SIMULATION OF ISOCHRONOUS STORAGE RING FREE ELECTRON LASER

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Résumé.- On réalise une simulation unidimensionnelle d'un laser à électrons libres dans un anneau de stockage travaillant de façon isochrone pour un ensemble de paramètres visant à simuler le laser à électrons libres proposé pour l'anneau de stockage BESSY de Berlin.

Abstract.- A one dimensional simulation of a free electron laser in a storage ring operating close to isochronisme is performed for a set of parameters to match the free electron laser proposal for the Berlin storage ring BESSY.

The isochronous storage ring free electron laser (ISRL) |1| is potentially a high power tuneable laser of high efficiency. It solves in a natural way the problem of beam heating present in an ordinary storage ring free electron laser |2,3|. The ring isochronisme ensures repetive laser interaction with correlated initial conditions. Synchrotron radiation damping then leads to a stable density modulated electron beam in the ponderomotive potential of the free electron laser (FEL) that converts efficiently cheap radiofrequency (RF) energy into optical energy. Deacon |4| analyses the motion of a particle in a prescribed optical field, i.e. an amplifier. There are regions in longitudinal phase space extending a fraction of a wavelength where bound motion is possible, and particles can be trapped. Large small signal gain is found, and oscillator operation is possible. In this paper the slowly varying amplitude and phase approximation for the optical field together with the single particle pendulum equation |5| is used for a self consistent numerical simulation of the oscillator. Transverse effects are not considered.

The motion of an electron injected at z=O parallel to the z-axis into the wiggler field (constant wiggler strength)

$$\vec{B}(z) = \hat{y}\, B_o \cos k_o z \qquad (1)$$

in the presence of an optical field with electric component

$$\vec{E}_r(z,t) = \hat{x}\, E(z,t) \cos(k_r z - \omega_r t + \phi(z,t)) \qquad (2)$$

is governed by the pendulum equation

$$\ddot{\zeta} = -\Omega^2 \sin(\zeta+\phi) \qquad (3)$$

ϕ is the phase of the optical field. ζ is the phase difference of a monochromatic wave of wave number k_r and the transverse motion of the particle. It is related to the particle's position by

$$z(t) = \beta_o ct + \zeta/k_r \qquad (4)$$

where $\beta_o c$ is the longitudinal velocity on resonance

$$\beta_o = k_r/(k_r + k_o) \qquad (5)$$

The pendulum frequency of a linear wiggler is

$$\Omega^2 = \frac{e\,E\,B_o}{\gamma^2 m^2 c^2} \tag{6}$$

The field equations are

$$\frac{\partial E}{\partial z} + \frac{1}{c}\frac{\partial E}{\partial t} = 2\pi\ \frac{eK}{\gamma}\ \sin(\zeta+\phi)\ \ \delta(\vec{r}-\vec{r}_i(t))$$

$$E\left(\frac{\partial \phi}{\partial z} + \frac{1}{c}\frac{\partial \phi}{\partial t}\right) = 2\pi\ \frac{eK}{\gamma}\ \cos(\zeta+\phi)\ \ \delta(\vec{r}-\vec{r}_i(t)) \tag{7}$$

The right hand side is the transverse current of the i-th particle. The wiggler strength is $K = eB_o/(mc^2k_o)$. γ is the relativistic factor.

In the arc of the ring the particle experiences a longitudinal shift due to a length mismatch δ of ring circumference and resonator length, and due to first and second order momentum compaction |6,4|. The mean energy loss by synchrotron radiation is balanced by the energy gained at the synchronous RF phase. Off this phase there is a net energy change per turn in accordance with the (constant) slope of the RF potential. Quantum effects of synchrotron radiation add a random amount of energy normally distributed around zero with fractional width Q_{exc}. Radiation damping causes an energy change proportional to the deviation from ring synchronous energy.

The formulation of the problem involves approximations, but none of them restricts the applicability to a large number of wiggler periods |7| (cf. below table 1). However, due to correlated initial conditions of subsequent passages effects may accumulate that are safely negligible in a single pass situation. In a crude approximation these effects can be dealt with by increasing the quantum excitation in the ring beyond its natural value.

Table 1: Parameters of reference case

Laser:	
wiggler period	100 cm
number of wiggler periods	3
wiggler strength K	15.6
resonance wave length	100 μ
fractional optical loss	0.06
Ring:	
1^{st} order momentum compaction	-0.000 5
2^{nd} order momentum compaction	0.02
ring circumference	62 m
slope of RF potential	-255 V/cm
number of turns to damp	1500
fractional quantum excitation Q_{exc}	$1\cdot 10^{-5}$

The behaviour of the system is investigated in the vicinity of the so called reference case. The parameter values listed in table 1 match the proposed ISRL |8| for the Berlin storage ring BESSY |9|. Among the distinct features there is the small number of wiggler periods (cf. above), and the small value of the first order momentum compaction factor. It remains to be verified wether ring operation sufficiently close to transition energy is feasible. To keep computation time within resonable limits the number of turns to damp is reduced from its natural value of the order of several 10^5. The excitation is increased beyond proportion by a factor 100 to account for the approximations of the model (cf. above).

Usually 256 particles representing a total charge of $5.12 \cdot 10^6$ e are followed for typically 20 000 round trips. The optical field is digitized into bins approximately 3/4 wavelength long. Initially the particles are normally distributed in longitudinal phase space near the synchronous RF phase with fractional energy spread of $1.2 \cdot 10^{-4}$, and the optical field is zero.

The evolution of the intracavity pulse energy is shown in fig. 1. Rapid variations are superimposed over the slow evolution. The pulse moves along the z-axis towards negative values, i.e. higher RF voltage, thereby reducing the trap size. Expelled particles perform a large scale synchrotron motion, and become retrapped near the synchronous RF phase (see lower part of fig. 2). Thus while the particle distribution continuously evolves the pulse energy saturates.

The pulse shape is characterized by an envelope raising towards the trailing edge with multiple narrow spikes (see fig. 2). The details presumably depend on the (random) initial conditions. The Fourier transform shows a broad distribution in accordance with the narrow spikes.

The dependence of pulse energy on first order momentum compaction factor α (fig. 3) is better described by a $|\alpha|^{-1}$ dependence than by α^{-2} as predicted by a linear analysis |4|. The points on the cavity length detuning curve (fig. 4) stress the sensitivity to small deviations from synchronisme.

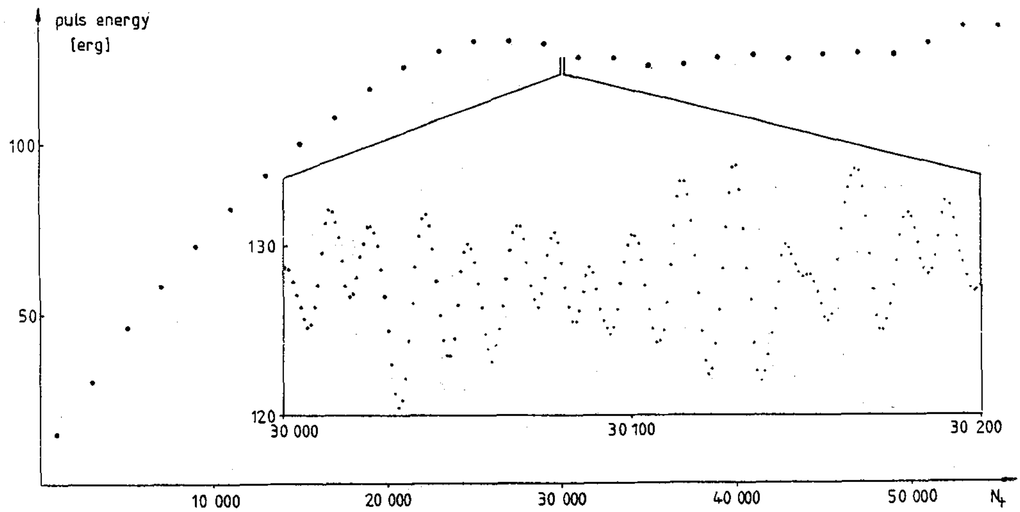

Fig. 1: Intracavity pulse energy versus number of turns through the system. The main graph shows averages over 2000 turns, the insert shows the rapid variations from turn to turn.

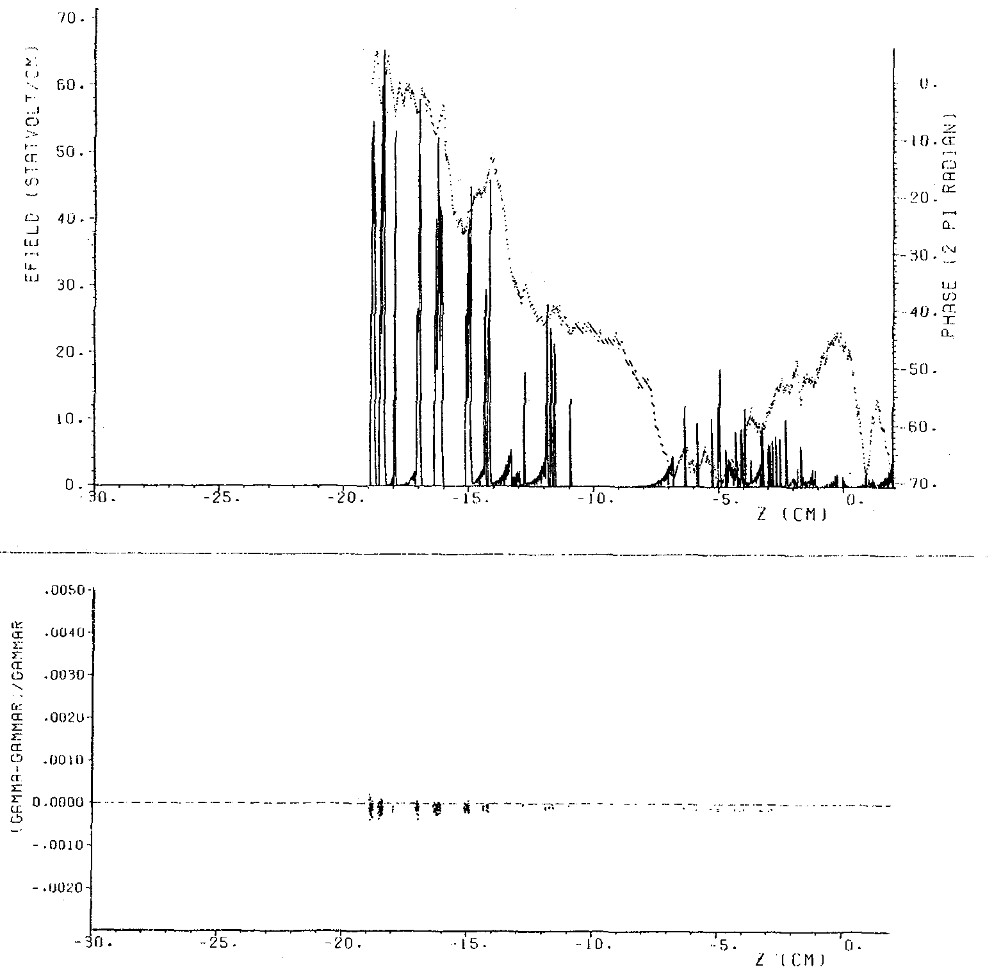

Fig. 2: Upper frame: Amplitude (continuous line) and phase (dots) of optical pulse after 30 000 turns.
Lower frame: Longitudinal phase space diagram after 30 000 turns. Each dot represents one particle. The zero of the vertical axis is the laser resonance energy that coincides with the ring synchronous energy.

Summarizing we have combined the classical single particle pendulum equation with the slowly varying amplitude and phase approximation to describe self-consistently the isochronous storage ring free electron laser. Computer simulations show both fast and slow evolution exhibiting some kind of limit cycling behaviour. The pulse energy exceeds 10^{-5} J. Comparing the laser power to the RF power supplied to the particles efficiencies greater than 80 % are observed. These numbers are lower than obtained in ref. 8. However, here a momentum compaction larger by a factor 2.5 is used, and allowance is made for approximations of the model.

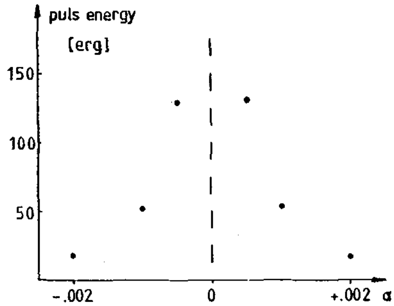

Fig. 3: Intracavity pulse energy at saturation versus first order momentum compaction factor α. α = 0 corresponds to the transition of the ring which is an unstable operating point.

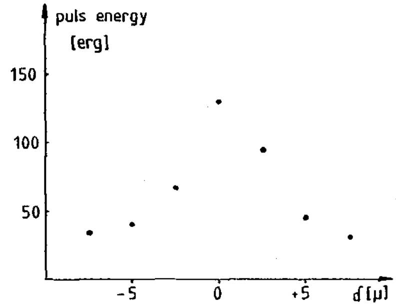

Fig. 4: Intracavity pulse energy at saturation versus optical resonator length detuning δ. Positive δ corresponds to a resonator round trip time longer than the ring round trip time.

The spontaneous spectrum of a single pass through the wiggler is broad and of low coherence due to the small number of periods. However, no problem with turn-on is expected, since the isochronisme of the ring together with the synchronisation (fig. 4) ensures correlated initial condition from turn to turn thus simulating an infinitely long FEL.

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