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LINAC TECHNOLOGY FOR FREE-ELECTRON LASERS

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Résumé- On passe en revue la technologie des accélérateurs linéaires de puis-
sance élevée destinés aux lasers à électrons libres.

Abstract- Electron linear accelerator technology for high-power, short-wave-
length free-electron lasers is reviewed.

I. INTRODUCTION

The first successful free-electron laser (FEL) experiments1–5 used linear
accelerators as their source of high-energy electrons. Perceived limitations in
linac performance have led experimenters to propose free-electron lasers using
electron beams in storage rings6 and even in betatrons and synchrotrons.7
Experiments have been proposed that use low-voltage high-current beams8 to
explore free-electron laser interaction in the collective or Raman regime. The
purpose of this paper is to concentrate on the properties of high-energy electron
linear accelerators for use in free-electron lasers operating principally in the
Compton regime. To fix our focus somewhat, we shall consider electron energies in
the 20- to 200-MeV range and consider requirements for high-power free-electron
lasers operating in the 0.5- to 10-μm range. Thus we will not discuss
Cockroft-Walton type accelerators nor pulsed-diode machines.

We present first a little motivational analysis that will introduce the two prin-
cipal types of linacs and indicate the utility of both for free-electron laser
work. At the same time we hope to point out that the accelerator and its design is
an integral part of the free-electron laser system, and not just a given entity
around which laser designs must be based. We direct our preliminary remarks toward
high-power free-electron laser amplifiers and oscillators and deduce some desirable
characteristics of the linacs that deliver electron beams for these devices.

For a high-power amplifier (that is, a high-gain single-pass system) to work, we
must have a high-power electron beam. Since the individual electron energy is
fixed by the wiggler design and operating optical wavelength, this means that we
are interested in high beam currents. The length of the electron beam bunch is
only constrained by the requirement that the electron beam-optical beam slippage be
small compared to the electron beam length, that is, ⁠≤<⁠-bunch⁠≥⁠λ⁠s⁠N⁠w⁠, where λ⁠s⁠ is
the optical wavelength, and N⁠w⁠ is the number of periods in the wiggler. High

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Efficiency extraction of the electron energy in the single pass of the amplifier dictates that we operate in the nonlinear regime, that the wiggler be tapered, and that initial optical field amplitudes should be high to trap the maximum number of electrons. The repetition rate of the linac is determined by the application and may not necessarily be particularly high (for example, in heavy ion fusion applications the rep rate may be a few hertz).

For an oscillator application, we need not require large gains because since the photons in the system pass through the wiggler region many times and can interact with many electron pulses. The gain need only be sufficient to overcome cavity losses and to provide whatever energy is coupled out of the oscillator system. The repetition frequency of the electron beam pulses is, however, determined in this mode of operation by the round-trip time of the optical pulse in the optical resonator; that is, for typical laboratory dimensions we are talking about some 10's to 100's of nanoseconds between electron pulses. Thus we are talking about pulse repetition frequencies from 10 to 100 MHz or higher. Again, the electron bunch length must be $\gg \lambda_s N_w$ (for example, a 100-period wiggler with $\lambda_s = 1 \text{ \mu m}$ implies $\lambda_{e\text{-bunch}} = 0.1 \text{ mm}$).

Fortunately for our field of endeavor, both the high peak-current requirement of the amplifier and the high pulse-repetition frequency requirement of the oscillator can be met by present-day linac technology, although not necessarily by the same machine. As we shall subsequently describe in more detail, both the induction linac and the rf linac appear to have significant roles to play in high-power free-electron laser technology. A few quick statistics here may be in order to orient the reader to the characteristics of these two rather different types of linear accelerator. The most advanced induction linac at the present time is nearing completion of construction near Livermore, California. It is the Advanced Test Accelerator (ATA) constructed by the Lawrence Livermore National Laboratory. The ATA will have a 50-MeV electron energy, a 70-ns, 10-kA electron pulse with a 1-kHz rep rate for short bursts, with an overall 5-Hz average rep rate. Thus the peak power of the ATA beam will be 500 GW; the short-term average power can be as high as 35 MW, while the long-term average is 175 kW.

The most advanced rf electron linac at present is the Stanford two-mile accelerator at the Stanford Linear Accelerator Center (SLAC). The SLAC machine is capable of accelerating to 20 GeV a 100-mA beam of 1.5-\(\mu\)s duration at a repetition rate of 360 Hz. Each 20-GeV micropulse contains $5 \times 10^8$ electrons and has a 3-ps duration. Thus the peak power (delivered by a micropulse) is 5 TW, the average power in the 1.5-\(\mu\)s pulse is 2 GW, and the long-term average power in the SLAC beam can be as high as 1 MW.

While most electron rf linacs use a travelling wave accelerating structure (see Sec. III below) such as that of the SLAC accelerator, an attractive alternative for high-power cw operation lies in a high-impedance standing-wave structure of the type used at the high-energy end of a proton linac such as the Los Alamos Meson Physics Facility (LAMPF) accelerator. This machine provides an average current of 1 mA at a kinetic energy of 800 MeV. At the end of the linac, a micropulse approximately 20 ps in duration contains $5 \times 10^8$ particles, for a peak current of the order of 40 A, with a 201.25-MHz repetition frequency. During the time the machine is accelerating, nominally 6% of the time, the average current is 16 mA, and the long-term average current is 1 mA. Thus the LAMPF peak beam power is 32 GW, the short-term average power is 12.8 MW, and the long-term average is 800 kW.

II. INDUCTION LINACS

An induction linac consists of an injector and a series of acceleration modules each of which presents a relatively constant, essentially electrostatic, accelerating voltage to the beam. This voltage is usually maintained across the accelerating gap for some few nanoseconds (depending on the geometry) without the use of
magnetic materials, some hundred or so nanoseconds using ferrite cores, and some few microseconds using ferromagnetic or metglas cores. The use of the various core materials is dictated primarily by cost and space considerations. Each module is energized in time so that it has achieved its desired voltage before the arrival of the beam. Figure 1 (from Ref. 9) shows a conceptual development of the accelerator module idea. Figure 2 shows a more or less realistic induction-linac acceleration-module design. The "voltage" $V = \int E dz$ across the accelerating gap can be maintained for a time $T = \Delta \Phi/V$ where $\Delta \Phi$ is the magnetic flux change achievable in the core. For a magnetic core material this flux change is $(B_{\text{saturation}} + B_{\text{remanant}}) \times \text{core area}$. For an air core this time is usually related to the time required for an electromagnetic wave to make a trip from the feed point to the module wall and back to the gap.

For the high currents and high peak powers (for example, the 2.5-GW power per ATA module delivered to the beam) used in past and present induction-linac designs, pulsed-power technology has been used to provide both the high voltage and high currents usually used. A typical charging circuit will use a modest (30-kV) resonant charging circuit that will in turn be used in a resonant charging transformer circuit to charge a Blumlein line typically to a few hundred kV. The Blumlein line, when a high voltage switch (typically a spark gap) is closed, will deliver a constant voltage pulse to a properly matched load for a time equal to twice the time it takes an electromagnetic wave to travel the length of the line. For long pulses, in order to keep the Blumlein line dimensions reasonable the line may be filled with a dielectric, such as water. For even longer pulses (in the usec range) the Blumlein pulse-forming line would be replaced by a lumped-element pulse forming network (PFN).

A somewhat simplified analysis (from Ref. 10) of the action of the induction-linac module, fed by a transmission line of characteristic impedance $Z_0$, uses the concept of a gap impedance, $Z_g$, which for high frequencies is given by

$$Z_g = 60 \, \text{g/a ohms} \quad ,$$

where $g$ is the gap length and $a$ is the gap radius. The voltage across the gap will result from three terms: the incident voltage from the feed line $V_0^+$, the voltage reflected from the gap, $V_r^+ = V_0^+(Z_g - Z_0)/(Z_g + Z_0)$, and the voltage wave generated by the beam current $V_b^+ = I_b(Z_g Z_0)/(Z_g + Z_0)$ since the beam "sees" the gap impedance paralleled by the transmission-line impedance. The efficiency of the induction-linac module can be defined as the ratio of the power delivered to the beam to the power delivered to a matched load. This efficiency is

$$\eta = \frac{V_{\text{gap}} I_b}{(V_0^+ + V_r^- + V_b^-) I_b} = \frac{V_0^+ Z_g}{Z_g^2 + \frac{Z_0}{Z_g}} = \frac{Z_g}{Z_g + \frac{Z_0}{Z_g}} \left( 2 - \frac{I_b Z_0}{V_0} \right) \left( \frac{I_b Z_0}{V_0} \right) .$$

When maximized, this efficiency is

$$\eta_{\text{max}} = \frac{Z_g}{Z_g + Z_0} .$$
which can be made close to unity by choosing $Z_0 \ll Z_g$. The achievement of such a high efficiency, however, would require a precise matching of the voltage and current waveforms, and a specific design value of the beam current.

A somewhat different sort of induction linac embodied in the RADLAC design is shown in Fig. 3. After the central radial conductor has been charged, half the double gap is shorted by a coaxial array of spark gaps, so that a net voltage is seen across the gap. This voltage is constant until the wave generated by the shorting action of the spark gaps reaches the unshorted gap (the cavity is tapered to maintain a constant impedance). In the absence of dissipation in the cavity, the net voltage across the double gap is as shown in Fig. 4.

Present day technology permits an average accelerating gradient of approximately 1 MV/m (RADLAC = 3 MV/m), with a repetition rate approaching 1 kHz. Table I, taken from Ref. 9, gives parameters for most of the induction linacs built to date. At high repetition rates, switch lifetime becomes a problem. With some care, an energy spread $\Delta E/E$ of 0.1% can be achieved readily. Reference 10 gives switch characteristics, and Ref. 9 is a good review of induction-linac technology, including information on the effects of various core materials. Reference 12 discusses high beam-current accelerator research.

III. RF LINACS

The number of rf electron linacs in the world is of the order of 1000 (Ref. 13), and they are as diverse as their applications, which range from medical therapy to industrial radiography to nuclear physics. G. Loew13 has given a comprehensive review of these machines. The basic principle of operation is simply that an electron should be acted upon by an electromagnetic wave that, on average, presents a constant phase to the electron. If one considers confining the electromagnetic wave to the interior of a waveguide, then a modification of the waveguide must for be made to lower the phase velocity of the wave, since $v_{\text{phase}} > c$ in an empty waveguide of uniform cross section. In many rf electron linacs, this modification takes the form of loading with disks; that is, the disk-loaded waveguide supports electromagnetic waves with $v_{\text{phase}} \leq c$ that can be used to accelerate electrons.

An alternative approach to acceleration is to arrange for a series of resonant cavities energized with rf power and properly phased so that a given electron is acted upon by the same value of electric field in each cavity it transits. A linac constructed this way is referred to as a standing wave linac, and this is the accelerator approach upon which we shall concentrate in this paper.

By far the most popular standing wave accelerating structure in electron linacs is the side-coupled structure invented at Los Alamos. This structure is a development resulting from the observation that a chain of coupled resonators having a $\pi/2$ phase shift per cell is relatively insensitive to tuning errors. In such a structure every other cavity is unexcited, thereby contributing not at all to acceleration and cutting the average accelerating gradient in half. The average gradient can be increased by reducing the length of the unexcited cells (the bi-periodic system) or better yet, moving these unexcited cells off to the side (the side-coupled structure). Figures 5 and 6 (from Ref. 15) shows the conceptual development just outlined and a cutaway view of a typical side-coupled structure.

A figure of merit often defined for a linac structure is known as the (uncorrected) shunt impedance per unit length

$$Z_u = \left[ \frac{1}{E} \int E_z dz \right]^2 \text{ ohms per unit length}$$

\[ (4) \]
where $E_z$ is the axial electric field distribution at peak fields, $l$ is the length of one period of the structure, and $P/l$ is the power per unit length dissipated in the structure walls. Clearly this impedance parameter is a measure of the ability of a structure to provide an accelerating field for a given power dissipation.

An electron crossing a cavity with field given by $E_z(z) \cos \omega t$ will gain an energy

$$W = \frac{2}{g/2} \int E_z(z) \cos \omega t \, dz$$

if the field is at maximum strength when the electron is in the center of the cavity ($z=0$). In this expression, $g$ is the accelerating gap length and $v$ is the electron speed. If $E_z$ is independent of $z$, as in a TM$_{010}$ mode of a pillbox cavity neglecting the bore hole, this energy gain is

$$W = \frac{2}{g/2} \int E_z \cos \omega t \, dz = E_z g T$$

namely the "gap voltage" $E_z g$ times the transit-time factor $T = \sin(\omega g/2v)/(\omega g/2v)$. More generally, the transit-time factor may be defined as $\sin^2(\omega g/2v)/(\omega g/2v)$. More generally, the transit-time factor may be defined as $16$

$$T = \int E(z) e^{j\omega z/v} \, dz \int E(z) dz$$

or with more elaborate expressions if transverse effects are included.

The occurrence of the transit-time factor in the expression for energy gain makes the more meaningful definition of shunt impedance per unit length

$$Z_{sh} = Z u \frac{g^2}{2v} = E_z g T$$

Designers of accelerator structures have been working for many years to increase the shunt impedance as much as possible. Various coupling schemes have received theoretical and experimental treatment, and this is an area of active interest at present. Table II reproduced from Ref. 17 should give an idea of the nature of this effort.

Other parameters used to characterize a given accelerating structure are the quality factor $Q$ of a cell, and $Z_{sh}/Q$, where $Q \equiv \omega W/P$ with $W$ the energy stored in the resonator and $P$ the average energy dissipated. For a pillbox cavity, Wilson$^{16}$ gives the following relations:

$$Q = \frac{G_1}{R_s} \propto \omega^{-1/2}$$

$$Z_{sh}/Q = \frac{G_2 T^2}{\lambda} \propto \omega$$
\[ Z_{sh} = \frac{G_1 G_2 T^2}{\lambda R_s} \propto \omega^{1/2} , \]  
(11)

where \( R_s = (\omega \mu_0/2\sigma)^{1/2} = \pi \delta/\lambda \) is the surface resistance

(\( Z_0 = \) impedance of free space = 377 \( \Omega \), \( \sigma = \) conductivity of cavity walls and
\( \lambda = 2\pi c/\omega \)). The quantities \( G_1 \) and \( G_2 \) are functions of the geometry

(\( p_{01} = \) the first root of \( J_0(x) = 0 \)):

\[ G_1 = \frac{p_{01}}{2} \left( \frac{L}{b+L} \right) Z_0 = 453 \left( \frac{L}{b+L} \right) \text{ ohms} , \]  
(12)

\[ G_2 = \frac{4Z_0}{p_{01} J_1'(p_{01})} = 967 \text{ ohms} , \]  
(13)

\[ T = \frac{\sin (\pi L/\lambda)}{(\pi L/\lambda)} , \]  
(14)

where \( L \) is the length of the pillbox cavity, and \( b \) is the radius.

Finally, the resonator filling time, \( T_f \), which measures how rapidly fields build up in time when the driving voltage is applied, is

\[ T_f = \frac{2Q_L}{\omega} = \frac{2Q}{\omega(1+\beta)} , \]  
(15)

where \( Q_L \) is the loaded \( Q \) of the resonator, and \( \beta \) is the cavity coupling constant\(^{16} \) (usually \( \approx 1 \)).

Table III from Ref. 18 gives the frequency dependence of various parameters characterizing rf linear accelerators, and the normal preference in frequency for those parameters.

IV. LIMITATIONS

The energy spread that can be achieved in an rf electron linac is limited by the variation in rf accelerating field along the length of the bunch, and by single-bunch beam loading. For a Gaussian bunch of rms length \( \sigma_Z \), riding on the crest of an rf wave with wavelength \( \lambda \), the first effect gives

\[ \frac{\Delta V}{V} \approx \pm \frac{1}{2} \left( \frac{2\pi \sigma_Z}{\lambda} \right)^2 . \]  
(16)

The second effect, which depends on the charge per bunch, is due to the wake potential produced at the tail of the bunch by charge at the front of the bunch. The relative energy spread due to this effect is

\[ \frac{\Delta V}{V} \approx \pm \frac{k_1 q8(\sigma)}{E_a} , \]  
(17)
where $E_a$ is the accelerating gradient, and $k_1$ is a structure constant given by

$$k_1 = \frac{E_a^2}{4w} = \frac{\omega r}{4} \ . \tag{18}$$

Here $w$ is the energy stored per unit length, and $r/Q$ is the $Z_{sh}/Q$ per unit length. Note that $k_1$ scales as $\omega^2$. For the SLAC disk-loaded structure, $k_1 \approx 20$ $\Omega$/ps at 2856 MHz. Finally, $B(\sigma)$ in Eq. (17) is a function of bunch length that can be computed if the longitudinal wake potential is known for a given structure (the longitudinal wake potential is simply the potential as a function of distance following a unit point charge passing along the axis of the structure). For the SLAC structure, $B(\sigma_z)$ is 6.0 for $\sigma_z = 0$ and 3.0 for $\sigma_z = 1$ mm.

By adjusting the phase of the center of the bunch to ride an appropriate distance ahead of the crest of the rf accelerating wave, the rising slope of this wave can partially compensate for the decelerating wake potential. By this means the energy spread given by Eq. (17) can be reduced by perhaps a factor of 5, but with some loss in the mean energy of the bunch.

The transverse emittance (see Appendix) of an rf linac beam is limited by cumulative beam breakup and by single-bunch emittance growth. Cumulative beam breakup is a complex phenomenon in which off-axis bunches at the front of a long train of bunches interact with the (usually) lowest frequency dipole mode in the structure to produce a deflection away from the axis for the following bunches. Significant emittance growth can be expected when the quantity

$$\frac{I_0 zc t(r_j/Q)}{V'\lambda_b^2}$$

becomes comparable to unity. Here $I_0$ is the average current during the bunch train, $V'$ is the accelerating gradient, $z$ is the accelerator length, $c$ is the length of the bunch train, $\lambda_b$ is the wavelength of the breakup mode, and $r_j/Q$ is the average transverse shunt impedance per unit length of the structure. For breakup in the constant gradient SLAC structure, $\lambda_b \approx 1.4 \lambda$ and $r_j/Q \approx 130$ $\Omega$/m. For a uniform (not constant gradient) disk-loaded structure,

$$\frac{r_j}{Q} \approx \frac{100\Omega}{\lambda} \ . \tag{19}$$

Single bunch emittance growth is again due to the transverse force produced by off-axis charges at the front of the bunch acting to deflect the tail of the bunch. If a bunch is undergoing betatron oscillations with a wavelength $\lambda_b$, the amplitude of the oscillations at the tail of the bunch will be greater than that at the head of the bunch by the ratio

$$A \approx \frac{qzw_l(2\sigma_z)\lambda_b}{8\pi V_o} \ , \tag{20}$$

where $q$ is the total bunch charge, $z$ is the length of the accelerator, $V_o$ is the average energy and $w_l(2\sigma_z)$ is the transverse dipole wake potential at $2\sigma_z$. Details of the dipole wake are given in Ref. 15. As an example, for the SLAC disk-
loaded structure \( w_l = 3 \times 10^{15} \) V/Cm\(^2\) at \( 2\sigma_x = 2 \) mm. Note that the amplitude of the dipole wake per unit length of structure varies as \( \omega^3 \) at time \( t \sim \omega^{-1} \).

Induction linacs also suffer from the beam breakup instability. Although the induction-linac structure is not intentionally resonant, there exist TM\(_{\text{inm}}\)-like resonances in practical structures that can deflect the beam from its desired axial position. While the transverse shunt impedance for these modes can be deliberately lowered, the high currents employed make the beam breakup instability a troublesome feature of linac operation.\(^{19}\)

The energy spread in an induction linac arises principally from two sources: time variation of the gap voltage and the beam potential. Present day practice will permit \( \Delta \gamma/\gamma \) from the former source to be readily adjusted to about 0.1%. The \( \Delta \gamma/\gamma \) due to the beam potential (Ref. 20) has the value \( 301/\gamma \gamma E_0 \) for a uniform-density beam of current \( I \) and particle-rest energy \( E_0 \) in electron volts. For short transport distances with laminar flow, it is possible that the space-charge potential could be counteracted at the source end with a segmented cathode with a radial potential variation, or (for an equipotential source) by dividing the beam at the load end among several electrostatically shielded channels or by neutralizing the space charge with other charged particles. For long transport systems with considerable betatron oscillation motion, the first two schemes are impractical, and a \( \Delta \gamma/\gamma \) of the order of that cited above may be considered to be a property of the beam.

V. APPLICATIONS TO THE FEL

Regardless of whether we have an induction linac or an rf linac as our source of high-energy electrons to drive an FEL amplifier, efficient conversion of the beam energy to radiation requires that both the energy spread \( \Delta \gamma/\gamma \) and the beam emittance be small. For good efficiency we want the quantity

\[
\frac{4 \Delta \gamma w_s^2}{1 + a_w^2} > \frac{\Delta \gamma}{\gamma}.
\]

that is, the bucket height should be at least of the order of the energy spread. In this relation \( a_w \) and \( a_s \) are the usual electromagnetic vector potentials of the wiggler and optical fields, respectively, multiplied by \( e/m_0 c \) (MKS units). Thus for good efficiency of operation in an amplifier, the input laser intensity required (\( \propto a_w^2 \)) will be proportional to the fourth power of the energy spread; that is,

\[
\text{input laser} \propto (\Delta \gamma/\gamma)^4.
\]

For oscillator operation this same dependence on energy spread relates to the circulating intensity in the optical resonator.

The electron beam quality as measured by its emittance (see Appendix) plays an important role in the trapping process. It can be shown that the (unnormalized) emittance gives rise to a minimum effective energy spread

\[
(\frac{\Delta \gamma}{\gamma})_{\text{effective}} = \frac{\varepsilon a_w}{\gamma a_s^2},
\]

(21)
so that the optical intensities of the amplifier input laser and the oscillator circulating beam may depend strongly on the emittance.

Both induction linacs and rf linacs at present satisfy the Lawson-Penner scaling law, \( \pi \varepsilon = 50 \pi I^{1/2}/B \gamma \). It does not seem unreasonable at this time, there being apparently no fundamental limit other than the cathode emission limit, to expect that with careful research and development this emittance can be improved by an order of magnitude or so.

APPENDIX

Emittance

The concept of emittance provides one measure of accelerator beam quality. In transverse phase space (\( x-x' \) or \( y-y' \) where the prime indicates derivative with respect to distance along the beam propagation direction) the beam occupies a certain area. This area, divided by \( \pi \), is called the emittance, denoted by the symbol \( \varepsilon \). The factor \( \pi \) results from the practice of circumscribing the actual phase-space area, which may look like a bow tie or an S-shaped figure or something more complex, with an ellipse. If the ellipse is upright, as it will be at a beam waist or antiwaist, the phase-space area will be given by \( \pi x_0 x'_0 \), where \( x_0 \) and \( x'_0 \) are the ellipse semiaxes. If the transverse forces acting on the individual particles in the beam are all linear, then, in the absence of acceleration, the emittance can be shown to be a constant of the motion. If the beam is accelerating, then the emittance decreases by the factor \( B \gamma \); that is, the normalized emittance

\[
\varepsilon_n = \frac{\text{phase space area}}{\pi} B \gamma
\]

is constant.

In electron linacs one source of emittance is the thermal motion of the electrons in the cathode. A finite cathode temperature gives rise to a distribution of electron energies transverse to the direction of emission. The emittance resulting from this thermal distribution of electron velocities is given by (Ref. 20, p. 221)

\[
\gamma B \varepsilon = 2R(kT/m_o c^2)^{1/2},
\]

where \( R \) is the cathode radius.

The current drawn from the cathode will be proportional to the cathode area, that is, \( I^{1/2} \propto R \). Using a cathode current density 10 A/cm\(^2\) and a cathode temperature giving \( kT = 0.1 \text{ eV} \), we can write Eq. (A.2) as

\[
\pi \varepsilon = 1.6 \pi I^{1/2}(A)/B \gamma \text{ mm•mrad}.
\]

Any grid present will distort electron trajectories, giving transverse motion that serves to increase the emittance. Any nonlinear forces resulting from focusing and extraction elements, from space-charge fields, and from the accelerating fields, may also cause the emittance to increase.

An empirically determined law relating emittance to average current in electron linacs states that the emittance satisfies

\[
\pi \varepsilon = 50\pi I^{1/2}(A)/B \gamma \text{ mm•mrad}.
\]
This relationship is widely known as the Lawson-Penner scaling law and holds, within factors of 2 or 3, for a large number of electron linacs. Emittances obeying this law are well over an order of magnitude greater than the emittance due to the cathode alone, indicating that careful attention to details in electron gun design and the design of focusing and accelerating systems may bear fruit in terms of significant reduction of emittance.

Fig. 1. Conceptual model of an induction module.

Fig. 2. Induction acceleration cavity and voltage generator.
Fig. 3. RADLAC: radial line geometry.

Fig. 4. Unshorted gap voltage vs time.
Fig. 5. The $\pi/2$-mode operation of a resonant cavity chain.

Fig. 6. The side-coupled cavity chain. The accelerating cavities are shaped for maximum shunt impedance, and the coupling cavities are staggered to reduce asymmetries introduced by the slots.
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<tr>
<td>Rep Rate (PPS)</td>
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<td>1</td>
<td>5</td>
<td>1/3</td>
<td>1000 Burst</td>
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<td>Number of Switch Modules</td>
<td>1500</td>
<td>250</td>
<td>10/200</td>
<td>54</td>
<td>24</td>
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<tr>
<td>Core Type</td>
<td>Ni-Fe Tape</td>
<td>Fe Tape</td>
<td>Ferrite</td>
<td>Ferrite</td>
<td>Water</td>
<td>Oil</td>
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<tr>
<td>Switch</td>
<td>Thyratron</td>
<td>Spark Gap</td>
<td>Spark Gap</td>
<td>Spark Gap</td>
<td>Spark Gap</td>
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<tr>
<td>Module Volt</td>
<td>250kV</td>
<td>400kV</td>
<td>250kV</td>
<td>400kV</td>
<td>500kV</td>
<td>1.75MV</td>
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<tr>
<td>Core Volt</td>
<td>22kV</td>
<td>40kV</td>
<td></td>
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<tr>
<td>Accel Length</td>
<td>210m</td>
<td>250m</td>
<td>10/53m</td>
<td>40m</td>
<td>3m</td>
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<tr>
<td>Structure</td>
<td>Disk and washer</td>
<td>Side coupled</td>
<td>On-axis coupled</td>
<td>Ring coupled</td>
<td>Coaxial coupled</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>-----------------------------------</td>
<td>-----------------</td>
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<td>-----------------</td>
<td>--------------</td>
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<tr>
<td>$ZT^2$ (M2/m)</td>
<td>90</td>
<td>59</td>
<td>55</td>
<td>59</td>
<td>55</td>
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<tr>
<td>Coupling (%)</td>
<td>50</td>
<td>5</td>
<td>11$^b$</td>
<td>18</td>
<td>12$^b$</td>
<td></td>
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<tr>
<td>Outer radius (cm)</td>
<td>16.8</td>
<td>9.0$^a$</td>
<td>9.0</td>
<td>14.7</td>
<td>12.9</td>
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<td>Cooling</td>
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<td>medium</td>
<td>easy</td>
<td>medium</td>
<td>easy</td>
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<tr>
<td>Ease of manufacturing</td>
<td>difficult</td>
<td>difficult</td>
<td>easy</td>
<td>medium</td>
<td>easy</td>
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<tr>
<td>Material cost (% of total structure cost)</td>
<td>2.5</td>
<td>2</td>
<td>2</td>
<td>5.7</td>
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<td>Vacuum conduction</td>
<td>high</td>
<td>fair</td>
<td>low</td>
<td>fair</td>
<td>low</td>
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<tr>
<td>Direct beam excitation of higher order axially symmetric modes in accelerating (A) and coupling (C) cells</td>
<td>unknown</td>
<td>A only</td>
<td>A and C</td>
<td>A only</td>
<td>A only</td>
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<tr>
<td>Direct beam excitation of beam break-up (BBU) modes (not calculable)</td>
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<td>A only</td>
<td>A and C</td>
<td>A only</td>
<td>A only</td>
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<td>BBU mode propagation</td>
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<td>unknown</td>
<td>between unknown</td>
<td>no neighboring cells</td>
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</table>

$^a$Does not include side couplers.
$^b$Could be larger.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Frequency preference</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shunt impedance per unit length (r)</td>
<td>( f^{1/2} )</td>
<td>X</td>
</tr>
<tr>
<td>RF loss factor (( Q ))</td>
<td>( f^{-1/2} )</td>
<td>X</td>
</tr>
<tr>
<td>Filling time (( t_f ))</td>
<td>( f^{-3/2} )</td>
<td>X</td>
</tr>
<tr>
<td>Total RF peak power</td>
<td>( f^{-1/2} )</td>
<td>X</td>
</tr>
<tr>
<td>RF feed interval (( \lambda ))</td>
<td>( f^{-3/2} )</td>
<td>X</td>
</tr>
<tr>
<td>No. of RF feeds</td>
<td>( f^{3/2} )</td>
<td>X</td>
</tr>
<tr>
<td>RF peak power per feed</td>
<td>( f^{-2} )</td>
<td>X</td>
</tr>
<tr>
<td>RF energy stored in accelerator</td>
<td>( f^{-2} )</td>
<td>X</td>
</tr>
<tr>
<td>Beam loading ((-dV/di))</td>
<td>( f^{1/2} )</td>
<td>X</td>
</tr>
<tr>
<td>Peak beam current at maximum conversion efficiency</td>
<td>( f^{-1/2} )</td>
<td>X</td>
</tr>
<tr>
<td>Diameter of beam aperture</td>
<td>( f^{-1} )</td>
<td>X</td>
</tr>
<tr>
<td>Maximum RF power available from single source</td>
<td>( f^{-2} )</td>
<td>X</td>
</tr>
<tr>
<td>Maximum permissible electric field strength</td>
<td>( f^{1/2} )</td>
<td>X</td>
</tr>
<tr>
<td>Relative frequency and dimensional tolerances</td>
<td>( f^{1/2} )</td>
<td>X</td>
</tr>
<tr>
<td>Absolute wavelength and dimensional tolerances</td>
<td>( f^{-1/2} )</td>
<td>X</td>
</tr>
<tr>
<td>Power dissipation capability of accelerator structure</td>
<td>( f^{-1} )</td>
<td>X</td>
</tr>
</tbody>
</table>

Notes:
- a. For direct scaling of modular dimensions of accelerator structure.
- b. For same RF attenuation in accelerator section between feeds.
- c. For fixed electron energy and total length.
- d. For fixed total length.
- e. When limited by cathode emission.
- f. When limited by beam loading.
- g. Approximate; empirical.
REFERENCES


