(E')}$$

From (3)

$$\frac{dx}{dE} = -\mu_o \Big/ S(E)$$

The divergence of $J_\epsilon(x)$ represents the energy deposition rate per unit volume (or, strictly, the energy loss rate per unit volume) of the test particles at a point x, W(x). Assuming all energy is deposited locally, we have

$$W(x) = \mathrm{div}\, J_\epsilon(x) = - \frac{1}{\mu_o} \left[S(E) + \bar{E}(x)\Sigma_a(\bar{E}) \right] \exp - \int_{\bar{E}(x)}^{E_o} \frac{\Sigma_a(E')\, dE'}{S(E')} \qquad \ldots\ldots.(5)$$

The first term in eqn. (5) is the Coulomb collisional energy loss rate per unit volume from the beam, whilst, the second term is the energy removal rate per unit volume from the beam due to the absorption process represented by Σ_a. Depending on the physical nature of these processes, this may or may not result in a local energy deposition. Fission, for example, would produce a non-local effect, where the fission fragments would have their own energy-range relationships which could be quite different from the ions in the primary beam. Finally, the exponential term in eqn.(5) represents the probability that the particle has travelled a distance x without being absorbed. The development in this paper continues for $\Sigma_a = 0$ and $\mu_o = 1$. Competition from fission processes will be considered in a subsequent paper.

The extension of eqn.(5) to a non-absorbing inhomogeneous material produces the result ($\mu_o = 1$).

$$W(x) = - S(E)$$

where now $\bar{E}$ (x) is the solution to the equation

$$\frac{d\bar{E}(x)}{dx} + S(\bar{E},x) = 0$$

which may be obtained straight forwardly, for the initial conditions $E = E_o$, $x = 0$, using a Runge-Kutta technique.

3.Extension to multiple beams

In fig. 2 we consider the i^{th} beam of an array of beams traversing the target, and evaluate the total energy deposition rate per unit volume at a point $\underset{\sim}{r}$. The component of the i^{th} beam passes through an element of area $d\underset{\sim}{S} = \underset{\sim}{n}dS$ at a position $\underset{\sim}{a}_i$ in the direction $\underset{\sim}{\Omega}_o$, and traverses the distance L_i to $\underset{\sim}{r}$. Each beam is considered to have cylindrical symmetry. A vector $\underset{\sim}{u}$ is defined, perpendicular to $\underset{\sim}{\Omega}_o$, measured from the centre line of the beam, $\underset{\sim}{a}_i = \underset{\sim}{a}_{io}$, to define any radial variation of intensity the beam may possess.

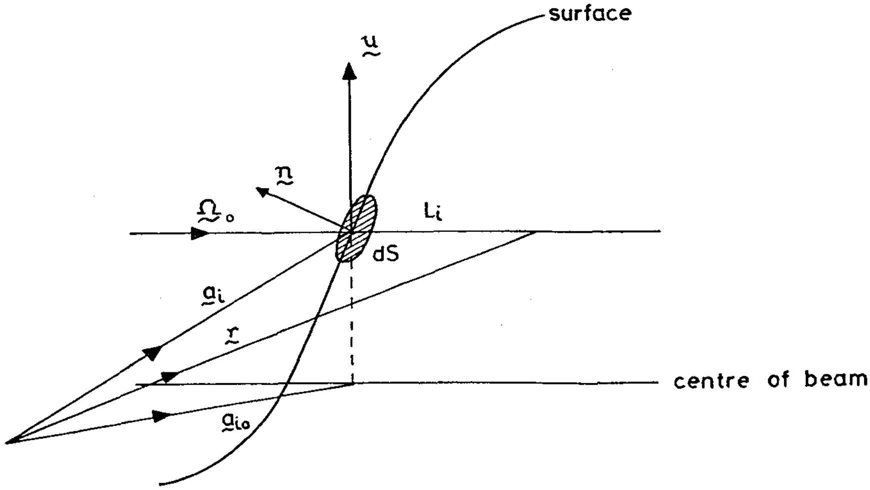

Fig. 2 Vector description for an array of beams.

If $j^i(a_i)$ denotes the ion beam current entering the surface at a_i (in ions per unit area per unit time) then we can use the result of eqn. (4) to write the energy deposition rate per unit volume at r for the i^{th} beam, $W_i(r)$, as

$$
\begin{aligned}
W_i(r) &= \nabla\cdot\bar{E}(r)\; j^i\;(a_i) \\
&= j^i(a_i)\cdot\nabla\bar{E}(r) \\
&= -j_o^i\; S(\bar{E})
\end{aligned}
$$

where $j_o{}^i = n_i\; v_o\; n\cdot\Omega_o$ for a beam of particle density n_i, initial velocity v_o at a_i. The energy E now satisfies the relationship

$$L_i = \int_{\bar{E}}^{E_o} \frac{dE'}{S(E')}$$

We further assume that

$$j^i\;(a_i) = j_o^i\;(a_{io})\;\exp - (u_i\cdot u_i/k^2)$$

That is, the beam has a gaussian radial intensity distribution, centered about the beam axis of symmetry. The total energy deposition function is now

$$W(\underset{\sim}{r}) = - \sum_i j_o^i \, (\underset{\sim}{a}_{io}) \exp - \left\{ \underset{\sim}{u}_i \cdot \underset{\sim}{u}_i / k^2 \right\} S(\bar{E}) \qquad \ldots\ldots(6)$$

4. Application to spherical geometry

For a spherically symmetric target the most convenient coordinate system is that placed at the centre of the sphere, as illustrated in fig. 3.

Then

$$L_i = \underset{\sim}{r} \cdot \underset{\sim}{\Omega}_o + \left[(\underset{\sim}{r} \cdot \underset{\sim}{\Omega}_o)^2 + R^2 - r^2 \right]^{\frac{1}{2}}$$

and

$$u^2 = r^2 - (\underset{\sim}{r} \cdot \underset{\sim}{\Omega}_o)^2$$

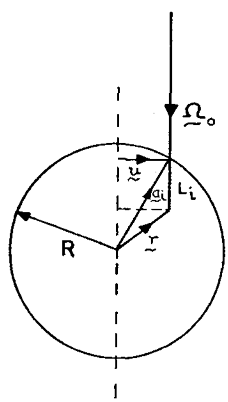

Fig. 3 Coordinate system at centre of spherical target.

Evaluation of (6) is now reduced to providing a prescription for the direction cosines, $\underset{\sim}{\Omega}_o = (\Omega_x, \Omega_y, \Omega_z)$, for each beam.

Probably the most convenient and useful approach is to use the archimedean symmetry of a cube, inscribed in the sphere. One set of beams, six in total, can be orientated perpendicular to each of the six sides of the cube with the direction cosines $(\pm 1,0,0)$, $(0 \pm 1,0)$ and $(0,0,\pm 1)$. The second symmetry set is the beams lying along the lines joining opposite vertices of the cube, giving a further eight beams with direction cosines $(\pm 3^{-\frac{1}{2}}$ $\pm 3^{-\frac{1}{2}}$, $\pm 3^{-\frac{1}{2}})$. A further twelve symmetric beams may be added by joining the midpoint of each edge of the cube to the centre point, with the cosines $(\pm\sqrt{2}, \pm\sqrt{2}, 0)$, $(0, \pm\sqrt{2}, \pm\sqrt{2})$ and $(\pm\sqrt{2}, 0, \sqrt{2})$ where $\pm$ signs are to be taken independently. Combination of these symmetry groups will therefore allow configuration of 6, 8, 12, 14, 18, 20 and 26 beamlets on target.

The results of a series of typical calculations are presented for a 6-beam and a 26-beam irradiation of a homogeneous solid carbon sphere of radius R = 1 mm. Each beam has a gaussian half-width (k in eqn. (6))equal to the radius of the target and is normalised to unit current ($j_0^i = 1$ in eqn.(6)) The ions have the slowing down power S(E) shown in fig. 4 which represents a hypothetical heavy ion of energy 10 GeV with a range of 0.24 mm in graphite.

Figure 5 displays the energy deposition function W(r) in a diametral plane for a 6-beam irradiation. The plane is chosen to be perpendicular to one pair of oppositely directed beams, and contains the maximum overlap effect of the incident beams.

A number of observations can be made. Firstly, the deposition distribution with the minimum number of beamlets contains gross modulations with amplitudes varying by up to a factor two, a result which is repeated in planes parallel to the

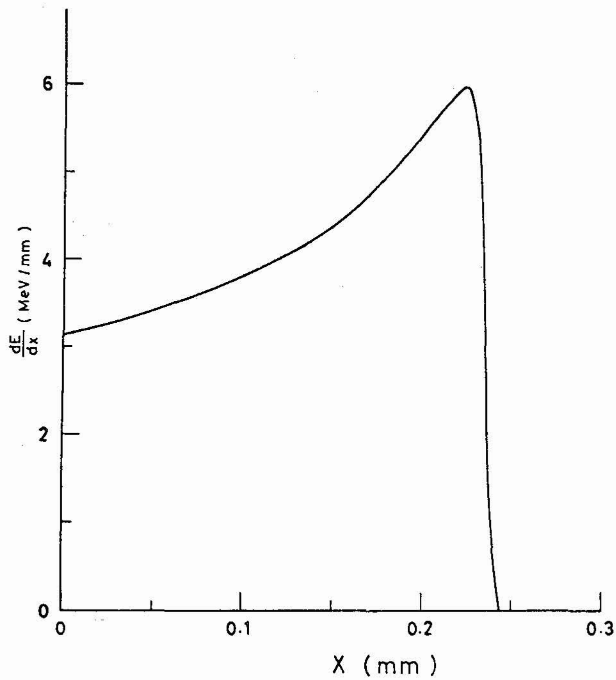

Fig. 4 The slowing down power S(E) used in calculations.

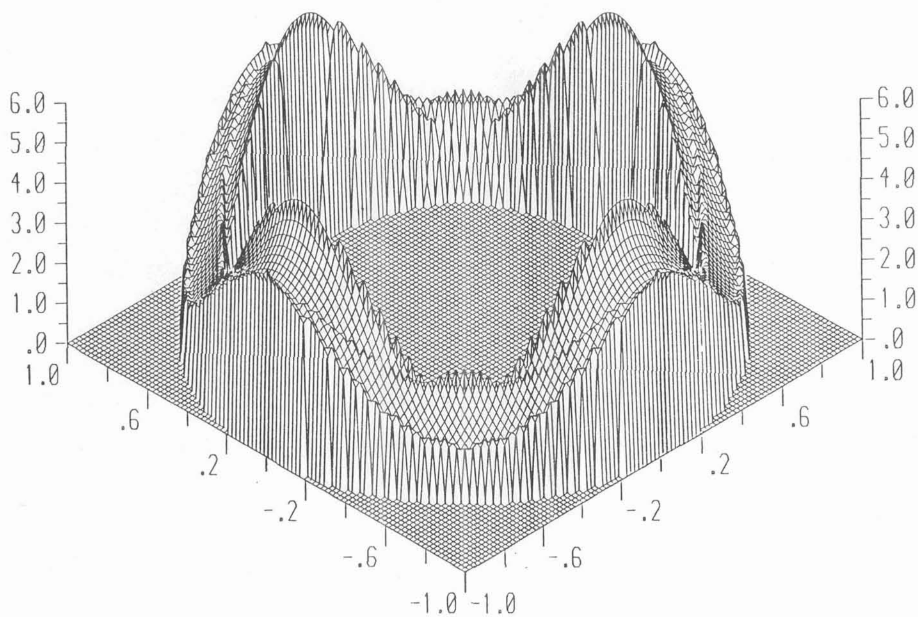

Fig. 5 Deposition function (in arbitrary units) in diametral plane for a 6-beam irradiation.

diametral plane. Secondly, fine structure is evident in the distribution because of the penetration of the two beams perpendicular to the plane at distances $r > (R^2-\lambda^2)^{\frac{1}{2}} = 0.94$ mm, where λ is the ion range. Fine structure is also produced by overlap of the remaining four beams. This effect is shown in figure 6, which is the distribution contained in area ABCD of figure 7.

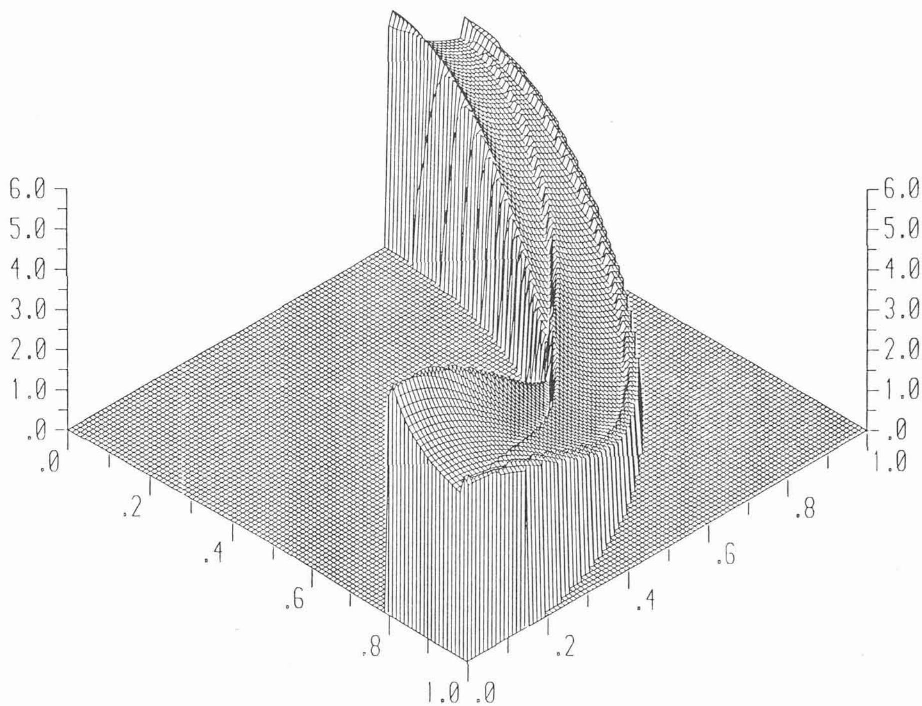

Fig. 6 Deposition function (in arbitrary units) in one quadrant (area ABCD of fig 7) of diametral plane for a 6-beam irradiation.

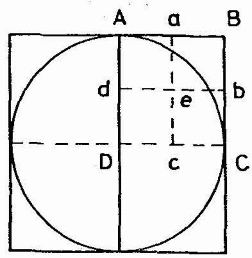

Fig. 7 Definition of various areas of diametral plane.

Figures 8 and 9 contrast the energy deposition function in the same plane in area aBbe (fig 7) for a 6-beam and 26- beam irradiation, respectively. It is seen for the latter case that the fine-structure fluctuations have disappeared although the gross variation persists. Similar effects occur in planes parallel to the diametral plane considered.

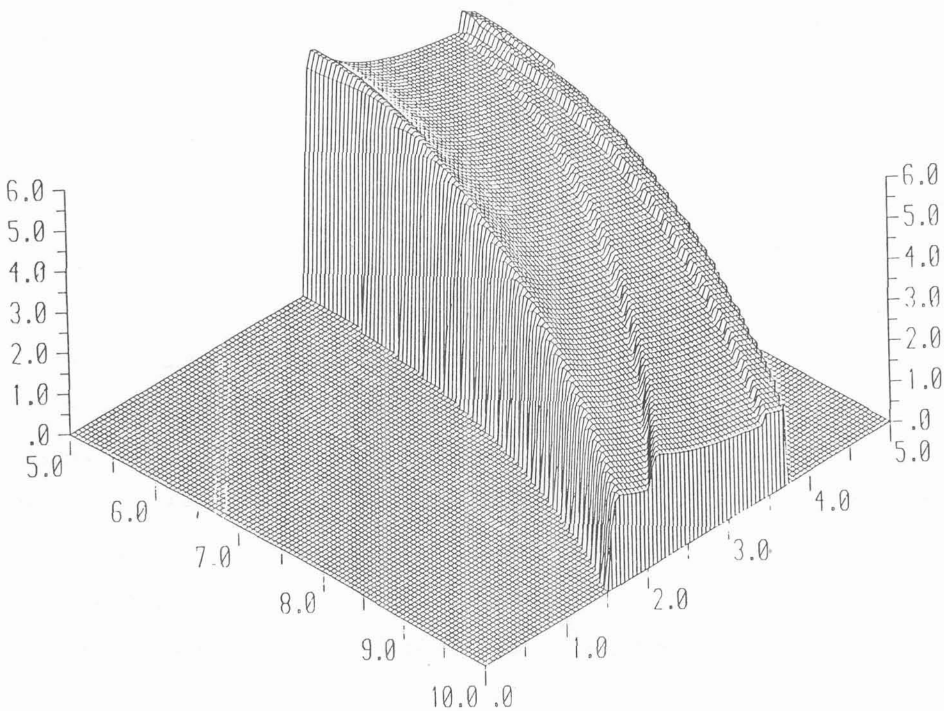

Fig. 8 Deposition function (in arbitrary units) in area aBbe (cf. fig 7)for a 6-beam irradiation.

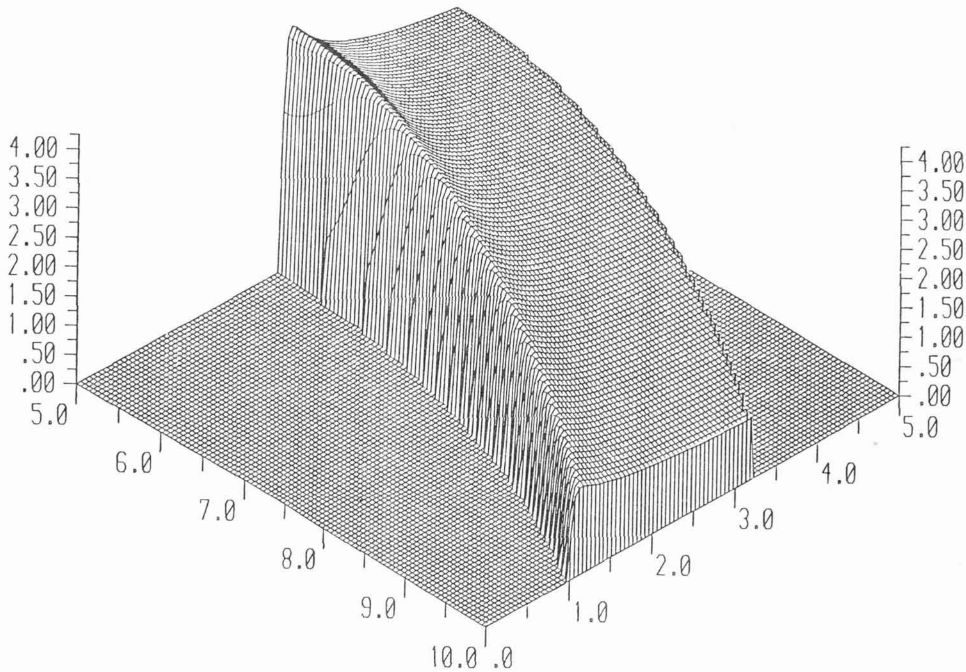

Fig. 9 Deposition function (in arbitrary units) in area aBbe (cf. fig 7)for a 26-beam irradiation.

5. Conclusions

It is apparent from these results that in evaluating target performance the beamlet geometry is as important as the detailed description of the slowing down processes contained in S(E). A few-beam irradiation produces rapidly fluctuating energy deposition profiles, which are significantly smoothed when more than about 20 beams are used, as is the case for the HIBALL target design (1). Quantitative estimates of the effects of these fluctuations require a detailed analysis with a suitable eulerian hydrodynamics code. Moreover, it appears that a full three-dimensional approach would be necessary. Some preliminary two-dimensional analysis of this problem has been reported by

Buchwald et al[2] but the present results would seem to indicate that considerable ambiguity could exist as a result of the particlar choice of planar slice taken through the target for such two dimensional modelling. Additionally, the shape of $W(\underset{\sim}{r})$ would not remain invariant during the compression stage of the target, particularly if localised ion range shortening or lengthening occurs in regions of high temperature and density.

Acknowledgement

This work is part of a program in ion beam fusion supported by the Science and Engineering Research Council.

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