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RYDBERG ATOMS AND RADIATION IN A RESONANT CAVITY I. THEORY

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Résumé.— Nous analysons dans ce papier divers aspects de l'interaction d'atomes de Rydberg avec une cavité électromagnétique. Nous montrons qu'un ensemble d'atomes couplés à un mode unique du champ à une température finie se comporte comme un "objet quantique unique" manifestant des effets interprétables en terme de statistique de Bose ou de mouvement brownien. Nous décrivons l'amplification transitoire du champ thermique par un échantillon d'atomes de Rydberg et étudions l'évolution de la statistique des photons émis en fonction du temps. Nous montrons également qu'un atome de Rydberg isolé dans une cavité résonnante évolue suivant un régime oscillant ou amorti, suivant les valeurs respectives des constantes de temps du couplage atome-champ et champ-parois de la cavité. Nous analysons cet effet comme une modification des propriétés d'émission spontanée de l'atome induite par la présence de la cavité. Des expériences destinées à vérifier ces effets sont en cours et seront décrites dans le papier suivant.

Abstract.— We analyze in this paper various aspects of the interaction of Rydberg atoms with resonant electromagnetic cavity. We show that an ensemble of Rydberg atoms coupled to a single field mode at a finite temperature behaves as a single quantum object exhibiting features of Bose-Einstein statistics and Brownian motion. We analyze the transient amplification of the blackbody field by a Rydberg sample and study the change of the photon emission statistics as a function of time. We also show that an isolated Rydberg atom in a cavity undergoes either an oscillating or a damped evolution according to the respective values of the atom to field coupling and cavity damping times and we analyze this effect in term of a modification of the vacuum field modes surrounding the atom. Experiments to check these effects are under progress and will be described in next paper.

1. Introduction.— Rydberg atom and cavity as a test system for the study of simple quantum electrodynamical effects

Theoretical models describing two-level atom systems interacting with a single mode of the electromagnetic field lie at the heart of modern quantum optics ⁽¹⁾⁽²⁾⁽³⁾⁽⁴⁾⁽⁵⁾. By considering various initial conditions for the atomic system and for the state of the field, such models are used to understand a variety of simple and important effects : optical nutation and Rabi oscillation of various kinds, statistics of laser and maser emission, superradiance ... Due to the intrinsically small atom to field coupling, most of these effects require very large atom or photon numbers to be observed in an actual experiment. When these numbers are reduced to small absolute values, the time constant of the atom-field evolution becomes indeed usually much longer than other characteristic times in the problem (atomic relaxation or cavity mode damping time), so that models involving a single electromagnetic mode are not realistic to describe the experimental situation in these cases. For this reason, some simple effects predicted by the theory on very small atomic system have not been observed so far. Among them, let us quote for example the modification of the spontaneous emission rate of a single atom in a resonant cavity ⁽⁶⁾, the oscillatory energy exchange between an isolated atom and the cavity mode ⁽⁷⁾, the disappearance and quantum revival of optical nutation signals induced on a single

atom by a resonant field $(^8)(^9)$.

There is now a real possibility of observing all these effects with the development of Rydberg atom experiments in resonant electromagnetic cavities. There are at least three features of Rydberg systems which make them particularly attractive for this kind of studies :

(i) Rydberg atoms are much more strongly coupled to radiation than ordinary atoms. The electric dipole matrix elements between neighbour levels are, for principal quantum numbers n around 30, about three orders of magnitude larger than in ground or low excited states. This makes the intrinsic coupling of a single atom to a cavity mode very strong.

(ii) Rydberg atoms are resonant for millimeter wave transitions, which correspond to reasonably large low-order-mode electromagnetic cavities, insuring a rather long interaction time -in the μs domain- for an atom propagating at thermal velocity across the cavity.

(iii) Rydberg atoms have relatively long spontaneous emission times, which means that their coupling to the other modes of the field can be neglected during the interaction time with the cavity-selected single mode.

For these reasons, Rydberg atoms and cavities constitute quite new radiative systems, in which atom-field coupling reveals itself at an unusual scale. The ultimate goal of experiments in this area will be to study the spectacular single atom effects mentioned above. Even before this goal is reached, there is already a whole range of new interesting effects which have been observed and analyzed : superradiant behaviour has been obtained with atomic systems made of very small number of atoms $(^{10})$, Rabi nutation effects have been induced by the presence of very small number of photons in the cavity $(^{11})$, thermal radiation effects have been detected in which the atomic system behaves as a Bose gas exchanging a very small amount of energy with the field $(^{12})$... It is the purpose of this and the next communication to describe the present status of the research at Ecole Normale Supérieure on these systems made of only "a few atoms and a few photons in interaction". In this first paper, we present the theoretical framework allowing us to describe the system and we analyze some simple and important effects. In the next contribution, we describe the experimental techniques developed to study the evolution of the Rydberg atoms in the cavity and we present the results of some recently performed experiments.

2. Outline of the quantum description for an N-two level atom system coupled to a single electromagnetic mode

We consider an ensemble of two level atoms interacting with a single field mode (practically an eigenmode of a Fabry-Perot cavity resonant with the atomic transition). The atoms are coupled to the field via the well-known electric dipole interaction :

$$V = \hbar\Omega \sum_i [a D_i^- f(\vec{r}_i) + a^+ D_i^+ f^*(\vec{r}_i)] \quad (1)$$

a and a^+ are the annihilation and creation operators for the field, $D_i^\pm$ the raising (lowering) dipole operators which respectively excite (or desexcite) atom i from its lower to upper state (or vice versa); $\vec{r}_i$ is the position of atom i , where the field amplitude is $f(\vec{r}_i)$. The strength of the atom-field coupling is measured by the parameter :

$$\Omega = \frac{d}{\hbar} \sqrt{\frac{\hbar\omega}{2\epsilon_0 V}} \quad (2)$$

which is the Rabi nutation frequency associated to the field of a single photon in

the mode of frequency ω (d : electric dipole matrix element of the transition; $\mathcal{V}$: effective volume of the cavity). For sake of simplicity, we assume in the following that all atoms see the same field [$f(r_i) = 1$ for all r_i 's].

The basic feature of the intrinsically symmetrical interaction V is to couple collectively and coherently the atoms to the field ⁽⁴⁾⁽⁵⁾. When a photon is absorbed, the excitation is shared by the whole atomic system considered as a single quantum object. Figure 1 shows the relevant atom and field levels : the harmonic oscillator-like levels of the field, labelled 1, 2, ... n, ... correspond to increasing photon occupation numbers, whereas atomic levels 1, 2, ... N are collective states in which one, two, ... N atoms have absorbed radiation. Due to the atomic indiscernability in this absorption process, each of these atomic states is non degenerate, a basic feature arising from the fact that the atoms are coupled to a single field mode. At resonance ($\omega_0 = \omega$), the field and atomic scales of Figure 1-a have identical steps. As a result the coherent system evolution governed by V is a combination of elementary processes in which the two subsystems jump in opposite directions along their respective scales.

Figure 1 : Schematic description of the N-atom-single field mode system : the two energy scales represent respectively the atomic and field states, coherently coupled by an interaction proportional to Ω . The field is damped into a thermal reservoir with a rate ω/Q .

For a more realistic description of this evolution, it is necessary to take also into account dissipative processes. In Rydberg atom experiments, the atoms relaxation can generally be neglected during the cavity atom interaction time, so that it is a good approximation to include in the picture the relaxation of the field alone : the harmonic oscillator representing the field is supposed to be coupled to a thermal bath reservoir at temperature T (describing for example the cavity walls). The effect of this coupling acting alone is to bring the field mode to thermal equilibrium within a characteristic time Q/ω (Q : cavity quality factor). This process is described by a master equation for the density matrix of the field, whose equilibrium value is the well-known Bose-Einstein distribution. The N-atom field evolution results from a competition between the coherent atom-single mode coupling (characteristic time $1/\Omega$) and irreversible field damping (characteristic time Q/ω). This evolution is also described by a master equation for the combined system density matrix. We will not give here this equation, but only discuss rapidly various effects predicted by its resolution for different initial conditions.

3. Single atom effects

Let us first consider the case of a single atom ($N = 1$). The energy levels of the combined atom and field system when the coupling V is negligible ($\Omega = 0$) are sketched on Figure 2-a, while Figure 2-b presents the eigenenergies of the system taking V into account. These "dressed" states ⁽¹³⁾ consist of a ground level $|g, 0\rangle$ representing the atom in its lower state $|g\rangle$ with no photon, and of a succession of two level manifolds separated from each other by the energy $\hbar\omega$ of a photon. The

n^{th} manifold is made of two dressed eigenstates $|\pm n\rangle$, which are linear superpositions of the unperturbed $|e, n\rangle$ and $|g, n+1\rangle$ levels (e : excited atomic state) :

$$|\pm n\rangle = \frac{1}{\sqrt{2}} \left[|e, n\rangle \pm |g, n+1\rangle \right] \quad (3)$$

The energy separation between these two levels is $2\hbar\Omega\sqrt{n+1}$. The atomic system evolution can be very simply described in this dressed atom basis :

Figure 2 : Single two level atom-single field mode system at resonance ($\omega_0 = \omega$)
 a) Energy levels in the uncoupled case ($\Omega = 0$)
 b) Dressed energy levels taking the coupling Ω into account.

3.1. Spontaneous effects

Let us assume that the atom is prepared at time $t = 0$ in level $|e\rangle$ in an empty cavity mode ($n = 0$, $T = 0$ K). If the cavity damping effect is neglected ($Q = \infty$), the dissipation-free system evolution corresponds to a quantum mechanical oscillation between the unperturbed combined states $|e, 0\rangle$ and $|g, 1\rangle$. In the dressed atom picture, this oscillation appears as the evolution of a coherent superposition of sub-states in the $n = 0$ manifold. The initial state $|e, 0\rangle$ is indeed a superposition of the levels $|\pm 0\rangle$ and $|\mp 0\rangle$ (see Equ. 3), which evolves with time at frequency 2Ω . When cavity damping is added, this nutation effect competes with the irreversible photon escape. Two regimes should be considered : if $\Omega > \omega/Q$, the probability $P_e(t)$ of finding the atom in state $|e\rangle$ undergoes a damped oscillation (Figure 3-a) (the order of magnitude of this oscillation for typical cavities with $Q \sim 10^4$ and Rydberg transitions around $\lambda \sim 1$ mm is $\Omega \sim 10^4 - 10^5$ Hz). If $\Omega < \omega/Q$, $P_e(t)$ is irreversibly damped according to an exponential-like law with a rate $\Gamma_{\text{cav}} = 4\Omega^2 Q/\omega$ (Figure 3-b). This result can be given a very simple interpretation. Replacing Ω by its

Figure 3 : Single two-level atom in a resonant cavity. a) oscillation of the probability $P_e(t)$ in case of weak cavity damping ($\omega/Q = 0.2 \Omega$; $T = 0$ K)
 b) irreversible decay of $P_e(t)$ in case of strong cavity damping ($\omega/Q = 5 \Omega$; $T = 0$ K)

expression [Equ. 2], one can write indeed $\Gamma_{\text{cav}} = \frac{3}{4\pi^2} \frac{Q\lambda^3}{\mathcal{V}} \Gamma_{\text{sp}}$ where $\Gamma_{\text{sp}} = \frac{8\pi^2}{3} \frac{d^2}{\hbar\epsilon_0\lambda^3}$ is the spontaneous rate of emission between levels $|e\rangle$ and $|g\rangle$ in free

space. The above relation between Γ_{cav} and Γ_{sp} had been predicted long ago by Purcell⁽⁶⁾ who had noticed that the number of oscillator modes per unit frequency interval available for spontaneous emission is equal to $\frac{8\pi^2}{3} \frac{v^2}{c^3} \mathcal{V}$ in free space and to Q/v in a resonant cavity, the ratio of these two quantities accounting precisely for the factor $\Gamma_{\text{cav}}/\Gamma_{\text{sp}} > 1$. The fast damping of $P_e(t)$ in the cavity is thus nothing but the modification of free space spontaneous emission induced by the presence of the cavity walls which changes the boundary conditions for the vacuum field surrounding the atoms. Note that the opposite effect (suppression of spontaneous emission in a non-resonant cavity) has recently been discussed by D. Kleppner in the context of Rydberg atom experiments⁽¹⁴⁾.

3.2. Blackbody radiation induced effects

At a finite temperature $T \neq 0$ K, the above effects are complicated by the occurrence of absorption and induced emission of thermal photons stored in the cavity. The atomic system evolves according to an oscillatory or to an irreversibly damped transient regime towards the final state distribution corresponding to thermal equilibrium at temperature T . The nature of this transient regime depends upon the average number $\bar{n} = [e^{\hbar\omega/k_B T} - 1]^{-1}$ of photons and upon the cavity damping rate ω/Q .

In the case of weak cavity damping ($\Omega \gg \omega/Q$), this transient regime can be expanded along a sum of elementary Rabi oscillation terms. The modulated part of $P_e(t)$ can be written as⁽¹⁵⁾⁽¹⁶⁾:

$$P_e^{(\text{mod})}(t) = \sum_n e^{-n\hbar\omega/k_B T} \left(\frac{1 - e^{-\hbar\omega/k_B T}}{2} \right) \cos(2\Omega \sqrt{n+1} t) e^{-\Gamma_n t} \quad (4)$$

In this expression, $\Gamma_n(t)$ is a damping parameter proportional to ω/Q and depending upon n . Equation (4) merely expresses the fact that the atomic system oscillates in a field in which the number of photons is a random quantity obeying to the Bose-Einstein statistics. The time dependance of Equation (4) has been studied in details in the limiting case of a perfectly reflecting cavity⁽⁹⁾ ($\Gamma_n = 0$). It has been shown that the distribution of Rabi frequencies results in an apparently random oscillating behaviour. For large n values, the oscillation first collapses very quickly, and then revives at much longer delays. This effect is typical of a chaotic quantum field, in the sense that it is not obtained with a semi-classical description of a random Gaussian field. Rydberg atoms seem to provide the only chance of observing it, although the experiment is difficult since in a real cavity, the damping process will rapidly wash out the oscillations before their revival could occur.

4. Collective behaviour of N-atoms in the cavity at $T = 0$ K

The single atom effects described above are easily generalized to N atom systems. Let us consider first the evolution of an ensemble of N two level atoms starting at time $t = 0$ from the upper state $|\psi_{\text{at}}(0)\rangle = |e, e, e, \dots e\rangle$, in a resonant cavity at $T = 0$ K. At first, spontaneous emission in the cavity mode makes the atomic system decay down its ladder of eigenstates and at the same time, increases the field excitation. If the cavity damping is negligible ($\omega/Q = 0$), this process is followed by a subsequent phase in which the photons are reabsorbed and the atomic system reexcited and so on ... This oscillation⁽³⁾⁽⁴⁾ can be portrayed as a Rabi nutation in the field radiated by the atoms and stored in the cavity. The exact shape of the oscillation depends upon N , with an apparently complicated beating pattern for relatively small N values⁽¹⁶⁾ (see Figure 4). In the presence of cavity relaxation, this oscillation is damped. It can be observed as long as $\Omega \sqrt{N} > \omega/Q$. On the contrary, for strong damping or small atom numbers ($\Omega \sqrt{N} < \omega/Q$)

the atomic system energy irreversibly decays according to a monotonous law⁽¹⁷⁾. The decay process is governed by a characteristic collective decay rate $\Gamma_R^{-1} = 4\Omega^2 N Q / \omega$ which is equal to N times the single atom cavity enhanced decay rate Γ_{cav} . In the same way as the single emission rate is enhanced by the presence of the cavity, the collective superradiance rate of the N atom system, which is equal to $N\Gamma_{\text{sp}}$ in free space, is increased by the factor $(3/4\pi^2) Q \lambda^3 / \mathcal{V}$ and becomes equal to $N\Gamma_{\text{cav}}$ in the cavity (see section 6).

Figure 4 : $N = 5$ atoms in a resonant cavity without damping at $T = 0$ K : time evolution of the average atomic energy (the system starts from a fully excited state at time $t = 0$). Note the complicated beating pattern due to Rabi nutation in the time varying self-radiated field.

5. Collective absorption of blackbody photons in the cavity at $T \neq 0$ K⁽¹²⁾

Let us consider now the problem of blackbody radiation absorption by the N -atoms in the cavity. The atomic system is initially in its lower state $|g, g, \dots, g\rangle$ and the field at thermal equilibrium at a temperature $T \neq 0$ K (Bose-Einstein distribution with average photon number $\bar{n} \neq 0$). As time evolves, the atomic system gains energy at the expense of the mode, which is itself replenished by thermal photon emission from the thermal reservoir. The way equilibrium is approached depends upon N and, as in the single atom case, upon the values of $\bar{n}$ and ω/Q . Whatever the exact details of the transient regime are, it is in any case clear that the system will tend towards a distribution given by the Boltzmann law for the atomic energy scale at temperature T , i.e. an equilibrium corresponding exactly to the distribution of the photon number in the field. In other words, the average number of excited atoms at equilibrium is

$$\Delta N = \bar{n} = [e^{\hbar\omega/k_B T} - 1]^{-1} \text{ which is close to } k_B T / \hbar\omega \text{ (in the Rayleigh-Jeans$$

limit). This limit is independent of the size of the sample as soon as $N > \bar{n}$. We thus find the peculiar result that the thermal energy absorbed by N atom in the cavity is not the sum of the energy that would be absorbed by N independent atoms. This comes from the fact that the atoms are not absorbing independently from each other the radiation in the cavity, but rather collectively. The existence of collective energy levels for the atomic system does indeed imply strong dipole-dipole correlation between the atoms. These correlations, which are physically produced by the spatial coherence of the thermal field in the cavity mode, make the system behave as a single quantum object made of indiscernable particles. It thus obeys, as does the field, a Bose-Einstein statistics. It is well known that a gas of bosons reaches thermal equilibrium by absorbing much less energy than a gas of independent atoms. This explains the saturation of the absorption process to a very small limit corresponding to an excitation precisely equal to the number of blackbody photons in the cavity mode ... We give in next section another interpretation of this phenomenon in term of a Brownian motion of the collective atomic polarization in the cavity.

6. Outline of the Bloch vector description for an ensemble of N -atom coupled to a single cavity mode

The full quantum mechanical description in terms of a state vector or a density operator for the compound atom + field system is convenient only for relatively small systems, practically N of the order of a few units at most. For much larger N 's, the number of states to keep track of becomes prohibitive. Fortunately, the

system evolution can then be alternatively described in a classical way, using the concept of Bloch vector (¹⁸). We present here qualitatively this point of view, and limit ourselves -for sake of simplicity- to the case of a purely symmetrical coupling to the field ($f(r_i) = 1$ for all r_i 's). The starting point in this model is to describe each two-level atom as a spin 1/2 particle and to remark that the N atom system, fully symmetrical by atom exchange is isomorphous to a $J = N/2$ angular momentum evolving in an abstract space. This is the Bloch vector, whose "up" and "down" positions correspond to the fully excited and deexcited atomic states respectively. During the system evolution, the angle θ made by the Bloch vector with the "up" direction changes in time and the system evolution is determined by the $\theta(t)$ law. The population difference between the excited and deexcited states is equal to $N \cos\theta$ whereas the global atomic electric dipole is proportional to $N \sin\theta$. The evolution of θ can be derived rigorously by starting from the Heisenberg equations of motion for the operators a , $a^\dagger$, $D_j^\pm$ and by going to a classical limit valid for large N's and amounting to neglecting the commutators of these operators. This equation can also be inferred by intuitive arguments partly based on energy conservation requirements (¹⁹). At $T = 0$ K, one finds for $\theta(t)$ the following equation :

$$\frac{d^2\theta}{dt^2} + \frac{\omega}{2Q} \frac{d\theta}{dt} - \frac{\omega}{4QT_R} \sin\theta = 0 \quad (5-a)$$

where T_R^{-1} has been defined above. This equation describes the evolution of a fictitious pendulum damped by a viscous drag term $|\frac{\omega}{2Q} \frac{d\theta}{dt}|$ in a potential well proportional to $\cos\theta$. If the system is initially fully excited ($\theta = 0$), it is in an unstable equilibrium state. The departure from this equilibrium is generally insured, for Rydberg transitions in the mm wave domain by blackbody radiation (see below). For $T = 0$ K, the departure from equilibrium comes from the fluctuations inherent to spontaneous emission. It can be shown (¹⁹) that these fluctuations are correctly described by assuming a small random non zero initial value for θ , obeying a Gaussian statistics. The evolution of the system down from this initial state depends upon the respective value of ω/Q and T_R (i.e. ω/Q and $\Omega/\sqrt{N}$). If $\omega/Q < T_R^{-1}$, the decay occurs according to an oscillating regime. If on the contrary $\omega/Q > T_R^{-1}$, one can neglect in Equation (5.a) the $d^2\theta/dt^2$ term and this equation becomes :

$$\frac{d\theta}{dt} - \frac{1}{2T_R} \sin\theta = 0 \quad (5-b)$$

This equation describes the irreversible evolution of an overdamped pendulum which monotonically decays from 0 to π without undergoing any oscillation. One thus retrieves in this classical model the general conclusions of the full quantum description (section 4).

7. The Brownian motion of the atomic polarization on the Bloch sphere for $T \neq 0$ K

In the overdamped regime at $T \neq 0$ K, Equation (5-b) has to be completed by an extra term accounting for the effect of a small chaotic radiation field acting on the atomic polarization. If $\bar{n} \ll N$ (much less blackbody photons than atoms in the cavity), the action of this field is only important in the case where θ is close to 0 or π so that $\sin\theta$ in Equation (5-b) can be replaced by θ or $\pi - \theta$ (as soon as θ departs from these values, the $\sin\theta/2T_R$ term, which describes the effect of the self radiated field in the system, starts to drive the atomic evolution and the thermal field can be neglected). One can thus restrict the description of the effect of thermal radiation on linearized equation obtained from Equation (5-b) by replacing $\sin\theta$ by θ or $\pi - \theta$. Since the thermal field in the mode can be decomposed along two uncorrelated random out of phase components, one has also to allow for a free rotation of the Bloch vector in the azimuthal directions around the "up" axis.

This amounts to defining two independant partial Bloch angles θ_X and θ_Y . If the Bloch vector is initially in the up position (figure 6-b), θ_i obey the following linearized equation ($i = X, Y$) :

$$\frac{d\theta_i}{dt} - \frac{\theta_i}{2T_R} = \frac{d \mathcal{E}_B^i(t)}{\hbar} \quad (6-a)$$

in which $\mathcal{E}_B^i(t)$ are two out of phase random components of the blackbody field in the cavity. If, on the contrary, the Bloch vector is initially deexcited (fig. 6-c), one defines $\alpha_i = \pi - \theta_i$ and the α_i obey the equation :

$$\frac{d\alpha_i}{dt} + \frac{\alpha_i}{2T_R} = \frac{d \mathcal{E}_B^i(t)}{\hbar} \quad (6-b)$$

Equation (6-b) is formally identical to the Langevin equation of a Brownian particle of velocity α_i , and mass $m = N\hbar\omega/2c^2$ subjected to a restoring force ($-\alpha_i / 2T_R$)

and to a fluctuating force F_i such that $F_i(t) / mc = d \mathcal{E}_B^i(t) / \hbar$. The average energy of this Brownian particle, $\hbar\omega \frac{N\bar{\alpha}^2}{2} = N/2 (\bar{\alpha}_X^2 + \bar{\alpha}_Y^2) = 1/2 m(V_X^2 + V_Y^2)$ tends, according to equipartition theorem, towards $k_B T$. As a result, the number of excited atoms is limited by $k_B T / \hbar\omega$ and we find again the result of the quantum model (section 5). The absorption of the blackbody field in the cavity can thus be pictured as the Brownian motion of the atomic polarization immersed in a bath of thermal photons ... In the case of a system starting from $\theta = 0$ (equation 6-a), the system undergoes a similar Brownian motion, this time with a "positive drag force", i.e. an amplifying diffusion process. The expectation value of θ^2 , instead of reaching a steady state, increases exponentially according to the law :

$$\frac{N\bar{\theta}^2}{2} = (\bar{n}+1) \left[e^{t/T_R} - 1 \right] \quad (7)$$

Moreover, the fluctuations of θ^2 around $\bar{\theta}^2$ reproduce identically the fluctuations of the number of photons n around $\bar{n}$, so that the statistics of the Bloch angle is given by a Gaussian law :

$$P(\theta^2) d(\theta^2) = \frac{1}{\bar{\theta}^2} e^{-\theta^2 / \bar{\theta}^2} d(\theta^2) \quad (8)$$

with an average value $\bar{\theta}^2$ increasing exponentially with time (see Equation 7). This result holds only during the early stage of the evolution, when $\bar{\theta}$ is small.

8. Statistics of superradiant emission triggered by the blackbody field

We conclude this analyzis of the Bloch vector model by a derivation of the Bloch vector average evolution and fluctuations in a superradiant process down the $\theta = 0$ initial position. It is convenient to divide the system evolution into two stages ⁽²⁰⁾⁽¹⁶⁾ : an initial linear phase near $\theta = 0$, described as a Brownian diffusion, and a subsequent non linear phase, during which the main part of the emission occurs and the effect of the blackbody field can be neglected. These two steps are connected at an arbitrary time $t_0 = qT_R$ ($q =$ a few units). Let us consider a particular realization of the process, corresponding to a specific value θ_0 of θ at time t_0 . The subsequent evolution of the system from time t_0 on is described by the non linear equation (5-b) with the initial condition $\theta(t_0) = \theta_0$. After a straightforward integration, one finds for this case :

$$\cos \theta(t) = -\tanh \left[\frac{t - t_D(\theta_0) - t_0}{2T_R} \right] \quad (9)$$

$$\text{with } t_D(\theta_0) = -2T_R \text{Log}(\theta_0/2) \quad (10)$$

This "hyperbolic-tangent solution" shows that the atomic energy is irreversibly damped after a delay of the order of $t_D(\theta_0)$ within a time of a few T_R . Due to the θ_0 fluctuation, this delay itself fluctuates from one realization of the experiment to the next, the "average" system evolution being obtained by replacing in Equation (9) θ_0 by its average mean square root value $\bar{\theta}_0 = \sqrt{n+1} e^{t/2T_R}$. One finds an average delay $\bar{t}_D = T_R \text{Log} \frac{N}{n+1}$.

The fluctuations of θ around its average at time t are clearly related to the one of the number of photons emitted by the system between times 0 and t , which is equal to $\frac{N}{2} (1 - \cos \theta)$. Let us call $P(n,t)$ the probability for this number to be equal to n at time t . After a straightforward calculation taking into account Eqs (8), (9), (10), one finds ⁽²¹⁾:

$$P(n,t) = \frac{1}{(\bar{n}+1)} \left(\frac{N}{N-n} \right)^2 e^{-t/T_R} \times \exp \left[-\frac{N}{(\bar{n}+1)} \frac{n}{N-n} e^{-t/T_R} \right] \quad (11)$$

Figure 5 represents the $P(n,t)$ distribution at various times, either shorter or longer than t_D . At the beginning of the emission, the photon statistics clearly obeys a Bose-Einstein law characteristic of an amplified thermal field, whereas at later times, the non-linear character of the system evolution equations leads to a probability distribution with a maximum peaked at a $n \neq 0$ value (Poisson-like statistics typical of a coherent field).

Of course, all the properties described in this paper are not a priori restricted to Rydberg atoms. Clearly, the orders of magnitude of the atom to field coupling make them however impossible or very difficult to observe on ordinary atomic systems. In the next communication, we describe experiment performed on Rydberg atom in which we have verified several aspects of the theory outlined above. Researches are in progress to check other predictions of the theory, in particular those concerning the behaviour of single atoms in cavities.

Figure 5 : Probability $P(n,t)$ of emission of n photons between times 0 and t as a function of n for different t values, equal respectively to : a) $0.58 t_D$; b) $0.65 t_D$; c) $0.71 t_D$; d) $0.88 t_D$; e) $0.95 t_D$; f) $1.22 t_D$.

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