ATTENUATION OF SECOND SOUND IN SOLIDS
K. Weiss

To cite this version:
K. Weiss. ATTENUATION OF SECOND SOUND IN SOLIDS. Journal de Physique Colloques, 1981, 42 (C6), pp.C6-531-C6-533. <10.1051/jphyscol:19816155>. <jpa-00221231>

HAL Id: jpa-00221231
https://hal.archives-ouvertes.fr/jpa-00221231
Submitted on 1 Jan 1981
ATTENUATION OF SECOND SOUND IN SOLIDS

K. Weiss*


Abstract. — The small-amplitude plane-wave solutions of the Boltzmann-Peierls equation for phonons in solids are classified and the propagation properties of second sound are discussed in a simple way. Specifically the transition (e.g. with rising temperature) from the case where momentum non-conserving phonon collision processes are much slower than the N-processes (second sound regime) to the case where the U-processes dominate (heat diffusion regime) is examined in detail.

0. Introduction. — Second sound in solids [1] is a density wave of the phonons which can be detected as a temperature wave propagating through the sample. It is not easy to create the experimental conditions [2] which allow these observations because the rate $\tau_N^{1}$ of phonon-phonon collisions which conserve momentum and energy must be large compared to the frequency $\Omega$ of the wave (local thermal equilibrium) while the rate $\tau_{\text{diss}}^{1}$ of the collisions which do not conserve these four quantities must be low compared to $\Omega$. This "window condition" $\tau_N^{1} \gg \Omega \gg \tau_{\text{diss}}^{1}$ puts rather severe restrictions on the experimental conditions: the sample must only contain very few impurities, dislocations, etc., it must be fairly large to avoid excessive boundary scattering, and the temperature must be low (but not too low because otherwise ballistic propagation results in a pure sample) in order to ensure that Umklapp (U) processes are much less frequent than Normal (N) processes.

The consequences of this last condition are the topic of the present paper. We will therefore assume that the U-processes are faster than all the other dissipative processes which we will ignore. Our discussion will be concerned with the transition (with rising temperature) from the regime where $\tau_N^{1} \gg \tau_U^{1}$ (second sound) to the regime $\tau_N^{1} \ll \tau_U^{1}$ (diffusion). It is based on the Boltzmann-Peierls equation [3] in the relaxation time approximation to which we will apply a general method [4] which allows the small-amplitude plane-wave solutions of such an equation to be classified.

1. The Boltzmann-Peierls Equation in Relaxation Time Approximation. — In the absence of external forces the kinetic equation for phonons with group velocity $v(p) = \partial \epsilon / \partial p$ ($\epsilon =$ phonon energy) in a pure and large crystal in the relaxation time approximation is [5]

$$
\left( \frac{\partial}{\partial t} + v(p) \cdot \nabla \right) f(p) = -\frac{1}{\tau_N} (f(p) - f_{L,N}(p)) - \frac{1}{\tau_U} (f(p) - f_{L,U}(p)).
$$

(1)

$f_{L,N}(p)$ and $f_{L,U}(p)$ are the local equilibrium distribution functions towards which $f(p)$ develops as a function of time in the two limiting cases that all collisions conserve momentum and energy (N) or energy only (U). They are

* Present address: HILTI A.G. FL-9494 Schaan, Fürstentum Liechtenstein.
Here $T_1(T^s,t)$ and $T_2(T^s,t)$ are the local temperatures in the two cases and $\overline{u}(T^s,t)$ is the local drift velocity established in the case that only $N$-processes are driving the system towards equilibrium. The relaxation times $\tau_N$ and $\tau_U$ are assumed not to depend on the phonon energy and we will restrict the discussion to phonons of one polarization. At the end we will comment on both of these assumptions.

The five unknown local parameters $T_1$, $T_2$, and $\overline{u}$ (the hydrodynamic variables) can be determined for a given $f(\vec{p})$ from the five conservation laws which must be imposed on the collision terms. If we define for any function $g(\vec{p})$ the average $<g> = \frac{8\pi}{3} \int d^3p g(\vec{p})$

these conservation laws can be written as

$$\frac{1}{\tau_N} (f - f_{L,N}) e = \frac{1}{\tau_N} (f - f_{L,N}) \vec{p} = 0 \quad \text{and} \quad \frac{1}{\tau_U} (f - f_{L,U}) e = 0$$

(4)

In what follows we will be interested in small-amplitude plane-wave solutions about total equilibrium $f_0(\vec{p})$ with (complex) frequency $\Omega$ and real wave vector $K$ and we will restrict the discussion to the hydrodynamic regime, i.e.

$$|\Omega| \tau \ll 1 \quad \text{and} \quad |K\overline{v}| \tau \ll 1 \quad \text{with} \tau^{-1} = \tau_N^{-1} + \tau_U^{-1}$$

(5)

where $\overline{v} = <v f_0>/ <f_0>$ is the mean phonon velocity.

After expanding the hydrodynamic variables about their total equilibrium values we are now ready to apply the classification scheme of Ref. 4 to discuss the physical properties of the solutions of Eq. (1) as the ratio $\tau_N/\tau_U$ varies from values much smaller than one to very large values.

2. Transition from second sound to the diffusive regime. As shown in Ref. 4 the relevant information needed to interpret the modes resulting from an analysis of the Boltzmann equation as sketched in the previous section is the number of collisional invariants, i.e. conserved quantities belonging to the fastest relaxation mechanism. If $\tau_N^{-1} \gg \tau_U^{-1}$ this number is four and therefore the analysis in Ref. 4 implies that four of the five nontrivial solutions belonging to the five conservation laws Eqs (4) are compatible with the conditions of Eqs (5). The fifth solution belongs to a collisional mode which implies that $T_1(T^s,t) = T_2(T^s,t)$ on a time scale $\tau \approx \tau_N$ which is much faster than the time scale of the hydrodynamic modes. Solving the fourth order polynomial for the four solutions to second order in the small quantities of Eqs (5) yields two purely damped transverse modes and, more interestingly, the two propagating damped second sound modes

$$\Omega = \Omega_0 (\pm 1 + \Omega_0^{2/5} \frac{1}{\Omega_0 \tau_N} \frac{1}{2} \frac{1}{\Omega_0 \tau_U})$$

(6)

where $\Omega_0 = Kc \sqrt{3}$ and $c$ is the velocity of ordinary sound. The damping is seen to consist of two contributions: the first occurs because the rate at which the $N$-processes establish local equilibrium is, of course, not infinite while the second one is due to the $U$-processes which tend to destroy the collective (oscillating) drift velocity because they do not conserve momentum. Both contributions are small compared to one and a weakly attenuated second sound wave results.
As the temperature of the sample is increased the rate of U-processes increases much faster than the rate of N-processes. Therefore as $\tau_U$ becomes of the same order as $\tau_N$, the damping $\frac{i}{2}(\Omega \tau)^{-1}$ becomes comparable to the velocity (which is lower in this case) of the strongly damped wave. At still higher temperatures we have $\tau_U^{-1} \gg \tau_N^{1}$ and only one (i.e. the number of collisional invariants of the U-processes) of the five solutions remains compatible with the conditions (5). The other four solutions imply (on a time scale of the order of $\tau_U \ll \Omega^{-1}$) $T_1(\vec{x}, t) = T_2(\vec{x}, t)$ as before and in addition $\vec{u}(\vec{x}, t) = 0$, which is plausible from the physics involved. The only remaining solution describing a collective mode is

$$\Omega = i\Omega_0^2 \tau_U$$

i.e. a purely diffusive mode in the phonon gas: heat diffusion in the crystal.

**Concluding Remarks.** — We have made two assumptions which are not in fact necessary for the argument as we have developed it: we have considered phonons of only one polarization and relaxation times which were independent of the phonon energies (and of the polarization, of course). Dropping these assumptions still allows to carry the argument through, and as far as phonons of more than one polarization are concerned earlier results (obtained by different methods) are reproduced [1,6,7].

Admitting energy dependent relaxation times introduces the interesting possibility of time-dependent local equilibrium distribution functions [4] but does not change the classification of the modes as given in this paper.

**References**