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C. Tang, P. Sprangle

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A TWO-DIMENSIONAL THEORY OF PULSE PROPAGATION IN THE FEL OSCILLATOR

C.M. Tang and P. Sprangle

Naval Research Laboratory, Washington, D.C. 20375, U.S.A.

Abstract.

We present an axial symmetric analysis of the free electron laser (FEL) oscillator in the low gain regime. The electron beam consists of short pulses where the axial pulse shape is arbitrary and the transverse profile is Gaussian. The radius of the electron beam is taken to be much smaller than the radius of the radiation beam. We will consider the case where the resonator is designed to operate in the Gaussian TEM\(_{oo}\) mode. The portion of the stimulated radiation of interest is a superposition of the Gaussian mode, i.e., the vector potential of the radiation pulse can be written as

\[
\mathbf{A}_R(r,z,t) = \left(2\pi\right)^{-\frac{1}{2}} \int dk \, \mathbf{A}_{oo}(k,t) G_{oo}(r,k,z) e^{i(kz - \omega c t)},
\]

where \(G_{oo}(r,k,z)\) is the normalized complex amplitude associated with the TEM\(_{oo}\) mode and \(\omega_o\) is the resonant laser frequency. The equation governing \(\mathbf{A}_{oo}\) can be summarized in a rather compact form,

\[
\frac{\partial}{\partial t} \mathbf{A}_{oo}(k,\tilde{z}) = \alpha_{oo}(k,\tilde{z}) + \int_0^T dt \int_{-\infty}^{\infty} d\xi \, \gamma^{-1}(\xi,\tilde{z}) h(\xi) e^{-i\gamma(\xi,\tilde{z},t')} F(k,\tilde{z},t')
\]

where \(F(k,\tilde{z},t')\) is the self-consistent complex filling factor, \(\alpha\) is a constant, \(h(\xi)\) is the arbitrary axial electron profile, \(\xi\) is the axial position of the electron relative to the center of the \(N\)th electron pulse at \(t = t_N\), \(t_N\) is the time the \(N\)th electron pulse entered the wiggler, and \(\gamma(\xi,\tilde{z},k,t)\) is the phase of the electron.

The particle dynamics of the electrons enter the calculation through the phase \(\gamma\). Taking a constant wiggler as an example, the phase equation can be written as

\[
\frac{d^2}{dt^2} \gamma(\xi,\tilde{z},k,t) = \alpha_{oo} e^{i\phi_{oo}} - \int_0^\infty dk \, \mathbf{a}_{oo}(k,\tilde{z}) g_{oo}(0,k,\tilde{z}) \sin[\gamma(\xi,\tilde{z},k,t) + \phi_{oo}(0,k,\tilde{z})] e^{i\phi_{oo}},
\]

where \(\mathbf{a}_{oo} = \mathbf{a}_{oo} e^{i\phi_{oo}}\) and \(\phi_{oo} = \phi_{oo} e^{i\phi_{oo}}, \gamma = (kz + \omega_o) + \phi(\xi,\tilde{z},k,t) / (k_z + k + \omega_o / c)\) is the axial electron velocity, \(\zeta = \int_0^t dt' \gamma(\xi,\tilde{z},k,t') + \xi\) is the axial position of the electron at time \(t\), the \(\gamma\) over the variables stand for Langrangian variables which are functions of \((\xi,\tilde{z},t)\), and \(\alpha_{oo}\) are constants.

Equations (2) and (3) form a complete set of nonlinear self-consistent equations which govern the dynamics of the radiation pulses in the oscillator.

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