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MACROSCOPIC ROTORS AND GRAVITATIONAL EFFECTS

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Abstract.—Astronomical bodies have, in the past, provided essentially the only macroscopic basis for studies of gravitation by means of rotations. Now new technology provides the possibility that laboratory rotors may be made more precise than astronomical ones. This article surveys the properties of some of both types of rotors and describes several laboratory experiments for tests of General Relativity.

Introduction.—Except for astronomical bodies, we seldom think of precision rotors as tools for studying gravitation. Three exceptions can be mentioned. 1) The Hughes-Drever experiments and particles as precision rotors took advantage of the privileged status of these rotors as noninertial frames in order to test for the possible anisotropy of space. 3) The Stanford Relativity Gyroscope will provide a test of General Relativity predictions of spin-orbit and spin-spin coupling of the gyro to the earth's rotation in a satellite orbit. Here the gyroscopic properties, not pure rotation, are what is of interest. 3) Demerey used the decay of a spinning rotor in an experiment aimed at testing what is now known to be an invalid theory of gravitational radiation.

This last test is closest in experimental method to those to be discussed here, although these will have radically different objectives from that one. The objectives of concern here are, instead, mostly related to a series of Machian concepts which reach beyond Einstein's General Relativity. This includes tests for changes in the Newtonian gravitational number, G, and the related question of spontaneous, cosmological matter creation. We also will discuss a test for non-metric relativity.

The temporal and spatial scales of these "Machian" phenomena are huge, and Earthbound schemes must use ultra precision to cope with the narrow spans available to us locally. The first two parts of this article will consider such a question, and discuss in what ways it makes any sense to compete in the laboratory against the reaches of astronomy.

In succeeding sections we will describe three experiments which use precision laboratory rotors to measure gravitational effects—two of them in progress.

Astronomical and Laboratory Rotors.—Dicke and subsequently Muller, have used ancient observations of eclipses, etc. to provide a long temporal baseline for measuring the tidal deficit in the secular decay of the lunar orbital motion as a possible means of estimating G. Precision of the earth's rotational and orbital motion is of much concern in the time-keeping aspects of such calculations.

Van Flandern has used lunar occultations in combination with atomic-clock timing of the secular decay of the lunar orbit for the same reason. This does not involve considerations of precision of rotational motion, but it does indicate the direction in which modern studies of such effects are going. A particular example is the way the precision of the atomic clock makes up for the short temporal baseline available in the time span since its invention and use. In addition, these ...
experiments exploit the new way of considering Dirac's ideas about changing $G$. In particular, his use of two metrics, atomic and cosmological, make comparison of the clock time and lunar orbit time (corrected for tidal effects) a direct test of this version of predictions for changing $G$.

Reference 13 lists a number of other such tests, the basis of which will be discussed in the following section.

Although even galaxy clusters have been involved in considerations of $G$, we will use the earth as a more tangible precision spherical rotor for comparison with a laboratory rotor (of 5 cm radius). Table I lists some of the pertinent mechanical properties of these two rotors.

Table I. Mechanical properties of astronomical and laboratory rotors of interest.

<table>
<thead>
<tr>
<th>Property</th>
<th>Earth</th>
<th>Laboratory Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius (cm)</td>
<td>$6.4 \times 10^8$</td>
<td>5</td>
</tr>
<tr>
<td>Density (gm/cm$^2$)</td>
<td>5.5</td>
<td>8</td>
</tr>
<tr>
<td>Surrounding atmosphere*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) Gas density (molecules/cm$^3$)</td>
<td>$10^2-10^3$</td>
<td>$1.9 \times 10^{10}$ (at 10$^{-8}$ Torr)</td>
</tr>
<tr>
<td>b) Temperature (°K)</td>
<td>$10^3$</td>
<td>3</td>
</tr>
<tr>
<td>Momenta of inertia (gm-cm$^2$)</td>
<td>$8 \times 10^{4\delta}$</td>
<td>$4 \times 10^{4}$</td>
</tr>
</tbody>
</table>

* We arbitrarily choose a point ~ 500 km above the earth's surface. For purposes of gas drag illustration this rough choice will be adequate and in fact the notion of ordinary gas drag is quite crude in this case.

The behavior of these objects relevant to precision rotations is that concerned with their secular spindown and their fluctuations. The secular part is assumed to be exponential. That is, the angular velocity is

$$\omega = \omega_0 e^{-t/\tau^*},$$

where $\tau^*$ is a time constant containing all forms of drags,

$$\tau^* = \frac{I}{\alpha}.$$  

Here $I$ is the rotor moment of inertia and $\alpha$ is an effective overall drag coefficient, as in the following differential equation of a "free" rotor,

$$I\ddot{\omega} + \alpha \omega = 0.$$  

For present purposes we can take $\alpha$ to be composed of two parts, the gas drag $\alpha_g$ and the bearing drag, $\alpha_b$. In the case of an astronomical body the "bearing drag" will consist of nonconservative effects in coupling to other bodies. For the earth this is primarily tidal coupling to the moon and sun.

Other disturbances lead to both secular and fluctuational variations in rotor motions. The "ponderomotive effect" discussed by Braginsky is one such
Figure 1. Cylindrical laboratory test rotor in suspension. Actual rotors are of Zerodur glass-ceramic ultra-high stability material. Typical rotor mass = 250 g, moment of inertia = 1100 g-cm².

The differential heating of a rotor in a uniform unidirectional flux of electromagnetic radiation (the sun) leads to an increase in angular velocity. (This is listed as a negative time constant in Table II, below.)

Fluctuations in rotor angular velocity actually occur as a spectrum. For purposes here, we list only fluctuations at the fundamental rotational frequency for the earth: the length of day (lod) variation.¹⁶

Table II lists the rotational properties of the earth and laboratory rotors. In the latter case, small spheres¹⁷,¹⁸, 1 to 3 mm diameter, at high speeds, 10⁴ to 10⁵ Hz, are used for comparison although experiments to be discussed in following sections use longer, slower rotors.

Earth rotations actually serve as poor measurements for modern gravitational questions. For Ė, the earth rotations are used only indirectly, as measurements of secular lunar orbital decay. The large observed fluctuations¹⁶ and varying secular decay¹⁷ of the earth's rotation interfere greatly with the use of such data. As for direct measurement of Ė, the earth rotations would be inefficient. The earth acts as 10 to 17% of a rigid rotor²⁰,²¹. That is, its moment of inertia would react to Ė only 10 to 17% as much as an aggregate of freely orbiting bodies, because although it is gravitationally bound, its bulk compressibility is dominated by electrical forces of repulsion.

Calculations of the gas drag depend on the relation verified by Beams¹⁷,
Table II. Rotational properties of astronomical and laboratory rotors of interest.

<table>
<thead>
<tr>
<th>Property</th>
<th>Earth</th>
<th>Laboratory rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau^{*}_{g} )-Decay constant from gas drag (sec)</td>
<td>10(^{25})</td>
<td>6 \times 10(^{9})</td>
</tr>
<tr>
<td>( \tau^{*}_{b} )-Decay constant from bearing drag (sec)</td>
<td>(\sim 10^{17}) (ref. 16)</td>
<td>(\sim 10^{12}) (ref. 6)</td>
</tr>
<tr>
<td>(-\tau^{*})-Spin up time constant, pondromotive effect(^{13}) (sec)</td>
<td>(\approx 6 \times 10^{17})</td>
<td>(\infty)</td>
</tr>
<tr>
<td>( \tau^{*} )-observed (sec)</td>
<td>(\sim 10^{17}) (ref. 16)</td>
<td>(\sim 10^{10}) (ref. 17,18)</td>
</tr>
<tr>
<td>Observed rotational ( Q = \omega \tau^{*} )</td>
<td>(\approx 7 \times 10^{12})</td>
<td>(10^{14} - 10^{15})</td>
</tr>
<tr>
<td>Observed fractional fluctuations</td>
<td>(\approx 3 \times 10^{-8}) (ref. 16)</td>
<td>Not observed</td>
</tr>
</tbody>
</table>

\[
\tau^{*}_{g} = \frac{8g}{5z} = \frac{3p}{5z} (\frac{M}{2\pi RT})^{1/2},
\]

This can be put in different units,\(^{13,22}\)

\[
\tau^{*}_{g} = \frac{I}{\dot{\alpha}_{g}} = \left(\frac{4}{3} a k \sqrt{2\pi \mu kT}\right)^{-1} I,
\]

where \( k \) is the gas density, in molecules/cm\(^{3}\), \( \mu \) is the mass of the molecule, in gm, and \( T \) is the gas temperature, K. Thus for scaling of \( \tau^{*}_{g} \),

\[
\frac{\tau^{*}_{g}(\text{earth})}{\tau^{*}_{g}(\text{lab rotor})} = \frac{R_E}{R_L} \frac{\rho_E}{\rho_L} \frac{k_E}{k_L} \frac{T_E}{T_L} \frac{1}{\gamma},
\]

is the relevant ratio. For parameters in the tables, this takes the value \(\sim 10^{15}\).

In summary, both bearing drag and gas drag of the earth are much less than for the laboratory rotors of the past, although other variations also render the earth a rather ineffective precision rotor.

Improved Laboratory Rotors.—The rotors of Beams and Fremerey had three major faults: the excessive gas friction, excessive bearing friction, and high-speed stress-related instabilities.

From Eq. (5) it is seen that a vacuum of \(10^{-18}\) torr would be needed to achieve \(\tau^{*} \approx 10^{18}\) sec for rotors of a few cm size. This is essentially impossible, and a new scheme must be found to remove this limitation.

The method used in our experiments was first conceived by Beams\(^{23}\), although at that time he lacked instrumentation to develop and test it. In this method the gas is forced to rotate along with the rotor by means of an outer, corotating chamber. If the pressure is in the free molecule regime one can expect the gas drag on the inner rotor to approach zero as the rotors approach the same speed.
Two means have been used to keep the inner and outer rotors together. In Method I the outer cylinder rotates at constant velocity, locked to a cesium beam clock. The tiny decay of the inner rotor is such that an appreciable difference in angular velocity does not develop during the lifetime of any doable experiment.

In Method II a servomechanism senses slowdown of the outer rotor and drives it to keep it in phase with the inner one.

These "corotation protection" methods in principle and in practice provide a huge amount of gas drag reduction. They do, however, present a new series of problems related to coupling in the rotating reference frame, and their development has proved a considerable task.

The ferromagnetic suspensions which served as bearings for Beams and Fremerey exhibited limiting time constants at about $t_\text{b} \approx 10^{12} \text{ sec}$. This arises from the rotor spin coupling to the earth's spin, a "Coriolis torque" which causes a rotor in the northern hemisphere to hang off microradians toward the North. The noncolinearity of spin and suspension axes demands compensating eddy currents which have a tiny component in a direction such as to create drag. Surprisingly this effect is virtually independent of angular velocity.

Even if compensating schemes for the Coriolis torque could be devised, it is not clear that the ferromagnetic suspension is suitable for gravitational experiments. It is hard to imagine how the active servomechanism of such suspensions can fail to, through some form of mode coupling, remove or add energy to the rotor at the minute levels of interest. In one series of experiments we have observed some rotors to never lose speed, even in a $10^{-6}$ torr non-corotating gas atmosphere. In some cases this would correspond to addition of $\approx 10^{-15}$ watts in the rotational mode, presumably pumped in by a parametric mode coupling from the suspension.

Diamagnetic suspensions have been developed for superconducting spheres. The interest was for gyro work and to our knowledge the "lossless" rotational possibility of such rotors was never pursued. It is in this direction that the work of our laboratory almost surely must ultimately go. At the present, however, we are studying the problems of corotation in the more hospitable circumstances of the ferromagnetic suspension.

Finally, the stress and thermal instabilities of the rotors of Beams were a direct result of the high speeds of the rotors. Lower speed rotors, $\omega < 10$ rad/sec, of a few cm size can avoid these problems. At the same time, the moment of inertia goes up faster than the drag, another improvement.

Laboratory Measurement of $\dot{G}$.-A rotating Cavendish Balance can avoid the one major limitation for $G$ measurement, that of a variable gravity gradient at the balance. Since the desired sensitivity is better than $dG/G = 10^{-10} \text{ yr}^{-1}$, gradients at this level would be virtually impossible to avoid otherwise. This has been discussed at length. In essence the rotations, driven by an ultra-precise turntable, average out the torques imposed on the balance by non-rotating local gravity gradients.

To see the sensitivity needed and the character of these experiments the basis for interest in a possible variation of $G$ will be briefly discussed here.

Dirac developed his "large numbers hypothesis" along the lines of a two-metric concept of gravitation. An "atomic" or "quantum" metric applies to atomic events such as the ticking of an atomic clock. An "Einstein" or "cosmological" metric applies to large scale events. In this way the General Theory of Relativity can still remain valid, applying to the large scale events, even though it does not otherwise permit variable $G$, which comes out of the development. That is, variable $G$ only appears in atomic-measured events. This disengagement of the two metrics is a strange, but not new, concept.
Viewed in this way, the large numbers appear as dimensionless ratios, for example, of the largest (cosmological) physically defined distance to the smallest (quantum) distance: the "radius" of the universe to the radius of an elementary particle. Since the radius increases with time for our expanding universe this large number has a temporary present value, about $10^{40}$. At one early time it could have been unity.

By itself the above number is interesting only in its large size. Most dimensionless ratios in nature are only several orders of magnitude from unity. But another dimensionless ratio, this one of forces, has a nearly equal value of about $10^{40}$. This is the ratio of the electrical (quantum?) to gravitational (cosmological?) forces between a pair of electrons.

Dirac assumed these numbers were equal and were large not by accident but by growth. An alternate view could use the maximum size of an oscillating universe in the first ratio and thereby consider it a constant rather than growing ratio.

Taking Dirac's view, then, the gravitational force, $10^{40}$ times weaker, is diminishing when measured by atomic events. Or, the electrical force is increasing when measured by cosmological events. Such an idea has been carried further by Dirac in postulating that the number of baryons in the observable universe, about $10^{80}$, is then growing as $t^2$ when measured by atomic events. He has since devised another version of the concept in which, in an appropriate reference frame, matter is not being created. These ideas have been carried further by Canuto in his scale-covariant theory.

The attractiveness in such ideas lies partly in their Machian connection. They do lead to Scima's statement of Mach's Principle. Another attraction is in the hint of a connection to grand unification. If at very early times when pressures and temperatures were very great, and in the universe all forces were of nearly equal strength, an evolving ratio of electrical to gravitational would not be an untenable notion. Development of the electro-weak unification, grand unification, and the related prediction of a decaying proton have a growing edifice of theoretical interest about them with similar implication.

Thus the scale of growth of the large number ratios is given directly by the Hubble expansion rate of the universe, $H_0 \sim 2 \times 10^{10}$ yr. Consequently $G/G \sim 5 \times 10^{-11}$ yr$^{-1}$. This is the "target rate" for observation of a non-zero change in $G$. It should be noted that different versions of the concept lead to different connections between $G/G$ and the related changes in periods of orbits, or other potential observables. This is discussed, e.g. in Ref. 13.

The torque change in one year in a modest-sized experiment would be $3 \times 10^{-13}$ dyne-cm. This is 2000 times smaller than a signal torque at the limit of Dicke's equivalence principle measurement. And, it is $10^{8}-10^{9}$ times smaller than the torque for the best Cavendish balance measurements of $G$. The trade-off of absolute accuracy in metrology of those experiments for the extreme sensitivity and stability of the $\dot{G}$ experiment would offer hope of reaching the necessary level.

The experiment devised is complex. It is a rotating, superconducting, symmetrized, feedback Cavendish balance. While limits of nature, in principle, do not appear to be surpassed at the requisite level of sensitivity and stability, the possibility for difficult, practical problems seems enormous.

Matter Creation Experiment.—As brought out above, the large number hypothesis also predicts matter creation at a rate $\dot{M}/M = -2 \dot{G}/G \approx 10^{-10}$ per year. If $M$ is taken to be the mass of all baryons in the universe, and new ones are created uniformly throughout space, it is called "additive creation", and would be a rate so low as to affect rotations in an unobservable way. On the other hand, if matter is created where it already exists, each macroscopic body would increase in mass at the above rate. This is called "multiplicative creation" by Dirac.
If the matter is created in the simplest conceivable Machian fashion, i.e. in response to all the other matter in the universe, it would be in a state of rest for the frame of the average of all matter. In this case a rotor in an earthbound laboratory would increase in moment of inertia due to the added matter, and if sufficiently protected, would slow down at a measurable rate. We call this a dynamic measurement of matter creation. The particles created would, under the above assumption, act as a "wind" across the rotor of no observable consequence. (On the other hand, if matter was created in the state of motion of the local matter rather than the simpler Machian idea, it would not slow down the rotor.) An advantage of the dynamic method of measurement is that it is independent of the form of the creation (long a matter of discussion) as long as it stays in place once created in the rotor.

A rotor similar to that of Figure 1, but of Zerodur, weighing 250 gm and having moment of inertia $I = 1100$ gm cm$^2$, is spun up to $\omega_1$ rad./sec in a Method I corotation system. The primary apparatus for this first, room temperature dynamic test of matter creation is pictured in Figure 2.

A precision turntable drives the outer, quartz cylinder at constant speed. A special magnetic-averaging motor, phase-locked to a cesium beam clock, provides adequate speed constancy. A rotating electronic pump evacuates the chamber to $10^{-6}$ Torr. Optical systems observe mirrored surfaces on both the inner and outer rotors and provide timing of each period to about $10^{-7}$ sec. Averaging over $10^4$ period measurements is done through a computer-operated timing system which results in $10^{-11}$ sec. accuracy for the $10^4$ sec (2.78 hour) blocks. This is sufficient for present system testing.
Final runs, at the above "target rate" for matter creation would have to be months long if the above period averages were the only means of observation. However, an integrated phase lag of the inner rotor$^{13}$ of $2 \times 10^{-5}$ radians would occur every $10^6 \text{ sec}$ ($\approx 12$ days) and would be easier to measure.

At such high levels of sensitivity, of course, a large number of ordinary disturbances must be either removed or known and accounted for (e.g. averaged). For example, a 3600 kg granite block on Weber isolators, followed by a 270 kg block on damped pneumatic springs provides a two-stage mechanical isolation system.

A special set of problems arise from corotation. Coupling within the rotating reference frame applies secular and oscillating torques to the inner rotor. The "signal torque" which slows the rotor is about $10^{-13}$ dyne-cm at $H/M = 10^{-10} \text{ yr}^{-1}$. Imperfections in symmetry, with standard precision machining, would lead to gravitational coupling on the rotor with a torque of about $10^{-17}$ to $10^{-18}$ dyne-cm. For reference, thermal noise contributes a fluctuating torque of about $10^{-16}$ dyne-cm on such a rotor at room temperature for averaging times of $10^4 \text{ sec}$.

Thus far our experiments have largely been aimed at assessing the effects of unwanted coupling. Serious efforts have not yet been made to reduce ferromagnetic support bearing friction. (The support pole piece cannot rotate as it develops a "cranking" mode on the rotor.)

More ordinary coupling in the rotating frame consists of electrical and magnetic effects which are just now being studied. In one interesting experiment, the residual magnetism in a "nonmagnetic" clamp screw on the rotor coupled to the asymmetry of the small stray field (< $\frac{1}{4}$ gauss) of the electronic pump to produce oscillations about the synchronous rotation position. These had periods of 10 to 30 min. and amplitudes of about 300. They decayed at a rate of about $10^{-14}$ watts.

In other experiments we have put a pair of copper foil plates around the outer cylinder and applied a few kV potential to test electrical coupling. Remarkably strong coupling occurs, and confirms the need for conducting coatings on all surfaces.

Inertial Clock Experiment.-A sufficiently protected and appropriately interrogated rotor should be a good clock. If matter creation exists, such a clock would keep a particular kind of time, but this would presumably be smooth and predictable.

There could be many uses for such a clock, if it were made sufficiently precise and reliable. One use would be to take advantage of its particular nature from a fundamental point of view and attempt to search for nonmetric properties of Relativity.

Will and Nordvedt$^{36}$ have suggested a measurement of differential time-keeping of two different types of clocks, set side by side, as they progress around the axis on the surface of the earth in its rotation. There is a sinusoidal daily variation of the gravitational potential they experience as they move closer, then further from the sun. If non-metric effects apply, these might cause a differential red shift between clocks each of which obeys,

$$\frac{\Delta \nu}{\nu} = \frac{gh}{c^2} \left[ 1 + \sigma \nu^0 + \sigma \nu^1 \nu^0 + \cdots \right]$$

where $g$ is the mean gravitational field of the sun at their position, $h$ is their height variation, and terms in the bracket are the Lightman-Lee expansion in the so-called THM formalism.$^{37}$ In essence, the expansion has coefficients $\sigma_i$ of the order unity which depend on the nature of the clocks. A "magnetic clock", the hydrogen maser, would have one set. The "electrostatic clock", e.g. our
Figure 3. Primary apparatus for inertial clock. The main chamber, about 30 cm diameter, houses the shroud and minor rotors, seen through the hole in front. Arms at top house electronics for magnetic support and timing. Tube at lower left connects to vacuum pump.

The value of $g\hbar/c^2$ for diurnal variation would be about $10^{-12}$. The inertial clock, would have another. \(^{38}\)

Such an experiment has already been performed by Stein and Turneaure\(^ {39}\), comparing the hydrogen maser and the Superconducting Cavity Stabilized Oscillator (SCSO). As yet they have not had conclusive results.

Figure 3 shows our inertial clock experiment \(^ {40}\), and Figure 4 shows one version of the inner rotor for it, in suspension. The Method II corotation protection is used and the experiment is at room temperature. Double magnetic suspension\(^ {23}\) is used for the two bearings. Gas bearings have shown considerable roughness\(^ {34}\), and it is easier to obtain smooth servo properties with the outer "shroud rotor" bearing being magnetic also. The development of this was a considerable task, but it is now functional.

Thus far only room temperature tests have been carried out, and not under vacuum. The exaggerated viscous couplings between rotors, and between the shroud rotor and chamber have been useful in testing the behavior and comparison with the sets of coupled differential equations which the system obeys.\(^ {40}\)

The rotational servo is an interesting example of control. At present, the periods of the two rotors, shroud and inner, are measured separately and compared in a microcomputer.\(^ {40}\) The difference is converted to an analog signal which operates an eddy-current drive to speed up or slow down the shroud rotor, as needed.
Figure 4. Lower rotor of inertial clock in suspension with shroud removed. Position-sensing coil is immediately below rotor, and mounted on pole of position-correcting coil.

to keep it with the inner. This will be recognized as pure derivative feedback, and it permits wandering of the baseline. A new algorithm is being prepared to provide proportional, and possibly integral, feedback to remove this deficiency.

In the final clock of this type (at low temperature) the ultimate stability and precision will rest on the isolation from standard disturbances as well as those which come from interrogation of the inner rotor. Disturbances and undesired interrotor couplings constitute "leaks" across the servo control and are the matter of our most serious study. At present there appear to be no limitations, in principle, to achieving $Q \sim 10^{15}$ or higher.

Conclusion.—Laboratory precision rotations are a new field which presents interesting new possibilities for gravitational experiments, several of which have been described. At the same time, problems of new kinds appear, particularly in interrotor couplings for the needed corotation gas drag elimination. Room temperature experiments to date have shown some of these problems but have not exhibited evidence for prohibitive limitations in the main concepts. The virtues of high precision timing capabilities for rotational motion, particularly with averaging, have shown the potential for astonishing measurement sensitivity in such research.

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