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EFFECT OF SURFACE ROUGHNESS ON THE KAPITZA RESISTANCE

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Abstract.—The Kapitza resistance due to acoustic phonons crossing a rough planar interface from liquid He into an isotropic solid is investigated, and its characteristic function, viz. the penetration coefficient is calculated as a function of the frequency of the incident phonons and their angle of incidence.

The propagation of acoustic phonons across the interface between liquid HeII and a solid has been proposed as the mechanism giving rise to the thermal boundary (Kapitza) resistance at this interface. However, this process predicts a Kapitza resistance 10-100 times higher than the experimental results. This indicates the existence of additional heat conduction mechanisms. Among the possible candidates are the diffusely scattered phonons, which are observed in phonon reflection and transmission experiments. To account for the diffuse phonons, the theory of a rough interface has been introduced in some simplified versions. Here we re-examine the effects of roughness. We consider a system in which the region \( x_3 > \xi(x_1^+) \) is filled with liquid HeII, while the region \( x_3 < \xi(x_1^-) \) is filled by an isotropic elastic medium. We treat both media as non-dissipative.

The surface profile function \( \xi(x_1) \) is taken to be a stationary stochastic process with the following statistical properties. (1) \( \langle \xi(x_1) \rangle = 0 \); (2) \( \langle \xi(x_1) \xi(x_1') \rangle = \delta^2 \exp(-|x_1 - x_1'|^2/a^2) \), where \( \delta \) and \( a \) are the root mean square departure of the surface from flatness and the transverse correlation length, respectively.

The heat flux \( Q \) across the interface is given by

\[
Q = \frac{\mathcal{K} \varepsilon_o^2}{(2\pi)^2} \int \frac{dk}{k} n(k) \frac{\mathcal{K}}{P_B} \int_0^\pi d\cos \theta \cos \theta P(k, \theta),
\]

where \( n \) is Planck's function, \( c_o \) is the sound velocity in liquid He, \( P(k, \theta) \) is the penetration coefficient of a phonon with momentum \( \mathcal{K} \mathcal{K}^\perp \) incident from the liquid side at an angle \( \theta \) into the solid and is given by

\[
P(k, \theta) = \frac{\langle S^\text{trans} \rangle}{\langle S^\text{inc} \rangle}
\]
where $S_{\text{inc}}$ and $\langle S_{\text{trans}} \rangle$ are the normal components of the incident and the statistically averaged transmitted energy fluxes. These energy fluxes depend on the velocity potential $\phi(x,t)$ in the liquid and the displacement field $\mathbf{u}(x,t)$ in the solid which are, for $x_3>\zeta_{\text{max}}$,

$$\phi(\hat{x},t) = e^{-i\omega t} \left\{ e^{i\mathbf{k}_{\parallel} \cdot \hat{x} - i\mathbf{a}_{\omega}(k_{\parallel})x_3} + e^{i\mathbf{k}_{\parallel} \cdot \hat{x} + i\mathbf{a}_{\omega}(q_{\parallel})x_3} \right\}, \quad (3a)$$

and for $x_3<\zeta_{\text{min}}$,

$$(u_1(\hat{x},t), u_2(\hat{x},t), u_3(\hat{x},t)) = e^{-i\omega t} \int \frac{d^2q_{\parallel}}{(2\pi)^2} e^{i\mathbf{q}_{\parallel} \cdot \hat{x}} \left\{ -i\alpha_{2}(q_{\parallel})x_3 \times \right.$$ 

$$\times \left[ \hat{q}_{1A3} \hat{q}_{2A4} \hat{q}_{2A3} + \hat{q}_{1A4} \frac{q_{\parallel}}{\alpha_{2}(q_{\parallel})} A_{3} \right], \quad (3b)$$

where we have denoted $A_{j}(\hat{q}_{\parallel} | \hat{k}_{\parallel})$ ($j=1,2,3,4$) by $A_{j}$ for simplicity. In these expressions $\hat{q}_{\alpha} = q_{\alpha}/q_{\parallel}$, $\alpha = 1,2$, and $\alpha_{2}(q_{\parallel}) = [\omega^2/c_{p}^2 - 2]\frac{1}{2}$ for $q_{\parallel} > \omega/c_{p}$ and

$$i[q_{\parallel}^2 - \omega^2/c_{p}^2]^{1/2}$$

for $q_{\parallel} < \omega/c_{p}$, with $p = o,t,t,$ and $c_{o}, c_{t}$ and $c_{o}$ are the longitudinal, transverse sound velocities in the solid and the sound velocity in liquid He. These coefficients are determined by the boundary conditions, viz. the continuity of the normal components of the velocity and the stresses acting on the surface $x_3 = \zeta(\mathbf{k}_{\parallel})$, and the perturbative solutions for them can be written in the following forms

$$A_{j}(\hat{q}_{\parallel} | \hat{k}_{\parallel}) = a_{j}^{(0)}(\hat{q}_{\parallel} | \hat{k}_{\parallel}) (2\pi)^2 \delta(\hat{q}_{\parallel} - \hat{k}_{\parallel}) + \zeta(\hat{q}_{\parallel} - \hat{k}_{\parallel}) a_{j}^{(1)}(\hat{q}_{\parallel} | \hat{k}_{\parallel}) +$$

$$+ \int \frac{d^2q_{\parallel}}{(2\pi)^2} \zeta(q_{\parallel} - \hat{q}_{\parallel}) \zeta(\hat{q}_{\parallel} - \hat{k}_{\parallel}) a_{j}^{(2)}(q_{\parallel}, \hat{q}_{\parallel} | \hat{k}_{\parallel}) + \ldots \right\}, j = 1,2,3, \quad (4a)$$

$$A_{4}(\hat{q}_{\parallel} | \hat{k}_{\parallel}) = \zeta(\hat{q}_{\parallel} - \hat{k}_{\parallel}) a_{4}^{(1)}(\hat{q}_{\parallel} | \hat{k}_{\parallel}) + \int \frac{d^2q_{\parallel}}{(2\pi)^2} \zeta(q_{\parallel} - \hat{q}_{\parallel}) \zeta(\hat{q}_{\parallel} - \hat{k}_{\parallel}) a_{4}^{(2)}(q_{\parallel}, \hat{q}_{\parallel} | \hat{k}_{\parallel}) + \ldots \right\}, \quad (4b)$$

where the superscripts denote the order of the corresponding term in $\zeta(\mathbf{k}_{\parallel})$, and $\zeta(\mathbf{k}_{\parallel})$ is the Fourier transform of $\zeta(\mathbf{x})$. In terms of these coefficients, $P(k,\theta)$ can be separated into a specular part and a diffuse part

$$P(k,\theta) = P_{s}(k,\theta) + P_{d}(k,\theta), \quad (8)$$

where

$$P_{s}(k,\theta) = \frac{\rho}{\rho_{o} k \cos \theta} |a_{2}^{(0)}(k_{\parallel})|^2 \text{ Re} \left\{ [2\alpha_{2}(k_{\parallel}) - \frac{k_{\parallel}^2 - \alpha_{2}(k_{\parallel})^2}{k_{\parallel}^2} \alpha_{2}(k_{\parallel})^2] \right\}.$$
where $P$ and $P$ are the mass densities of the solid and liquid He, and the result \[ \langle \kappa'_s \kappa'_l \rangle = 6 \left( \frac{2n}{\pi} \right)^6 (\delta(k'_s + k'_l)g(k'_l)) \] has been used. We use an adaptive numerical integration scheme to evaluate the diffuse part of the penetration coefficient, which is shown in Fig. 1 for $s = a$, and will present the results for the Kapitza resistance elsewhere. In Fig. 1, the two peaks at the angles of 6.4° and nearly 90° come from the excitation of generalized Rayleigh waves and stoneley waves, respectively.

Fig. 1. The diffuse part of the penetration coefficient versus the angle of incidence for two different frequencies $\omega(=\omega_0 k)$.

References