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SURFACE ROUGHNESS KAPITZA CONDUCTANCE: DEPENDENCE ON MATERIAL PROPERTIES AND PHONON FREQUENCY

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Abstract.- A recent calculation of the surface roughness contribution to Kapitza conductance is compared with measurements on several different solid materials in contact with He II.

In a recent publication\(^1\) I have presented some results of a first order perturbation calculation of the effect of a statistically rough surface on Kapitza resistance. These results were compared with previously published data on Cu - He\(^3\), Cu - He II, and NaF-He II interfaces. Excellent agreement in magnitude, temperature dependence, and pressure dependence, was found at temperatures above 0.2 K for reasonably realistic values of the parameters (r.m.s. amplitude and correlation length) characterizing the roughness.

In this paper I compare the calculated Kapitza conductances for several materials with measured values compiled by Snyder,\(^2\) and also discuss the frequency dependence of the scattered phonons.

Let M indicate the wave polarization in the solid; M = l for longitudinal waves; M = σ, π for waves polarized perpendicular or parallel to the plane of scattering, respectively. Then, from reference 1, for an incident power per unit surface area per unit solid angle per unit frequency, \(\partial P_i/\partial \Omega_i\), at frequency \(\omega\) in the liquid, the fractional differential scattered flux into mode M in the solid is,

\[
D_M = \frac{\partial P_M/\partial \Omega_M}{\partial P_i/\partial \Omega_i} = k_M F_M G(\|k_M - k\|).
\]

\(k\) and \(k_M\) are the components of the wave vectors in the liquid and solid, respectively, parallel to the interface. The \(F_M\) are dimensionless expressions which are functions of all the various scattering and input angles as well as the velocities and densities. \(\partial P_M/\partial \Omega_M\) is the scattered power per unit surface area per unit solid angle per unit frequency.
$G(K)$ is the spectral density of the roughness modes with wave vector $K$ and amplitude $\xi_K$. For a simple exponential surface correlation function with correlation length $\ell$ and mean square amplitude $\langle \xi^2 \rangle$ ($\langle \rangle$ indicates an average over an ensemble of representations of the surface), $G(K)$ is given by

$$G(K) = \partial \langle \xi^2_K \rangle / \partial K^2 = 2\pi \frac{\langle \xi^2 \rangle \ell^2}{(1 + K^2 \ell^2)^{3/2}} . \tag{2}$$

The frequency dependence of $D_M$ is completely determined by the factor $k_M^4 G$. Let $H = (\| k_M - k \| )/k_M$, then $H$ is independent of frequency, and

$$k_M^4 G(k_M H) = 2\pi \frac{\langle \xi^2 \rangle}{\ell^2} \frac{(k_M \ell)^4}{[1 + H^2 (k_M \ell)^2 ]^{3/2}} . \tag{3}$$

Thus $D_M$ is directly proportional to the mean square of the perturbation parameter $k_M \xi$, and $\ell$ appears as a scale factor on the frequency.

The total power per unit surface area scattered into the solid is

$$Q_s = \sum_{M=\sigma, \pi, l} \int d\Omega_i \int d\Omega_M \int d\omega D_M (\partial \mathcal{P}_i / \partial \Omega_i) . \tag{4}$$

Its contribution to the Kapitza conductance is

$$h_K^s = \partial Q_s / \partial T , \tag{5}$$

and the total thermal resistance of the interface is

$$R = h_K^{-1} = (h_K^{AM} + h_K^s)^{-1} . \tag{6}$$

Fig. 1 shows the frequency dependence of $Q_s$ for single frequency inputs. At the low frequency end $Q_s \propto (k\ell)^4 \langle \xi^2 \rangle / \ell^2$ as expected from Eq. (2). However, for large $(k\ell)$ small values of $H$ are emphasized and $Q_s$ tends to vary as the integral of Eq. (2) (over $H dH$), i.e. $Q_s \propto (k\ell)^2 \langle \xi^2 \rangle / \ell^2$. Since, as shown by Fig. 1, the scattering probability increases monotonically with frequency, for thermal inputs the scattered frequency distribution is shifted towards higher frequencies. However, thru $H$ the amount of the shift is angle dependent; large angle inputs (from the helium) are shifted less than small angle inputs. These results are not dependent on the specific form of $G(K)$ used here. They hold for any normalized $G(K)$ which decreases monotonically with increasing $K$.

Snyder\textsuperscript{2} has tabulated the largest reported values of $h_K$ for several materials. I have chosen to compare the theory with these data (rather than, e.g. the smallest reported
values) because it is more likely that $h^s_K >> h^M_K$. The integrations required for evaluation of Eq. (5) were all computed numerically assuming $p_i$ proportional to a Debye density of states and equilibrium phonon populations. Of course, actual values of $<\zeta^2>$ and $\ell$ are not known so some choice has to be made. I have used the same values of $<\zeta^2>/\ell^2$ and $\ell/v_t$ ($v_t$ is the shear wave velocity) for all the materials.

Fig. 2 shows the relative values of $h^s_K$ plotted against relative $h^\text{max}_K$ (from Snyder$^2$). Obviously, we cannot expect exact agreement because of the unknown quantities $<\zeta^2>$ and $\ell$, however the general trend is correct.

Fig. 2: Relative values of scattering contribution to Kapitza conductance, $h^s_K$, versus largest measured values$^2$ of $h_K$ for the indicated materials in contact with He II.

References