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To cite this version:

S. Huard, D. Chardon. RAYLEIGH SURFACE WAVES (RSW) GENERATED BY PHOTOTHERMAL PROCESS : THEORY. Journal de Physique Colloques, 1983, 44 (C6), pp.C6-91-C6-96. <10.1051/jphyscol:1983614>. <jpa-00223173>

HAL Id: jpa-00223173
https://hal.archives-ouvertes.fr/jpa-00223173
Submitted on 1 Jan 1983

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RAYLEIGH SURFACE WAVES (RSW) GENERATED BY PHOTOTHERMAL PROCESS:
THEORY

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Résumé : Une étude théorique de l'excitation d'ondes de Rayleigh à la surface de solides isoropes par voie thermoélastique est présentée. Nous donnons ensuite une estimation du coefficient de conversion thermique acoustique.

Abstract : Thermoelastic excitation of RSW is theoretically investigated on isotopic solids. Thermal-acoustic conversion efficiency is evaluated.

It is well known that Rayleigh surface waves (RSW) can propagate on the surface of solids. Generally, such waves are excited by piezoelectric coupling on non-isotropic substrate. On isotropic materials a thermoelastic coupling appears as a suitable way to generate RSW. Recently, experimental investigations have been carried out with heat sources created by optical absorption /I,2/, but no complete theory of this effect exists.

In this paper, we present a monochromatic approach of this problem which leads to the exact determination of the displacement vector field, stresses and all other thermal and elastic variables. While we have mainly dealt with RSW, the method yields also bulk waves (compressional and shear waves). From thermoelastic equations /3/, which are coupled, solutions are expressed in terms of thermal mode and of compressional and shear modes as usual.

A one-dimensional spatial Fourier transform connected with suitable outgoing (or decaying) wave condition is used to solve the problem. In order to prove the small influence of the gas above the substrate on RSW, we have compared the solutions in two configurations : solid in vacuum and solid in a gas (air or helium). We have also theoretically studied the amplitude of RSW on different isotropic solids for the same heat source amplitude.

Moreover, numerical computations show that the thermal RSW conversion efficiency can be easily evaluated.

I - THEORY

When a light beam is focused on the surface of an absorbing solid, a part of the electromagnetic power is converted into heat. Moreover, if the light beam is sine modulated, elastic waves are generated by thermoelastic processes /4/. In order to simplify the mathematical treatment, only a line source along the z-axis is considered (Fig. 1).

When a solid is highly absorbing such as a metal or when the heat source appears on account of surface plasmon wave phenomena, the heat source can be reduced to a surface line source \( \delta(x) \delta(y) \). Every thermoelastic properties can be derived from the temperature field \( T(r,t) = T(r) \exp(i\omega t) \) and form the displacement vector \( u(r,t) = u(r) \exp(i\omega t) \). (As usual in harmonic time-dependent processes, complex
amplitudes of fields are used). In order to exhibit compressional waves and shear waves, the displacement vector is split in its longitudinal part \( u_L(\mathbf{r}) = \nabla \phi \) and in its transverse part \( u_T(\mathbf{r}) = \text{curl} \, \psi \) where \( \phi \) and \( \psi \) are the well-known elastic potentials.

Because of the symmetry of the line source on the solid, every field is \( z \)-independent; it comes that only the \( z \)-component of the transverse potential \( \psi \) is significant, so we shall note \( \psi_2 = \psi \).

Dealing with harmonic thermoelastic processes in isotropic solids, temperature and elastic potentials are coupled by the following equations /3/:

\[
(1.a) \quad \Delta T - \frac{i \omega}{\lambda_S} T - i \omega \eta \phi = -A \delta(x) \delta(y) \\
(1.b) \quad \Delta \phi + k_L^2 \phi = mT \\
(1.c) \quad \Delta \psi + k_T^2 \psi = 0
\]

where \( \lambda_S \) is the thermal diffusivity of the solid, \( k_L \) and \( k_T \) are respectively the wave number of the longitudinal (or compressional) and of the transverse (or shear) wave, \( \eta \) and \( m \) are the coupling coefficients both of them being proportional to the thermal expansion coefficient /5/. \( A \) is the amplitude of the line heat source. It must be quoted that \( T \) and \( \phi \) are directly coupled by eq. (1.a) and eq. (1.b) when \( \psi \) is related to \( \phi \) only through boundary conditions on the \( y = 0 \) plane. For solving thermoelastic equations (1), suitable conditions have to be chosen on the plane \( y = 0 \). Generally, when surface wave generation is studied, the surrounding air is ignored: for example, when surface elastic waves are induced on a solid by means of a piezoelectric transducer, the power loss in the air is negligible. But in the present case, a part of the heat diffuses into the fluid. It becomes necessary to take into account the thermal properties of the upper medium through the thermodynamic equations of the fluid /5/:

\[
(2.a) \quad \Delta T - \frac{i \omega}{\lambda_F} T + i \frac{C_p}{\gamma a} \frac{\omega}{\lambda_F} p = -A \delta(x) \delta(y) \\
(2.b) \quad \Delta p + \gamma k_F^2 p = \gamma \alpha k_F^2 T
\]

which are analogous to eq. (1). (It must be quoted that no viscosity effects are considered, so shear waves cannot exist in the fluid). In eq. (2), \( \lambda_F \) is the fluid thermal diffusivity, \( \gamma \) is the ratio of specific heats \( C_p/C_V \), \( a = (\partial p/\partial T)_V \) and \( k_F \) is the wave number of the sound waves.

The usual boundary conditions are the continuity of the temperature, of the heat flow, of the normal displacement \( u_N \) and the continuity of the stresses \( \sigma_{XY}, \sigma_{YY} \). At this point, it must be noted that the influence of the fluid (air) is weak because of the ratio of thermal conductivities \( k_S/k_F \), much greater than \( 1 \approx 10^4 \) for aluminum-air system. To evaluate the influence of the gas on the elastic surface waves generation, two resolutions have been carried out. First, we consider that vacuum...
is in the \( y > 0 \) half space and then we solve the real problem (solid-gas).

I-1 SOLID IN VACUUM

In this case, the stresses \( \sigma_{xy} \) and \( \sigma_{yy} \) vanish on \( y = 0 \) as also the heat flow towards the gas. To solve eq. (1) a \( \lambda \)-Fourier transform is used according the definition:

\[
\hat{T}(u,y) = \int_{-\infty}^{+\infty} T(x,y) e^{2i\pi u x} \, dx \quad \text{etc...}
\]

For a given \( u \), the problem is reduced to one dimension. The system (1) becomes:

(3.a) \[
\frac{\partial^2 \hat{T}}{\partial y^2} - (4\pi^2 u^2 + i \frac{\omega}{\lambda}) \hat{T} + \frac{i\omega}{\lambda} \hat{\phi} = -A \delta(y)
\]

(3.b) \[
\frac{\partial^2 \hat{\phi}}{\partial y^2} - \left( 4\pi^2 u^2 - k_L^2 \right) \hat{\phi} = -m^2 \hat{T} = 0
\]

in which we have used eq. (1.b) to substitute \( \Delta \phi \) in eq. (1.a). In this transformation we set

\[
\lambda^{-1} = \lambda_s^{-1} + \omega \eta
\]

Introducing \( \sigma_{\lambda}^2 = 4\pi^2 u^2 - k_L^2 \) and \( \sigma_t^2 = 4\pi^2 u^2 + i \frac{\omega}{\lambda} \), \( \hat{T} \) and \( \hat{\phi} \) can be easily expressed for \( y < 0 \):

(4) \[
\hat{T}_-(u,y) = a(u) e^{\sigma_t Y} + a_s(u) e^{\sigma_L Y}
\]

(5) \[
\hat{\phi}_-(u,y) = t_s(u) e^{\sigma_t Y} + \beta(u) e^{\sigma_L Y}
\]

where \( \sigma_t \) and \( \sigma_L \) are chosen so that only \( y \) decaying waves or outgoing waves exist in the solid. \( a_s \) and \( t_s \) are related to coupling coefficients by

\[
a_s = \frac{i\omega k_L^2}{(k_L^2 + i\omega/\lambda)} \quad \sigma_s \equiv \eta \lambda_k_L^2
\]

and

\[
t_s = \frac{m}{(k_L^2 + i\omega/\lambda)} \equiv -i \frac{m\lambda}{\omega}
\]

because \( \frac{\omega}{\lambda} \gg k_L^2 \) in the MHz frequency range. In a similar manner:

(3.c) \[
\hat{\phi}(u,y) = \gamma(u) e^{\sigma_t Y}
\]

where \( \sigma_t^2 = 4\pi^2 u^2 - k_T^2 \); \( \sigma_t \) will be chosen to satisfy the outgoing wave condition. The amplitudes \( a(u), \beta(u) \) and \( \gamma(u) \) of the thermal mode (\( e^{\sigma_t Y} \)), longitudinal mode (\( e^{\sigma_L Y} \)) and transverse mode (\( e^{\sigma_T Y} \)) are derived from boundary conditions. We obtain:

\[
a(u) = A \frac{\Delta \sigma_t}{\Delta} \quad \beta(u) = -A \frac{t_s \Delta \sigma_t}{\Delta}
\]

\[
\gamma(u) = \frac{4i\pi u t_s (8\pi^2 u^2 - k_T^2) (\sigma_t - \sigma_L)}{\Delta}
\]

with \( \Delta = \sigma_t \Delta_0 - a_s t_s \sigma_L \Delta_1 \)

in which \( \Delta_0 = (8\pi^2 u^2 - k_T^2)^2 - 16\pi^2 u^2 \sigma_L \sigma_T \)
and $\Delta_1 = (8\pi^2 u^2 k_T^2 - 16\pi^2 u^2 \sigma_{T}^2)$.

Because $\frac{\omega}{\rho} > kL$ and $|a_{st0}| << 1$, it appears that the first term in $\Delta$ is preponderant unless $\Delta_0$ vanishes. It is important to note that $\Delta_0 = 0$ yields the Rayleigh phase velocity along the $x$-axis $v_R/\sqrt{\rho}$. Using the expression $\hat{u} = \text{grad} \phi + \text{curl} \psi$ the $x$-Fourier transform $\hat{U}_x$ and $\hat{U}_y$ of $u_x$ and $u_y$ are easily obtained as also $\sigma_{xy}$ and $\sigma_{yy}$.

I-2 SOLID IN A GAS

The main modification referred to the preceding case lies in the boundary conditions. Now, the continuity of temperature, of heat flow and of normal displacement for $y = 0$ must be written. Moreover, the normal stress $\sigma_{yy}$ equals the gas pressure $p$ at $y = 0$.

From the likeness of eq. 1 and eq. 3 a thermal mode and an acoustical mode are also introduced in the gas; outgoing (or decaying) wave conditions are naturally taken into account. The boundary conditions lead to five linear equation which will yield the amplitude of each mode either in the solid or in the gas. The solution is numerically derived because no simple algebraic expression can be obtained. The x-Fourier transform of the displacement is calculated as in § I-1.

II - NUMERICAL RESULTS

II-1 RSW AMPLITUDE

In order to simplify the physical interpretation, we have calculated the displacement $u_x$ and $u_y$ on the $y = 0$ plane. Both $|\hat{U}_x|$ and $|\hat{U}_y|$ exhibit a sharp peak for $u = u_R$ such that $\Delta_0(u_R) = 0$ corresponding to the Rayleigh phase velocity.

![Fig. 2: Amplitude of normalized $U_y$ for vacuum and two gases](image)

While only $|\hat{U}_y|$ is plotted on Figure 2 numerical computations give the same shape for $|\hat{U}_x|$ and in particular the same width at half-maximum. It appears that the width $\Delta u$ is minimum when no gas is present; this configuration corresponds also to the highest peak. When a gas covers the solid, the amplitude of the RSW decreases as it propagates along the $x$-axis because of thermal losses. According to Fourier transform properties, the damping length is around $1/\Delta u$.

In every cases the narrowness of the peak near $u_R$ justifies the vacuum model even for thermoelastic excitation of RSW. From now on we shall use this model. Because of the outgoing surface wave condition, only positive spatial frequencies $u$ must be considered for $x > 0$ when the inverse Fourier transform is calculated. This operation gives the amplitude of $u_x$ and $u_y$ (for example at $t = 0$) versus $x$(Fig. 3).
As usual a π/2 out of phase exists between \( u_x \) and \( u_y \), and the wavelength observed is the Rayleigh wavelength. We have compared two substrates (aluminum and copper). A small difference appears on the width of the |\( \delta u_y \)| and |\( \delta u_x \)| peaks; on copper RSW decay slower than on aluminum.

### II-2 THERMAL-RSW CONVERSION EFFICIENCY

For evaluating the thermal-RSW conversion efficiency it is necessary to calculate the flux of the elastic Poynting vector through a \( x = \text{cste} \) plane for a narrow spatial frequency band \( \Delta u \) around \( u_R \).

The \( x \) component of the time averaged Poynting vector is

\[
P_x = \frac{1}{4} \left[ \sigma_{xy} \left( \frac{\partial u_y}{\partial t} \right)^* + \text{c.c.} \right]
\]

because \( \sigma_{xx} = 0 \)

ie

\[
P_x = \frac{1}{4} \left[ -i \omega u_y^* \sigma_{xy} + \text{c.c.} \right]
\]

Because |\( \delta u_y \)| and |\( \delta u_x \)| are narrow peaks, they can be approximated with \( \delta \) functions so that \( P_x \) becomes

\[
P_x = \frac{1}{4} \left[ -i \omega (\Delta u)^2 B_u^x(y) B_o(y) + \text{c.c.} \right]
\]

where \( B_u(y) \) and \( B_o(y) \) are the amplitudes of \( \delta u(y) \) and \( \delta_{xy}(u) \) for \( u = u_R \).

The \( x \) component of the Poynting vector is derived from \( \vec{\Phi} = \nabla \Phi + \text{curl} \, \vec{\Psi} \) and from expression (5) and (6). An evaluation of the \( P_x \) flux per unit length along the \( z \) axis

\[
\Phi_{RSW} = \int_{-\infty}^{0} P_x(y) \, dy
\]

leads to \( \Phi_{RSW} = e \Phi_0^2 \) with \( e = 10^{-8} \, \text{W}^{-1} \text{m} \) for aluminum. This result shows that a great part of the light absorbed is used to heat the substrate.
CONCLUSION

We have studied the KSW excitation by thermoelastic processes on isotropic substrates. When the solid is covered with a gas a resolution of thermoelastic equations and thermodynamic equations in the gas shows a weak influence of this last fluid. This analysis justifies that the gas is usually ignored when RSW are studied even in the case of thermoelastic excitation.

While experimental investigation often use pulse sources, we have dealt with harmonic sources. The knowing of the phenomena versus the modulation frequency then allows a spectral analysis of the pulsed case.

REFERENCES