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HOT ELECTRON POPULATION INVERSION AND BULK NDC IN SEMICONDUCTORS

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Résumé. - L'inversion de population due à la transition radiative entre les sous-bandes de trous lourds et légers donne une conductivité différentielle négative, dans la bande submillimétrique et dans l'infrarouge lointain. Les procédés de création d'une surpopulation de trous légers, dans des échantillons de Ge-p pur, à basse température, soumis à de forts champs électriques et magnétiques sont discutés ici. La surpopulation de la sous-bande de trous légers est due à l'accumulation de trous légers dans des régions spécifiques de l'espace des moments où l'énergie des trous est plus faible que celle du phonon optique. Des résultats obtenus par une simulation de Monte-Carlo sur cette surpopulation sont aussi présentés ici.

Abstract. - The population inversion of the radiative transition between light and heavy hole subbands results in negative differential conductivity in submillimetre and far infrared waveband. The means of producing of the light hole overpopulation in pure p-Ge samples at low lattice temperatures in strong electric and magnetic fields are discussed in the report. The overpopulation of light hole subband arises due to accumulation of light holes in the specific regions of the momentum space where the energy of holes is smaller than the optical phonon energy. The results of Monte Carlo simulation of the overpopulation are also presented in this report.

Valence band of many semiconductors consists of light ( \( t \) ) and heavy ( \( h \) ) hole subbands. If sufficient increase in the light hole concentration above its equilibrium value \( (m_t/m_h)^{3/2} \) is produced, then the population inversion of radiative transition between light and heavy hole subbands may arise (Fig. 1a). The overpopulation may lead to generation and amplification of far infrared and submillimetre radiation in bulk semiconductors. In the report we describe some ways of producing the overpopulation of light hole subband in crossed electric \( \vec{E} \) and magnetic \( \vec{B} \) fields, when hot holes strongly interact with optical phonons. The calculations and measurements refer to pure p-Ge as typical example of semiconductor. The overpopulation occurs in pure semiconductors at low lattice temperatures due to accumulation of light holes in the spindle region of momentum space. The energy of holes in this region does not exceed optical phonon energy. Even at low lattice temperatures \( T \ll \hbar \omega_0 / k \) hot holes in pure p-Ge strongly interact with optical phonons, so there is a difference in behavior of carriers with energies more or less than opti-
cal phonon energy $\hbar \omega_0$. In active energy region ($E > \hbar \omega_0$) holes can emit optical phonons spontaneously and the collision frequency is rather high: $\nu_0 \approx 10^{12} \text{s}^{-1}$. In passive energy region this scattering process is impossible and the collision frequency is defined by acoustical or impurity scattering. The impurity scattering in pure p-Ge samples is low and can be neglected in the facile calculations. Therefore the collision frequency in passive region is rather small. For spontaneous acoustical phonon emission

$$\nu_A \approx 2 \cdot 10^{10} \text{s}^{-1}$$

At appropriate values of electric field it is easy to satisfy streaming conditions:

$$\tau_0 \ll \tau_E \ll \tau_A$$

where $\tau_E = m^* V_0 / e E$ is the free flight time through passive region under the influence of electric field $E$;

$$V_0 = \sqrt{2 \hbar \omega_0 / m^*}$$

is the velocity on the boundary of passive region; $\tau_0 = \nu_0^{-1}$; $\tau_A = \nu_A^{-1}$.

Under streaming conditions there is a cyclic motion of holes with the period approximately equal to $\tau_E$ [1]. A hole almost without collisions transits through the passive region, penetrates into the active one and rapidly comes back because of the optical phonon emission. In the streaming without magnetic field the relative light hole concentration $n_\ell / n_h$ drops down to $(m_\ell^* / m_h^*)^2$ [2,3]. This drop arises as free flight time for light holes is less than that for heavy ones.

Thus, for the increase of light hole concentration above its equilibrium value it is necessary to fulfill the following condition: free flight time for light holes should exceed the corresponding value for heavy holes. This requirement can be achieved in strong crossed $E$ and $B$ fields, when free flight trajectories are closed and light holes may remain in the passive region during its free flight, while heavy holes keep streaming like motion (Fig. 1b). In this case
free flight time of light holes is \( \tau_A = \frac{\hbar}{V_A} \) and free flight time of heavy holes is \( \tau_{Eh} = \frac{m_h V_{oh}}{eE} \). For sufficiently strong electric fields \( \tau_{Eh} < \tau_A \) the condition \( \tau_{Eh} < \tau_A \) is easily fulfilled.

Free flight trajectories of holes in crossed \( \vec{E} \) and \( \vec{B} \) fields in the velocity space comprise the set of circumferences with common centre \( V_c = cE/B \). The position of the centre of circumferences is the same for light and heavy holes (Fig. 1b). The holes move along circumferences with cyclotron frequencies \( \omega_{Bh} = eB / m_h c \) and \( \omega_{B\ell} = eB / m_\ell c \); which are different for heavy and light holes \( \omega_{B\ell} \approx \delta \omega_h \) in p-Ge). If the boundary velocity of passive region \( V_0 \) exceeds \( V_c \) then in the passive region there exists a spindle region \( K \) of closed trajectories [5]. The ratio of volumes of spindle and passive regions \( V_K / V_p \) versus \( V_c \) is plotted in Fig. 1c. The energy of the carriers moving in the spindle region is always less than \( \hbar \omega_0 \), so free flight time of the carriers in spindle region is defined by acoustical scattering and is approximately equal to \( \tau_A \).

The boundary velocity of passive region for light hole subband
\[ V_{0\ell} = \sqrt{2 \hbar \omega_0 / m_\ell} \]
and for heavy hole subband. If velocity \( V_c \) satisfies the conditions \( V_{oh} < V_c < V_{0\ell} \) then the spindle region

\( K_\ell \) in the light hole subband is present, but heavy hole spindle region is absent [6,7] (Fig. 1b). In this case the holes may be accumulated in the spindle region of light holes \( K_\ell \) (cf. [5]) because free flight time of light holes in \( K_\ell \) is equal to \( \tau_A \). The motion of heavy holes looks like streaming. The average free flight time of heavy holes is approximately equal to \( \tau_{Eh} = m_h V_{oh} / eE \) and is smaller than \( \tau_A \).

If the electric field is large enough so that \( \tau_{Eh} < \tau_0 \) then it is possible to estimate the ratio of light and heavy hole concentrations \( n_\ell / n_h \) by the following formula:
\[ \frac{n_\ell}{n_h} = \left( \frac{m_\ell}{m_h} \right)^{3/2} \frac{V_k^\ell}{V_p^\ell} \frac{<\tau_A>}{<\tau_{Eh}>} \]
where \( V_k^\ell / V_p^\ell \) is the ratio of volumes of spindle and passive regions in light subband, this ratio is about 0.3 at \( V_c = V_{oh} \) (Fig. 1c). At helium temperature in pure p-Ge samples the ratio \( <\tau_A> / <\tau_{Eh}> \) is approximately equal to 50 in electric fields for which \( \tau_{Eh} \approx \tau_0 \). These estimations show that the accumulation of light holes in \( K_\ell \) region leads to the overpopulation of light hole subband. The relative concentration of light holes more than 10 times exceed equilibrium value \( ( m_\ell / m_h )^{3/2} \).
The overpopulation of light hole subband was simulated by Monte Carlo method. The spherical parabolic subbands were used in the calculations. Effective masses of light and heavy holes were chosen equal to \( m_e^* = 0.043 \) \( m_0 \) and \( m_h^* = 0.35 \) \( m_0 \). In this simulation optical, acoustical and ionized impurity scattering with intraband and interband transitions were taken into account. Acoustical scattering at \( T = 4 \) K was described by zero point approximation, and at \( T = 77 \) K by equipartition approximation. In the intermediate range of lattice temperatures, where such approximations are invalid strict calculations were made by modified self-scattering procedure [8] which described scattering statistically without any approximations.

Fig. 2: (a) Relative concentration of light and heavy holes, \( n_e/n_h \) is concentration of light holes, \( n_h \) - heavy ones, \( H = 18 \) kOe; (b) Hall and dissipative currents, \( j_H \) is Hall current, \( j_d \) is dissipative one, \( j_{oh} = e (n_h + n_e) V_{oh} \), \( H = 18 \) kOe; (c) Relation between \( n_e/n_h \) and \( j_H/j_{oh} \); (d) Experimental values of Hall and dissipative currents.

Simulation results are presented in Fig. 2a, b, c. The considerable increase of \( n_e/n_h \) takes place at \( V_c \approx V_{oh} \), when region \( K_h \) disappears (Fig. 2a). At \( T = 4 \) K the concentration of light holes may even exceed that of heavy ones. Fig. 2b shows the dependence of Hall \( j_H \) and dissipative \( j_d \) currents on \( E \) field. The dashed lines show the \( j_H \) and \( j_d \) currents for the single band model where only heavy
holes were taken into account. The characteristic feature of the overpopulation of light holes is the absence of the Hall current drop at $V_c \approx V_{oh}$ with the increase of field $E$. This behavior of $j_H$ is due to light holes. The average Hall velocity of light holes is almost equal to $V_c$ and exceeds the Hall velocity of heavy holes. This phenomenon can be used for experimental determination of $n_e/n_h$ by measuring $j_H$ at $V_c \approx V_{oh}$. Fig. 2c shows joint relation between $j_H/n_H$ and $n_e/n_h$. This relation follows from the results of simulations of overpopulation for different magnetic fields, lattice temperatures and concentrations of ionized impurities. A small value of heavy hole Hall velocity at $V_c = V_{oh}$ is responsible for this relation.

The measured Hall and dissipative currents are given in Fig. 2d. The measurements were taken in p-Ge samples with hole concentrations $\rho = 2 \cdot 10^{13} \text{cm}^{-2}$ at nitrogen and helium temperatures. The experiments were made at pulsed regime with low repetition rate. The measuring technique was described elsewhere [9]. At $T = 4 \text{K}$ in low electric fields the process of the impact ionization of impurities takes place, that is why the values of $j_H$ and $j_d$ are given for high enough fields when impurities are fully ionized. The comparison of experimentally obtained dependences of $j_H$ with the joint relation given in Fig. 2c shows that the concentration of light holes was 4 times higher than the equilibrium value [10]. Recently Komiyama et al. [11] (cf. [12]) has measured the dependence of the Hall field $E_H$ in p-Ge on the magnetic field $B$. He observed the increase in $E_H$ when $K_h$ and $K_t$ regions appeared, but the data on $n_e/n_h$ were absent.

The accumulation of light holes leads to overpopulation of light hole subband and to population inversion of the radiative transition $\ell \leftrightarrow h$ [6]. It is possible to make a facile calculation of the absorption coefficient of far infrared radiation in p-Ge under overpopulation of light hole subband. In these calculations free carrier absorption of heavy holes was described by Drude formula with characteristic scattering frequency equal to $\tau e^{-1}$. Transition $\ell \leftrightarrow h$ provides the negative term of the absorption coefficient. This term is proportional to the difference between distribution functions $f_h - f_\ell$. The ratio $f_\ell / f_h$ may be evaluated in accordance with density of states as $f_\ell / f_h \approx (n_e / m_e^{3/2}) / (n_h / m_h^{3/2})$. Therefore for the negative absorption coefficient it is necessary to obtain the considerable increase of $n_e / n_h$ above its equilibrium value $(m_e / m_h)^{3/2}$. Transition $\ell \leftrightarrow h$ is not resonant so the absorption coefficient is negative in wide frequency range from 40 $\mu$ to 500 $\mu$, but the abso-
lute value is rather small. The evaluation of absorption coefficient for wavelength $\lambda = 100 \, \mu m$ at the hole concentration $p = 10^{16} \, cm^{-3}$ gives the value $\alpha = 3 \times 10^{-2} \, cm^{-1}$. The direct numerical calculation of the absorption coefficient is of much interest.

Discuss now the possibility of producing the overpopulation of light hole subband in an intense microwave field of circular polarization.

\[ \bar{E} = E_0 ( \hat{x} \cos \omega t - \hat{y} \sin \omega t) \]

We consider the case of heavy hole cyclotron resonance $\omega = \omega_{bh}^h$, when heavy and light holes manifest different behavior. The motion of heavy holes at cyclotron resonance looks like streaming motion (cf. [13]) and the accumulation region in heavy hole subband is always absent (Fig. 3a). Light holes are magnetized because of $\omega_{be}^h >> \omega_{bh}^h$ and there is an accumulation region $K_t$ in the light hole subband (Fig. 3a). The characteristic velocity for the carrier motion in circular

\[ v/V_{oh} \]

\[ j_{oh} = e (n_e^h + n_h^l) V_{oh} \]
The polarized field is \( V_b = V_c/(1 - \omega/\omega_b) \), where \( V_c = CE_1/B \).

The relative volume of \( K_I \) region depends only on \( V_b/V_0 \) and is expressed by the same formula as in d.c. fields (Fig. 1c). It is only necessary to substitute \( V_c \) by \( V_b \). In high magnetic fields and at high frequency of the pump it is possible to enlarge the ratio \( V_k^e/V_p^e \) in comparison with d.c. fields where \( V_k^e/V_p^e \approx 0.5 \).

Fig. 3 and 4 show the results of Monte Carlo simulation for the a.c. field of circular polarization. The relative concentration \( n_e/n_h \) for a.c. field exceeds the maximum value \( n_e/n_h \) obtained for d.c. field at the same ratio \( \tau_A/\tau_b \) (Fig. 3b). The average heavy hole velocity is saturated with the increase of the amplitude of electric field \( E_1 \) as in the streaming (Fig. 4a). Heavy holes provide the resistive component of the current (Fig. 4b). The motion of light holes is of reactive kind. The average light hole velocity is approximately equal to \( V_b \). Light holes produce a large reactive component of the current Fig. 4b. The reactive component may even exceed the active one at cyclotron resonance of heavy holes because of the increase of the light hole concentration. This phenomenon may be used for experimental determination of the overpopulation of the light hole subband. The saturation of the resistive component of the current with the field increase results in the drop of the resistive conductivity at appropriate electric fields.

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