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SPONTANEOUS EMISSION OF UNDULATORS AND FREE ELECTRON LASERS

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Abstract - In this paper, we report the main characteristics of undulators assembled on an electron storage ring, operating as a synchrotron radiation source and as a free electron laser. It is shown that the spontaneous emission is a powerful, quasi-monochromatic and tunable source of light in the infra-red, visible, ultra-violet and X-rays range. First results on the stimulated emission in the visible range are also reported.

1. Introduction. - Since GINZBURG (1) proposed that relativistic electrons could emit intense, tunable and monochromatic radiation in a periodic electromagnetic structure, many theoretical works have been published on the spontaneous or stimulated emission by an undulator (2). Some experimental results have been obtained with linear accelerators (3, 4), synchrotrons (5) and more recently with storage rings (6, 7, 8). The first (and unique) operation of a free electron laser in the infra-red was obtained at Stanford (4, 9) in 1976 and a significant effort on the part of a number of groups worldwide has been applied to the development of these novel sources of light. In this paper we give a short description of the main features of these sources taking as an example the results obtained with the LURE undulator mounted on the storage ring ACO at Orsay.

2. Synchrotron radiation. - All charged particles emit electromagnetic radiation when accelerated (or decelerated) : electron storage rings are now powerful sources of synchrotron radiation. Using relativistic electrons an intense radiation is obtained in each bending magnet (Fig. 1) and we recall briefly the properties of this radiation.

a) The spectrum of emission is a "white spectrum" running from X rays to millimetric waves. This is illustrated on the figure 1 in the case of the ACO (540 MeV) and DCI (1.8 GeV) storage rings.

b) Divergence of the light beam is very small in the vertical plane and is given approximately by $1/\gamma$ where $\gamma = E/m_0c^2$. With an energy $E = 1.8$ GeV this divergence is about 0.4 mrad. (approximately as a good laser). But in the horizontal plane the divergence is large and the intensity of the useful radiation is limited by the aperture of the optical set-up.

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c) The beam has a pulsed structure (currently 1 ns) with a repeating frequency determined by the diameter of the ring (13 MHz for ACO).

3. Spontaneous emission of an undulator. — Schematically an undulator is a periodic magnetic structure which can be mounted on a straight section of a storage ring. Figure 2 shows a linearly polarized undulator, that is to say a series of magnets (+ + + ...) distributed along the z axis. In an ideal undulator the magnetic field is a perfect sinusoid:

$$B(z) = B_0 \sin \left( \frac{2\pi z}{\lambda_0} \right)$$

where $\lambda_0$ is the period of the undulator.

A relativistic electron which is propagating along the z axis has a small transverse velocity $v_x$, so that the electron trajectory is a sinusoid in the xz plane (Fig. 2c) with undulations of some microns. Then the electron can be considered as an oscillator travelling along the axis of the undulator and emitting an electromagnetic wave. In the direction of observation, the radiation is due to the interference of waves emitted by the same electron with a gradual phase displacement given by the difference of electron and photon velocities.

Main features of this emission are determined by the relativistic motion of the electron (Fig. 3). In a pseudo-rest frame travelling in the undulator the electron is a classical oscillator with a frequency $\omega = 2 \frac{mc\gamma}{\lambda_0}$, independent of the angle of observation. In the laboratory frame the frequency is determined by the Doppler effect which depends on the energy of electron, the amplitude of the magnetic field and the direction of observation. The fundamental wavelength is given by: 

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**Fig. 1**: Synchrotron radiation of a storage ring.

**Fig. 2**: A: Superconducting undulator. B: Distribution of the magnetic field along the axis of undulator. C: Electron trajectory and radiation in the $\theta$ direction.
where $\gamma = \frac{E}{m_0 c^2}$, $E$ is the electron energy and $m_0 c^2$ its rest mass energy (typically $\gamma = 300$ to 1000 for ACO),

$\lambda_0$ is the period of undulator (40 to 80 mm),

$K = \frac{eB \lambda_0}{2\pi m_0 c}$ depends on the amplitude of the magnetic field ($K = 1$ to 2 with 0.5 Tesla),

$\theta = \text{angle of observation with respect to the z axis}$. 

According to (1), a first interesting feature of this emission is the possibility to change the wavelength in a large spectral range by changing the energy of electron or the magnetic field.

A second feature is due to the relativistic projection of angles on the direction of motion. The angular divergence of the light beam is very small and is approximately $1/\gamma$, that is to say an aperture of a few milliradians.

Other interesting characteristics of this radiation are:
- the spectral width of the peak is near to $1/N$, $N$ being the number of periods of the undulators,
- the spectral brightness is expected to increase like $N^2$ with respect to the classical synchrotron radiation (if the electron beam aperture is small).

Such predictions have been verified with electron beams accelerated by linear accelerators or by synchrotrons. However the number of electrons stored into bunches by such machines is very small compared with that can be achieved with electron storage rings. With such machines, undulators should constitute extremely powerful, monochromatic and tunable sources of radiation. In this paper we present, as an illustration, some results obtained on the ACO storage ring at Orsay, first with a superconducting undulator and then with a permanent magnet undulator.

Figure 4A shows a picture of the light emitted by the permanent magnet undulator with $E = 240$ MeV and $K = 2$. One can see clearly the colored rings which correspond well to formula (1) and the most important part of radiation is concentrated within an angle of 2 milliradians. Two black regions can be observed, symmetric to the vertical plane of the magnetic field. They correspond to $\pm \gamma \theta = 1$; those directions in the pseudo-rest frame are the transverse horizontal directions where no emission is expected, electrons oscillating along this direction (see Fig. 3).
Fig. 4: A: Picture of the light emitted by the undulator ($E = 240$ MeV, $K = 2$),
B: Spectral distribution of the light ($E = 150$ MeV, $K = 1.2$) without angular selection (first order only).

Figure 4B shows the spectrum of the emission obtained without angular selection and with a 23 periods superconducting undulator. One can see that the most important part of energy is concentrated on a small spectral width ($d\lambda/\lambda = 1/10$).

Fig. 5: A: Spectral distribution of the light ($E = 150$ MeV, $K = 1.2$) through a small pin-hole. Experiment: full line; theory: dashed line.
B: Angular distribution of the light at 5300 Å ($E = 150$ MeV; $K = 1.5$)

Figure 5 A represents the spectral distribution measured for another energy with an angular resolution of $\Delta\theta = 0.2$ mrad. It looks very much like a $(\sin x/x)^2$ function due to the finite length of the undulator. Figure 5 B represents the angular distribution of light for a constant wavelength (symmetry of the ring comes from a lateral displacement of the $e^-$ beam towards the undulator axis). These results concern only the fundamental term of emission. But if the transverse velocity becomes large ($K > 1$), an intense emission is obtained for some harmonic frequency, so that we have a line spectrum $\lambda_1 = \sum_i \lambda_i$ where $\lambda_i$ is the fundamental term given by formula (1), and the theoretical spectral width is $d\lambda_1/\lambda_1 = 1/\pi N$. As an
example, with $K = 2$ the intensity of the $19^{th}$ harmonic is about 20% of the fundamental term, so that we have a quasi-monochromatic and tunable emission on a wide spectral range. With the permanent magnet undulator mounted on A.C.O. ($E = 150$ MeV - $540$ MeV, $\lambda_0 = 80$ mm) one can expect a useful emission between $50$ Å and $1.4$ µ (using D.C.I. at $1.8$ GeV, $\lambda = 4$ Å). At this time we have no results on the V-U.V. range, but we have measured seven harmonics between $2000$ Å and $1.4$ µ, and there is no doubt the V-U.V. emission will be obtained.

Similar results have been now obtained on several research centers (6, 7, 8) and in the future the undulators will become important source of radiation in the V-U.V. and X rays range.

4. Stimulated emission of an undulator. - Another important property of undulators is the possibility to obtain stimulated emission. In the pseudo-rest frame of the electron, by virtue of Lorentz transformation, the sinusoidal magnetic field $B_y$ becomes an electromagnetic field:

$$E_x^* = -\gamma v_L B_y$$
$$B_y^* = \gamma B_y$$

where $v_L$ is the longitudinal velocity of electrons. For relativistic electrons $v_L = c$, the field $(E_x^*, B_y^*)$ is like a pseudo-photon of frequency $\omega^* = \omega = 2\pi c y^*/\lambda_0$.

Spontaneous emission can then described as a Compton scattering between electrons and pseudo-photons. Now if we superimpose an external laser of frequency $\omega$ it is possible to produce stimulated Compton scattering into the laser beam, a process which can transfer some energy from the electrons to the laser (amplification) or vice-versa (absorption).

Another way to explain stimulated emission is the microbunching of electrons. Emission of coherent light by a bunch of incoherent electrons can be possible only if $\delta_e$, the length of the bunch is not too large with respect to $\lambda$. For most of storage rings, $3$ cm $< \delta_e < 30$ cm and a coherent emission in the visible range cannot take place. However, the undulator gives a transverse velocity $v_T$ to the electron, so that there is also a longitudinal force:

$$F_L = e \nu_T \times B$$

which derives from a sinusoidal potential. According to their optical phase, the electrons are accelerated or decelerated so that inside the bunch there are micro-bunches separated by an optical wavelength. Then the emission contains a small coherent part.

In fact, an exact calculation shows that:

- the gain is the first derivative of the spontaneous emission spectrum,
- the amplitude of gain is (10):

$$G \lambda^{3/2} N^3 I_p / \Sigma$$

where $I_p = \text{maximum current per bunch}$

$$\Sigma = \text{transverse section of a bunch}$$

Since the gain is a second order effect, its amplitude is relatively small (see below) and consequences of this formula are:

- Practically there is no chance to obtain a laser effect if $\lambda < 1000$ Å,
- the undulator must be as long as possible ($N^3$ factor),
- the storage ring must be constructed in order to obtain a maximum electron density.
Measurements of gain have been obtained on the ACO storage ring at two wavelength in the visible, 5145 Å and 4880 Å, and for several different magnetic fields (11). A CW argon laser was focussed to a waist at the center of the undulator and the laser beam was adjusted to travel coaxially with the electron beam. The laser is amplified at the frequency of the electron beam (27 MHz) and the signal was obtained by a demodulation technique at this frequency (Fig. 6).

![Diagram of gain measurements]

Fig. 6: Simplified schematic diagram of gain measurements

A plot of the gain measured as a function of energy is provided as figure 7. A spontaneous emission curve obtained under the same conditions is presented underneath for comparison. As expected the gain is related to the derivative of the spontaneous emission curve: we have an absorption on the low energy side and an amplification on the high energy side. The peak gain measured with the superconductivity undulator was about $3 \times 10^{-4}$ per pass. Similar results have been obtained with a permanent magnet undulator with a $G$ value of about $6 \times 10^{-4}$ at 6300 Å. These very small gain values are due to the fact the straight section available on ACO is rather short (1.2 m), so that the number of periods of undulators is $N = 20$ (see formula 2). Moreover, ACO is rather an old machine and the electronic density is not well adapted for this type of experiment. Higher values of gain would be obtained with future storage rings well adapted to the requirements of free electron lasers. The gain can be also increased through the use of an optical klystron (a more complicated structure with two undulators separated by a dispersive central part): theoretical predictions give an enhancement of gain by a factor of five or six for ACO and work is now under way to accomplish this.
5. Free electron laser. - The only free electron laser which has been made to operate in the weak space charge regime was made at Stanford on the superconductivity linear accelerator (4, 9). In this case, the undulator field has a helical symmetry, and consists of 160 periods of $\lambda_0 = 3.2$ cm. Emission was obtained in the 3.2 $\mu$m region with an electron beam energy near 43 MeV. The average power output was 2 watts in a pulse length limited bandwidth of 0.2 %. Work is currently under way to characterize the effects produced by the extremely short (~4 p sec.) electron and optical pulse lengths (12).

In the visible range several projects are now developed, but no oscillation has yet been reported. In the case of ACO an optical cavity has been mounted on the storage ring and preliminary experiments were made in order to test the properties of this cavity. Using the permanent magnet undulator to obtain oscillation at these extremely small gains requires the acquisition and preservation of a set of truly exceptional cavity mirrors. High reflectivity mirrors (losses of about $10^{-4}$ per mirror) are mounted inside the cavity in a vacuum of $10^{-10}$ torr. With this arrangement, losses of the cavity are due only to the mirrors. The length of cavity (5.5 m) is chosen in order to synchronize electron pulses and optical pulses with two bunches stored in the ring (Fig. 8).

Experiments are now running at Orsay and some preliminary results have been obtained:

- When the cavity is tuned, a small amplification takes place on the low frequency side of the spontaneous emission spectrum (and also a small absorption on the high frequency side).
- Unfortunately during the experiments, the mirrors are damaged. An enhancement of the optical losses by a factor of ten has been observed in about ten shifts (a shift is a 12 hours experiment period). The reasons of the damage of the coating is still unclear and experiments are now in progress to elucidate this point.

![Fig. 7: Gain and spontaneous spectrum vs energy.](image-url)
6. Conclusion. - Progress achieved during these last years show that emission of undulators mounted on electron storage rings will become an important source of radiation, particularly on the V-U.V. and X rays range. Up to now, these experiments used storage rings which have not been adapted to the requirements of synchrotron radiation. With suitable storage rings the following performances can be expected:

- Electron energy: 600 - 800 MeV
- Synchrotron emission: 40 kW
- Undulators:
  . period $\lambda_0 = 5$-20 cm
  . number of periods: $N = 20$-100
- Tunable spectral range: X rays to I.R.
  Total power: 0.1 to 1 kWatt
- Free electron lasers on the U-V range?

The experimental work presented in this paper are a result of a LURE-Stanford collaboration undertaken in 1979. These works were performed by: C. BAZIN, M. BERGER, M. BILLARDON, D.A.G. DEACON, P. ELLEAUME, Y. FARGE, J.M.J. MADEY, J.M. ORTEGA, Y. PETROFF, K.E. ROBINSON and M. VELGHE.

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