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TWO-DIMENSIONAL THEORY OF THE FEL AMPLIFIER FOR PULSED ELECTRON BEAMS*

C.M. Tang and P. Sprangle

Naval Research Laboratory, Washington, D.C. 20375, U.S.A.

Abstract.- We present a two-dimensional nonlinear self-consistent theory on the free electron laser (FEL) amplifier for a pulsed electron beam, where the pulse length is short compared with the length of the wiggler. The theory includes all available efficiency enhancement schemes.

The total radiation vector potential can be written as a sum of the input radiation and the excited radiation, i.e., \( \mathbf{A} = \mathbf{A}_{\text{in}} + \mathbf{A}_{\text{ex}} \). The excited radiation field can be written as a sum of Gaussian-like \( \text{TEM}_{00} \) modes with the spot size equal to the electron beam radius, i.e.,

\[
\mathbf{A}_{\text{ex}}(r,z,t) = \left( \frac{2}{k_{Dr}} \right) \left( \frac{w_{Dr}^2}{c^2} \right) \int_{\xi} \mathbf{d} \xi \mathbf{G}_{\text{ex}}(\xi, c(t-\tau), t) \mathbf{h}(\xi, c(t-\tau)) e^{i k_{Dr} (\xi z - \xi^2 / 2 r^2)}
\]

where

\[
\mathbf{G}_{\text{ex}}(r,z,t|\xi, \tau) = \left( \frac{z^2 + z_b^2}{r_b^2} \right)^{-1/2} \exp\left[ -\left( \frac{r}{r_b} \right)^2 \left( \frac{z^2 + z_b^2}{r_b^2} \right) \right] \cos\left[ \frac{w}{c} \left( \sqrt{z_b^2 + (r/r_b)^2} \xi - \frac{2}{w} \xi \right) \right]
\]

is the Gaussian \( \text{TEM}_{00} \) mode, \( z_b = \frac{r_b^2}{w_{Dr}} \) is the Rayleigh length associated with the electron beam, \( r_b \) is the radius of the electron beam, \( \gamma = \gamma_{Dr} \gamma_\gamma \), \( \gamma_{Dr} = (1 + (e^2 / m_0 c^2))^{1/2} \), \( \gamma_\gamma = (1 - \gamma_{Dr}^2)^{-1/2} \), \( \gamma_b \) is the axial velocity, \( w_{Dr} = (4\pi e^2 / m_0)^{1/2} \) is the plasma frequency, and \( \gamma_{Dr} \) is the peak plasma density, \( \xi \) and \( \xi_0 \) are the position of the electron relative to the center of the electron beam at time \( t \) and \( t=0 \) respectively, and \( \xi_0 \) is the initial phase of the electron in the ponderomotive well. The amplitude and wavenumber of the vector potential of the wiggler are \( \mathbf{A}_{\text{w}} \) and \( k_{Dr} \). The quantity \( \mathbf{h}(\xi, \tau) \) represents the axial electron pulse shape and \( \tau \) is the retarded time, which is obtained from the equation \( \xi_0 + z_b(\tau) + c(\tau-\tau) = \xi + z(t) \), where \( z(t) \) is the macroscopic location of the center of the electron beam at time \( t \). In deriving this, we have assumed a transverse Gaussian profile for the electron beam and taken the radius of the electron beam to be much smaller than the radius of the radiation beam, thus neglecting the transverse variation of the electron dynamics in the calculation of the phase \( \gamma \).

In the stationary resonant particle limit our integrals can be evaluated analytically for various axial electron pulse profiles. This formulation can be modified to study the FEL oscillator.

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