MAGNETIC DIFFUSE SCATTERING MEASUREMENTS USING POLARISED NEUTRONS

B. Rainford

To cite this version:
B. Rainford. MAGNETIC DIFFUSE SCATTERING MEASUREMENTS USING POLARISED NEUTRONS. Journal de Physique Colloques, 1982, 43 (C7), pp.C7-33-C7-43. <10.1051/jphyscol:1982704>. <jpa-00222311>

HAL Id: jpa-00222311
https://hal.archives-ouvertes.fr/jpa-00222311
Submitted on 1 Jan 1982

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
MAGNETIC DIFFUSE SCATTERING MEASUREMENTS USING POLARISED NEUTRONS

B.D. Rainford

Physics Department, Southampton University, Southampton S09 5NH, U.K.

Résumé. - Deux techniques différentes de neutrons polarisés peuvent être utilisées pour l'étude de la diffusion magnétique diffuse. La première, pour les alliages ferromagnétiques, implique un faisceau incident polarisé sans analyse de la polarisation diffusée. Cette méthode a été largement utilisée, surtout par le groupe d'Oak Ridge, afin d'étudier les défauts de moment magnétique dans les alliages binaires ferromagnétique de métaux de transition. La deuxième technique se base sur l'analyse de polarisation et a été utilisée pour l'étude des corrélations de spin dans les verres de spin concentrés, ainsi que des défauts de moment magnétique dans les alliages antiferromagnétiques. Dans le cas dernier les mesures montrent que l'hypothèse habituelle d'un défaut de moment collinaire n'est pas valable. On a trouvé que les corrélations de spin dans les verres de spin concentrés sont modulées d'une manière incommensurable.

Abstract. - Two distinct polarised neutron techniques are applicable to the study of magnetic diffuse scattering. The first, used for ferromagnetic alloys, employs a polarised incident beam without analysis of the outgoing polarisation. This method has been widely used, chiefly by the Oak Ridge group, to study moment disturbances in ferromagnetic binary transition metal alloys. The second technique involves polarisation analysis and has been used to study spin correlations in concentrated spin glass materials, and moment defects in antiferromagnetic alloys. Measurements on antiferromagnetic γ-Mn alloys show that the usual assumption of a collinear moment defect is not valid. The spin correlations in certain concentrated spin glass alloys are found to have an incommensurate modulation. This may result from the modification of the antiferromagnetic couplings by the atomic correlations in the alloys.

1. Introduction. - Measurements of diffuse magnetic scattering of neutrons from binary transition metal alloys enable us to probe the spatial fluctuations of the magnetic moments from site to site. In the case of ferromagnetic alloys the use of polarised neutrons can lead to greatly enhanced sensitivity in such measurements. In addition the character of the information derived from the data is somewhat different from that obtained using unpolarised neutrons, and is much more readily interpreted. The development of the technique of polarisation analysis over the last ten years has meant that it is now relatively easy to separate the magnetic component of the diffuse scattering in non-ferromagnetic alloys. This has led to studies of moment defects in antiferromagnets and the static spin correlations in spin glass and paramagnetic alloys. I will not attempt an exhaustive experimental review of this large field in the present paper, but rather concentrate on some recent results which demonstrate the way in which polarised neutrons have contributed to the study of diffuse scattering in transition metal alloys.

2. Ferromagnetic Binary Alloys - The sensitivity of the magnetic moments of transition metals to their local environment in alloys may be inferred from bulk magnetisation measurements and from neutron diffraction measurements on ordered alloys. For example in the compound Fe₃Al, diffraction measurements [1] show that Fe sites with
an Al neighbour have a lower magnetic moment than Fe sites whose nearest neighbours are all Fe atoms. In a disordered binary alloy $A_c B_{1-c}$, bulk measurements and diffraction can only give information about the mean atomic moments. Diffuse magnetic scattering, however, gives information about the fluctuations of the moments about their mean values. In principle one would like to be able to deduce how the moment of an $A$ or $B$ atom on the $i$th site changed both with its chemical environment (i.e., the number of $A$ or $B$ atoms in its nearest neighbour shell) and with its magnetic environment (i.e., the magnetic moments on its near neighbours). In practice the information which can be extracted from diffuse scattering data is more limited than this, though it is sufficient to show that both the chemical and the magnetic local environment effects are important.

For unpolarised neutrons the diffuse magnetic cross section is given by

$$\frac{d\sigma}{d\Omega} = N \left( \frac{Ze^2}{2mc^2} \right)^2 \left[ 1 - \frac{1}{\kappa} \left( \hat{\kappa} \cdot \hat{n} \right)^2 \right] T(\kappa)$$

where

$$T(\kappa) = \frac{1}{N} \sum_{m,n} e^{i \kappa \cdot (m-n)} \left< \frac{F_n(\kappa)(\mu_n - \bar{\mu})}{F_m(\kappa)(\mu_m - \bar{\mu})} \right>$$

$\kappa$ and $\hat{\kappa}$ are unit vectors in the directions of the scattering vector $\kappa$, and the magnetisation of the sample respectively. $\mu_n$ denotes the atomic moment at the $n$th site and $F_n(\kappa)$ the corresponding form factor. The triangular brackets indicate a configurational average. The factor in square brackets in (1) allows the magnetic cross section to be separated from the diffuse nuclear scattering, using the field switching technique in which difference counts are taken with the sample magnetised first perpendicular, then parallel, to the scattering vector. With polarised neutrons there is an extra term in the cross section:

$$\frac{d\sigma}{d\Omega} = 2N \left( \frac{Ze^2}{2mc^2} \right)^2 \frac{P_0}{\kappa} \left[ \hat{n} - \left( \frac{\hat{n} \cdot \hat{\kappa}}{\kappa} \right) \hat{\kappa} \right] \Sigma(\kappa)$$

where $P_0$ is a vector describing the polarisation of the incident beam and

$$\Sigma(\kappa) = \frac{1}{N} \sum_{m,n} e^{i \kappa \cdot (m-n)} \left< (b_m - \bar{b}) (F_n(\kappa) \mu_n - F(\kappa) \bar{\mu}) \right>$$

Here the usual geometry is to have the sample magnetised normal to the scattering vector, with the beam polarisation switched alternately parallel and antiparallel to the magnetisation. The polarised cross section depends linearly on the moment fluctuations, compared to the quadratic dependence seen in (2) for the unpolarised case. Consequently the polarised neutron technique is inherently more sensitive than the unpolarised one, as long as there is reasonable contrast between the coherent nuclear scattering amplitudes of the two alloy constituents. Separated isotopes may be used to enhance the difference in the nuclear scattering amplitudes.

The simplest case to consider is where there are no magnetic environment effects, so that all $A$ atoms have moment $\mu_A$ and all $B$ atoms have moment $\mu_B$. This assumption was made by Shull and Wilkinson [2] in their pioneering diffuse scattering work with unpolarised neutrons. In this case

$$T(\kappa) = c(1-c) \left( \frac{F_A(\kappa)}{F_B(\kappa)} \right)^2$$

and

$$\Sigma(\kappa) = c(1-c) (F_A(\kappa) \mu_A - F_B(\kappa) \mu_B)$$

By combining the value of $(\mu_A - \mu_B)$ obtained by extrapolating (5) or (6) to $\kappa = 0$ with measurements of the bulk magnetisation $\mu = c\mu_A + (1-c)\mu_B$, the separate atomic moments $\mu_A$, $\mu_B$ may be determined. Note, however, that in the case of unpolarised
data, there is a sign ambiguity which leads to two solutions. Collins and Forsyth \[3\] first used polarised neutrons to determine the sign of the moment difference in Fe Co and Fe Ni alloys. In the Fe Ni alloys they used the Ni 60 isotope since the coherent scattering amplitudes of natural Fe and Ni are almost the same.

In the case of dilute impurities in a ferromagnet, exact expressions may be obtained for the diffuse cross sections, since it may be assumed that all A atoms have identical moments and identical environments. The moment at the mth atomic site may then be written

\[ \mu_m = \mu_B + (\mu_A - \mu_B) p_m + \sum_r g(r) p_{m+r} \]  

where \( p_m = 1 \) if \( n \) is occupied by an A atom

\( = 0 \) if \( m \) is occupied by a B atom

and \( g(r) \) describes the disturbance in the moments of B atoms, distance \( r \) away from an A impurity. It follows that

\[ T(\kappa) = c(1 - c) \left( F_A(\kappa) \mu_A - F_B(\kappa) \mu_B + F_B(\kappa) G(\kappa) \right)^2 \]  

and

\[ \Sigma(\kappa) = c(1 - c) (b_A - b_B) \left( F_A(\kappa) \mu_A - F_B(\kappa) \mu_B + F_B(\kappa) G(\kappa) \right) \]  

It is readily shown that in the limit \( \kappa \to 0 \)

\[ T(0) = c(1 - c) \left( \frac{d\mu}{dc} \right)^2 \]  

and

\[ \Sigma(0) = c(1 - c) (b_A - b_B) \left( \frac{d\mu}{dc} \right) \]  

For polycrystalline samples

\[ G(\kappa) = \sum_i z_i \frac{g(r_i)}{\kappa r_i} \sin \frac{\kappa r_i}{r_i} \]  

where \( z_i \) is the coordination number of the ith shell, so that the polarised diffuse cross section has a forward peak (or dip) whose height is proportional to \( (d\mu/\mu dc) \) and width is of order \( \pi R^2 \), where \( R \) is the extent of the moment disturbance around the impurity. At large values of \( \kappa \), \( G(\kappa) \) is small and the cross section approximates to the Shull-Wilkinson result \(6\).

Between 1962-1969 G.C.Low and his colleagues \[4\] at Harwell investigated the structure of the moment defects around a wide range of both transition metal and non transition metal impurities in Fe and Ni. All of this work used the unpolarised beam technique. Undoubtedly polarised neutrons, if they had been available, would have been of great value in investigating difficult cases, like Fe with Rh, Pd, Os and Pt impurities, where \( (d\mu/\mu dc) \) is small and where \( G(0) \) and \( (\mu_A - \mu_B) \) have opposite signs.

Fig.1 shows some recent polarised neutron measurements by Najzar and Parette \[5\] on some dilute Fe alloys. The data agree well with the forward cross sections (arrows) estimated from \( (d\mu/\mu dc) \), and are consistent with a short range disturbance in the host.

Fig 1.

For concentrated alloys the analysis of the diffuse scattering is complicated by the wide range of local environments and the possibility of moment fluctuations on both atomic species of the alloy. Marshall \[6\] made the assumption that the moment perturbations at a given site are linear in the possible chemical fluctuations in its environment. Allowing for fluctuations in the moments of both species gives

\[ \mu_m = \mu_B + \sum_r g(r) (p_{m+r} - c) \]  

(13)
Fig. 1. Polarised neutron cross sections for dilute Fe alloys after ref. [5]. The arrows indicate the forward cross sections calculated from d\(\bar{\mu}/dc\).

\[
\text{if site } m \text{ is occupied by a B atom, or}
\]

\[
\mu_m = \mu_A + \sum_r h(r) (p_m r - c)
\]  

\[(14)\]

\[
\text{if site } m \text{ is occupied by an A atom. Here } \mu_A \text{ and } \mu_B \text{ are the mean atomic moments and } g(r), h(r) \text{ are the disturbances in the moments of B or A atoms at site } m \text{ due to the presence of an A atom distance } r \text{ away. The diffuse cross sections are then proportional to}
\]

\[
T(\kappa) = c(1 - c) F^2(\kappa) S(\kappa) M(\kappa) |^2 + \text{higher order terms}
\]  

\[(15)\]

\[
\Xi(\kappa) = c(1 - c) (b_A - b_B) F(\kappa) S(\kappa) M(\kappa)
\]  

\[(16)\]

\[
\text{for the unpolarised case, and}
\]

\[
M(\kappa) = \mu_A - \mu_B + (1 - c) G(\kappa) + c H(\kappa)
\]  

\[(17)\]

\[
\text{and } G(\kappa), H(\kappa) \text{ are the Fourier transforms of } g(r) \text{ and } h(r). \text{ For simplicity we have ignored the difference in the form factors for the two species. The factor } S(\kappa) \text{ allows for deviations from complete randomness in the alloy: for a polycrystalline alloy}
\]

\[
S(\kappa) = 1 + \frac{1}{\sum_i \alpha_i^2} \sum_i \sin kr_i \frac{\sin kr_i}{kr_i}
\]

\[
\text{where } \alpha(r_i) \text{ is the Cowley short range order parameter for the } i\text{th coordination shell. The terms ignored in (15) arise from the mean square fluctuations in the moments of each species and from higher order effects of the short range order. The } S(\kappa) \text{ factor can have a profound effect on measured cross sections and must be determined.}
\]
from the nuclear diffuse cross section, after subtracting the incoherent scattering:

\[ \frac{d\sigma}{d\Omega}_{\text{nuc}} = N c(1-c) (b_A - b_B)^2 S(\kappa) \]  

(18)

It can be seen that within the Marshall model the cross sections for concentrated alloys are still relatively simple, and that useful information may still be extracted from the data: \( M(\kappa) \) yields both the difference in the mean moments \( \mu_A - \mu_B \) and the combination \((1-c) G(\kappa) + c H(\kappa)\). Medina and Cable [7], following Balcar and Marshall [8], have considered the effects of going beyond Marshall's linear approximation by including moment perturbations due to many-site correlations. For example to include two site correlations an extra term

\[ \sum_{RR'} a(R,R') \left( p_{n+R} - c \right) \left( p_{n+R'} - c \right) \]  

(19)

is added to the linear terms in equation (13). It is found that these non-linear perturbations give rise to additional terms in the unpolarised cross section. However the cross section for polarised neutrons is still determined by the linear perturbations, so that \( E(\kappa) \) is still given by (16) (at least when the short range order is small). Furthermore Medina and Garland [9] have shown that in general

\[ \frac{d\mu}{dc} = S(0) M(0) \]  

(20)

so that the forward cross section for polarised neutrons may always be related to \( d\mu/dc \). It follows that polarised neutron data is much more amenable to direct analysis. However for a complete description of the diffuse scattering from an alloy system both techniques are required, since the presence of higher order or non-linear terms may be inferred by comparing polarised and unpolarised data. Thus \( M(\kappa) \) from a polarised neutron experiment may be used to calculate \( E(\kappa) \) using (15). Agreement between this calculated \( E(\kappa) \) and that measured in an unpolarised neutron experiment would indicate that these higher order and non-linear terms were negligible. This appear to be the case for Ni Cu alloys [7] and Ni Rh alloys [10]; the Ni Rh data is shown in Fig. 2 where the excellent agreement between \( M(\kappa) \) derived from polarised data (solid circles) and unpolarised data (open circles) may be seen. For some other Ni alloys, however, in particular Ni Cr, Ni V [11] and Ni Pt [12], it is found that \( M(\kappa) \) from unpolarised data is considerably larger than that derived from polarised data, indicating that non-linear terms are important. This is not surprising in the cases of Ni Cr and Ni V alloys, since Cr and V impurities create very large, long range moment perturbations in the Ni host \((d\mu/dc = -5.2 \mu_B \text{ per atom for Ni Cr in the dilute limit})\), and ferromagnetism is suppressed completely for \( c > 0.12 \). Cable and Medina [11] analysed the difference between the unpolarised and polarised data in terms of the Balcar and Marshall non-linear theory, including two-site correlations as in (19). To arrive at a few parameter fit, they assumed that \( a(RR') \) was only important if \( R \) or \( R' \) were first neighbours to an impurity. However this assumption was found to be inadequate for the more concentrated Ni Cr alloys. In the case of Ni Pt alloys [12] it is more difficult to understand the difference between the polarised and unpolarised data. Non-linearities of the type found in Ni Cr and Ni V are not expected, since the moment perturbations in Ni Pt are no larger than those in Ni Cu and Ni Rh, where there is no evidence for non-linearities. However there is a large degree of anticlustering, even in quenched Ni Pt alloys, and it is known that this short range order has a profound influence on the magnetism; for example, the disordered alloy Ni:5Pt:5 is ferromagnetic, while the ordered compound Ni Pt is an enhanced paramagnet [13]. It may be, therefore, that higher order terms connected with the short range order are important in the diffuse cross sections for Ni Pt alloys. In almost all of the Ni\(_{1-c}X_c\) alloys studied so far \((X = \text{Cu, Rh, Cr, V, Pt})\), \( M(\kappa) \) is observed to become sharply peaked in the forward direction with increasing concentration of \( X \). This shows that the range of the moment disturbances increases as the mean Ni moment decreases. The same behaviour is found in Fe\(_{1-c}X_c\) alloys [14].
Fig. 3 shows the development of $M(\kappa)$ with Pt concentration for Ni-Pt alloys [12]. In the vicinity of the critical concentration $c_F$ at which the ferromagnetism disappears, the range of the disturbances is very large, of the order of 10 Å. The form of the diffuse scattering near $c_F$ has been interpreted [15] as arising from a small concentration of extended polarisation clouds. A polarisation cloud is nucleated by the local moment formed at a local Ni-rich region; the local moment then polarises the highly susceptible atoms in the vicinity, rather as a local moment impurity in Pd polarises its environment, producing a giant moment. Hicks has shown that this picture is consistent with the polarised neutron data for Ni-Cu alloys near $c_F$ [16]. The change in the $\kappa$ dependence of $M(\kappa)$ with concentration is not expected from Marshall's theory, which essentially is only concerned with the chemical environment in the alloy. We conclude therefore that the diffuse scattering data for these Ni alloys gives direct evidence of the importance of magnetic local environment effects.

Fig. 4. Mean atomic moments for Fe-Cr alloys; experimental points from ref. [21], solid curves from CPA calculations of Sacchetti, ref. [20].

3. Comparison with Theory. - For dilute alloys the magnitude of the moments at the impurity sites and the perturbation in the host matrix may be largely understood on the basis of screening of the charge defect by the d electrons of the host [17], and, for large charge defects, by Friedel's model of virtual bound state formation [18]. Comly et al. [19] invoked exchange enhancement to explain why most impurities in Ni give rise to magnetic defects with the same long-range spatial extent.

For more concentrated alloys the mean atomic moments may be calculated using the coherent potential approximation (CPA) [20]. Fig. 4 shows the good agreement between the atomic moments determined by Aldred et al. for Fe-Cr alloys [21] and the CPA calculations of Sacchetti [20]. While it is possible, in principle, to predict the sensitivity of the atomic moments to the local chemical and magnetic environments.
from cluster CPA calculations [22], a detailed theoretical description of the $\kappa$ dependence of $M(\kappa)$ and the way it changes with concentration is much harder to achieve. Hicks [23] and Medina and Cable [7, 24] have instead adopted a phenomenological approach in which the moment of a B atom at site $m$ is expressed as

$$\mu_m = F(h_m, \nu_m)$$

(21)

where $h_m$ is the molecular field due to other magnetic atoms in the vicinity

$$h_m = \sum_{n \neq m} J_{mn} \mu_n$$

(22)

and $\nu_m$ is the number of A atoms in the local environment. Hicks assumes a particular form for the function $F$, viz:

$$\mu_m = \chi_m h_m / (1 + \beta h_m)$$

(23)

This is a simple saturating function of $h_m$, so that $\chi_m$ represents the initial susceptibility at the site, while $\chi_m / \beta$ represents the largest moment which can develop at the site. The effects of the chemical environment are included by writing

$$\chi_m = \chi_0 + \sum_{n \neq m} \Delta \chi (m - n) (1 - p_n)$$

(24)

where $\chi_0$ is the susceptibility of an isolated B atom and $\Delta \chi$ is the effect on $\chi_m$ of another B atom at site $n$. Medina and Cable make no assumption about the form of the function $F$, but make a Taylor expansion about $F(\langle h \rangle, \langle \nu \rangle)$:

$$\mu_m = \langle \mu_B \rangle + \frac{\partial F}{\partial h} (h_m - \langle h \rangle) + \frac{\partial F}{\partial \nu} (\nu_m - \langle \nu \rangle) + \ldots$$

(25)

where

$$\langle \mu_B \rangle = F(\langle h \rangle, \langle \nu \rangle)$$

Both approaches allow $M(\kappa)$ to be calculated in closed form with relatively few parameters. Hicks is able to determine these parameters by fitting the measured variation of $\mu$ with concentration. With the assumption of only nearest neighbour molecular field and chemical effects, the parameters give a good representation of the form of the measured $M(\kappa)$ over the whole concentration range for several Ni alloy systems. Medina and Cable prefer to fit the model to the measured $M(\kappa)$ and thereby derive two parameters, proportional to $\partial F/\partial h$ and $\partial F/\partial \nu$ which give a measure of the relative importance of the magnetic and chemical local environment effects. The results of applying this model to Ni Cu, Ni Rh and Ni Pd alloys have been reviewed by Cable [24]. Despite their phenomenological nature, these models have provided invaluable insight into the complex problem of local environment effects in concentrated alloys. It would be of great interest to make the connection between this approach and the cluster CPA theories.

4. Antiferromagnetic Binary Alloys. - In antiferromagnetic alloys it is not possible to influence the orientation of the magnetic moments significantly by an applied field. It is not possible, therefore, to use the field switching or polarisation switching techniques outlined in section 2. Instead polarisation analysis must be used to separate out the magnetic diffuse scattering from the nuclear disorder scattering. The simplest geometry to use has the polarisation of the incident beam switched parallel (+) and antiparallel (−) to the scattering vector at the sample. Of the scattered neutrons only those are detected whose polarisation is parallel (+) to the scattering vector. In this geometry the non spin flip cross section (+) contains only nuclear scattering while the spin flip cross section (−) contains the magnetic scattering plus some nuclear spin incoherent scattering. The latter gives a flat background which may be measured separately; in many elements it is negligible. Where it is significant (e.g. Cr alloys) it is safer to measure also the scattering for the polarisation normal to the scattering vector and use the difference cross sections. Most of the measurements made so far, by Hicks and his
co-workers at Lucas Heights have used the simple geometry. They use polycrystalline iron filters for polarising and analysing the neutron beam [25].

Assuming that all the moments are parallel or antiparallel to a unique direction $\hat{n}$, the spin flip cross section for the dilute alloy case is

$$\frac{d\sigma}{d\kappa} = N \left( \frac{\gamma e^2}{2mc^2} \right)^2 \left[ 1 - (\kappa \cdot \hat{n})^2 \right] T(\kappa)$$

with

$$T(\kappa) = c(1 - c) S(\kappa - \tau) \left[ F_A(\kappa)v_A - F_B(\kappa)v_B + F_B(\kappa)G(\kappa - \tau) \right]$$

where $v_A, v_B$ are the sublattice moments of the two alloy species and $\tau$ is a vector such that $\exp(i\tau \cdot R) = +1$ for $R$ on the up sublattice and $= -1$ for $R$ on the down sublattice, i.e. $\tau$ is the smallest magnetic reciprocal lattice vector. Note that $T(\kappa)$ is peaked around the magnetic reciprocal lattice points. With two exceptions [26,27] most of the measurements so far have been made on polycrystalline alloys. This is a pity, since the spherical averaging imposed by use of polycrystals decreases the information content of the data, since at a general scattering vector there are contributions to the cross section from more than one Brillouin zone. It also makes analysis of the data more difficult: corrections for multiple Bragg scattering can be large and it is not possible to use the sum rule

$$T(\kappa = \tau) = c(1 - c) S(0) \left( \frac{\partial v}{\partial c} \right)^2 \left[ F(\tau) \right]^2$$

as a direct check on the measured cross sections. Care must also be exercised in fitting spherically averaged data since the defect does not always have cubic symmetry. For example $\gamma$-Mn alloys are usually thought to have the type I antiferromagnetic structure (ferromagnetic (001) sheets, antiferromagnetically coupled). This is a tetragonal structure, and so the magnetic Brillouin zone has tetragonal symmetry. In addition the spin orientation factor $[1 - (\kappa \cdot \hat{n})^2]$ in (26) gives a vanishing cross section for $\kappa$ parallel to the moment direction (001). This factor was not taken into account in the analysis of the diffuse scattering from Mn Cu, Mn Pd [28] and Mn Zn [29] alloys. In fact recent data on single crystals of Mn Ni [27] and Mn Cu [26] alloys show large spin flip cross sections for $\kappa$ parallel to the (001) direction. These observations are inconsistent with the form of (26) and (27), so we must conclude that the underlying assumption of a collinear defect is incorrect for these alloys, and, presumably, for other $\gamma$-Mn alloys. The calculation of the diffuse cross section including the effects of non-collinearity is relatively straightforward. We find

$$\frac{d\sigma}{d\kappa} = N \left( \frac{\gamma e^2}{2mc^2} \right)^2 \sum_{\alpha\beta} (\delta_{\alpha\beta} - \kappa_\alpha \kappa_\beta) T^{\alpha\beta}(\kappa)$$

where $\alpha, \beta = x, y, z$ and

$$T^{\alpha\beta}(\kappa) = c(1 - c) S(\kappa - \tau) M^\alpha(\kappa) M^\beta(-\kappa)$$

$$M^\alpha(\kappa) = F_A(\kappa)v_A^\alpha - F_B(\kappa)v_B^\alpha + F_B(\kappa)G^\alpha(\kappa - \tau)$$

For dilute alloys having the type I AF spin structure, the moments in the vicinity of an impurity may cant away from the (001) direction, but by symmetry the canting must be along the radius vector to the impurity. In this case the cross section calculated using (29) still vanishes around (001). It appears then that the observed diffuse scattering cannot be understood on the basis of the assumed spin structure of $\gamma$-Mn. Cade and Young [30] have recently suggested that cubic $\gamma$-Mn alloys have a triple spin density wave leading to a non-collinear tetrahedral spin
structure. The structure factors for Bragg scattering in this structure are identical to those of a multidomain type I AF structure. It seems likely that the diffuse scattering in these $\gamma$-Mn alloys can be understood on the basis of this tetrahedral spin structure.

5. Spin Glass Alloys and Paramagnets. - The elastic diffuse scattering discussed above measures the effect of impurities on the time averaged moment distribution in alloys. On the other hand by measuring the total cross section, integrated over all energy transfers, it is possible to study the instantaneous (equal time or static) spin correlations. This is particularly appropriate in spin glass alloys or paramagnetic materials where there is no long range magnetic order. To effect a proper separation of the magnetic scattering from nuclear disorder scattering, polarisation analysis must again be used. The total spin flip cross section is given by an expression of the form of (29) with

$$T^{\alpha\beta}(\kappa) = \frac{1}{N} \sum_{mn} \langle S^\alpha_m S^\beta_n \rangle \left\{ 1 + i \kappa (m-n) \right\}$$

(30)

To apply this result in an experimental situation the quasistatic approximation must be valid, i.e. the inelasticity of the scattering must be small compared to the energy of the incident neutrons. This condition appears to be well satisfied in spin glass alloys at low temperatures, where the magnetic scattering is dominated by an essentially elastic response [31], corresponding to the extremely long relaxation times found in this limit.

In Cu Mn alloys, the canonical spin glass system, spin glass order persists up to 72% Mn. The freezing temperature increases with Mn concentration to a shallow maximum in the vicinity of 50% Mn. In moderately concentrated alloys the magnetic properties are quite sensitive to heat treatment, suggesting the importance of atomic short range order. Fig. 5 shows the magnetic and nuclear diffuse scattering, separated using polarisation analysis, in a series of quenched polycrystalline Cu Mn alloys.
alloys [32]. In the nuclear scattering there is a diffuse peak centred at the 
(\text{i}0) position, showing that considerable short range order is present even in 
quenched alloys. Analysis gives a positive value of $a(r_2)$, the Cowley SRO para-
eters for second nearest neighbours, at all concentrations, while $a(r_1)$ is negative 
below 60% Mn and positive above this concentration. The (\text{i}0) peak is also present 
in the magnetic scattering at all concentrations, showing that the spin correlations 
are antiferromagnetic for nearest neighbours and ferromagnetic for next nearest 
neighbours. The forward peak in the magnetic scattering for $c < 30\%$ Mn probably 
results from an enhancement of the ferromagnetic correlations between Mn due to the 
presence of short range order. Cable, Werner and co-workers [33] have recently 
studied the magnetic and nuclear diffuse scattering in single crystals of Cu Mn 
alloys which had been annealed to enhance the degree of short range order. Their 
results (Fig.6) show that the magnetic intensity peaks at incommensurate $(1,\text{i} \pm 6,0)$ 
type positions, corresponding to a long period modulation over distances of the 
order of ten times the lattice constant $a_0$. The period of the modulation is con-
centration dependent, varying from $6 a_0$ at 5% Mn to $3 a_0$ at 25% Mn. A similar 
behaviour was found in a disordered, but annealed Pd Mn alloy [31]. These results 
may be interpreted as arising from a tendency to long range commensurate antiferro-
magnetic order which is modified by the atomic correlations in the system. The 
static spin correlations in the spin glass phase just reflect the form of $\chi(\mathbf{k})$, the 
wavevector dependent susceptibility for the paramagnetic phase above the freezing 
temperature. For a pure system in mean field theory this is given by

$$\chi(\mathbf{k}) = \chi_0 \left[ 1 - \frac{2X_0}{(g_0 B)^2} J(\mathbf{k}) \right]^{-1}$$

where $\chi_0$ is the static susceptibility in the absence of interactions and $J(\mathbf{k})$ is the 
Fourier transform of the exchange interactions. $\chi(\mathbf{k})$ is therefore a maximum at the 
wavevector $X_0$ where $J(\mathbf{k})$ is a maximum. If the system ordered in a simple anti-
ferromagnetic structure, $X_0$ would be a commensurate wavevector in the Brillouin zone.

In a disordered alloy it is straightforward to show that

$$\chi(\mathbf{k}) = c \chi_0 \left[ 1 - \frac{2X_0}{(g_0 B)^2} \{ c J(\mathbf{k}) + (1 - c) \left[ J(0) + W(\mathbf{k}) \right] \} \right]^{-1}$$

where

$$W(\mathbf{k}) = \sum_n \alpha(n) J(n) e^{i\mathbf{k}.\mathbf{n}}$$

allows for the effects of short range atomic order on the magnetic interactions. In 
this case $\chi(\mathbf{k})$ is a maximum at the wavevector where $\{ c J(\mathbf{k}) + (1 - c) W(\mathbf{k}) \}$ is a 
maximum; clearly this wavevector will not necessarily coincide with $X_0$. The diffuse 
scattering will then peak away from the commensurate superlattice position. Further-
more the position of the maximum will be concentration dependent, in agreement with 
the change in modulation period observed in Cu Mn alloys.

$\chi(\mathbf{k})$ may be related directly to the total cross section $T^{\alpha\beta}(k)$ in (30) since it may 
be shown that

$$T^{\alpha\beta}(k) - T_{el}^{\alpha\beta}(k) = F(k)^2 \frac{\hbar}{\pi} \frac{1}{g_0^2 B^2} \int_0^\infty d\omega \coth \frac{\hbar \omega}{2T} \Im \chi^{\alpha\beta}(k,\omega)$$

where $T_{el}^{\alpha\beta}(k)$ is the strictly elastic component of the diffuse scattering, and 
$\chi(k,\omega)$ is the dynamical susceptibility. From the Kramers Kronig relations

$$\chi^{\alpha\beta}(k) = \frac{2}{\pi} \int_0^\infty d\omega \frac{1}{\omega} \Im \chi^{\alpha\beta}(k,\omega)$$

It can be seen that, as long as $\hbar \omega B << 1$ for all $\omega$,

$$\chi^{\alpha\beta}(k) = kT \left[ T^{\alpha\beta}(k) - T_{el}^{\alpha\beta}(k) \right] / F(k)^2$$

In disordered systems it is not always easy to determine the elastic component of
the diffuse scattering. This is especially true in spin glass alloys [31]. However in the paramagnetic state there is no elastic scattering and $X(k)$ may be determined directly, as long as the quasistatic approximation is valid, and $\omega B << 1$. In transition metal alloys it is often very difficult to guarantee that these conditions are satisfied, since the excitation spectrum often extends up to several tenths of an electron volt. These experiments may become feasible with the high energy neutrons furnished by the new generation of pulsed sources; for the present it is probably more worthwhile to investigate the dynamical response $X(\mathbf{q},\omega)$ in the range of energy transfers and wavevectors currently accessible.

References.

3. Collins M.F. and Forsyth J.B., Phil.Mag. 8, (1963) 401
27. Davis J.R., Cywinski R., Moze O. and Hicks T.J., (these proceedings)