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ATTENUATIONS OF PLANE WAVES IN DEFORMABLE, THERMO-ELECTRIC SOLIDS SUBJECTED TO A PRIMARY MAGNETIC FIELD IN AN ARBITRARY DIRECTION

Y. Ersoy

Department of Engineering Sciences, Middle East Technical University, Ankara, Turkey

Abstract.- The dispersion relation for electromagneto-thermo-mechanical (E-MTM) plane waves in electrically and thermally conducting, magneto-thermo-viscoelastic (MTVE) media is investigated with account for the effect of the external primary magnetic field. It is shown that several modes of E-MTM waves arise depending upon the direction of the magnetic field. Among them a particular attention is focused on a certain mode the phase velocities and the attenuation constants of which are obtained and some limiting values are discussed.

1. Introduction.- The propagation of E-MTM waves in materials subjected to an external magnetic field has been the field of interest during the last two decades, e.g. /1-3/. Because the disturbance of waves in media is of invaluable practical importance in seismology, acoustics, transmission lines, and etc. On the other hand, concerning the justification of a constitutive theory which has been developed by comparing with the results of experimental investigations, the solutions of the set of governing equations must be obtained at least approximately. Therefore, in the present investigation, a rather general dispersion relation for E-MTM plane waves propagating in magneto-thermo-viscoelastic (MTVE) solids is obtained, and some limiting values of the phase velocities and of the attenuation constants are discussed under certain assumptions. Therefore, dealing with the mutual effects of magnetic, strain and thermal fields, the following is emphasized by concerning the wave phenomena in isotropic solids: (i) The equations presented here are that in the linear theory of thermo-viscoelasticity (Kelvin-Voigt type) combined with the linearized Maxwell equations for the quasistatic magnetic field system. However, the pondermotive Lorentz force and the actual stress tensor for the soft ferromagnetic materials are taken into account. (ii) Due to the conductivity, the thermo-electric effects are also considered while the mechanical part of the present state stress tensor is not determined by the present strain alone, i.e. a short memory is included since viscous deformations are observed in magnetic materials at temperatures that can be appreciably below their melting points /4,p.52/. Thus we hope that a considerable insight in the role played by MTVE coupling, intermediate estimate of the damping because of the interacting fields, and utility in assessing the accuracy of an approximate theory are achieved.
2. Linearized Governing Equations of MTVE and Dispersion Relation:— The field equations presented here are the special case of the equations of motion and of constitutive equations obtained in [5]. More specifically, the mentioned equations are linearized under the assumptions that the material is subjected to a uniform primary magnetic field \( \vec{H} \) and undergoes infinitesimal deformations. Thus, eliminating the perturbed electric field \( \vec{E} \) and rearranging the terms, the following equations for the considered material are obtained [6]:

\[
\begin{align*}
\epsilon_{ijk} \mu_0 \nabla \times (h_{l,m} + j \mu_0 \alpha_m \alpha_l u_{l,j}) + \beta \theta,_{k,j} + \mu_0 \delta(\vec{H} \cdot \vec{A})_{t,j} + \mu_0 \delta(\vec{H} \cdot \vec{A})_{t,j} + \vec{H} = 0,
\end{align*}
\]

\( i, j, \ldots = 1, 2, 3 \)

\[
\begin{align*}
\epsilon_{ijk} \mu_0 \nabla \times (h_{l,m} + j \mu_0 \alpha_m \alpha_l u_{l,j}) + \beta \theta,_{k,j} + \mu_0 \delta(\vec{H} \cdot \vec{A})_{t,j} + \vec{H} = 0,
\end{align*}
\]

\( i, j, \ldots = 1, 2, 3 \)

\[
\begin{align*}
\begin{bmatrix}
\begin{array}{c}
\vec{h} \\
\vec{e} \\
\vec{u} \\
\rho \\
\lambda \\
\mu_1 \\
\mu_2 \\
\alpha \\
\beta \\
\gamma
\end{array}
\end{bmatrix}
\end{align*}
\]

where \( h \), \( e \) are the fluctuating magnetic and temperature fields, \( u \) the displacement, \( \rho \) the density, \( \lambda \) the magnetic susceptibility, \( \alpha \) the electrical conductivity, \( \lambda \) the thermal conductivity, \( \mu_1 \) and \( \mu_2 \) the elastic Léma constants, \( \alpha \) and \( \beta \) the viscous Léma constants, \( \gamma \) the thermal constant, \( \alpha \) and \( \beta \) the constants for thermoelastic and thermoelectric effects, \( \mu_0 \) the permeability of vacuum, \( \epsilon_{ijk} \) the alternating tensor, \( \alpha_t \) the partial derivative with respect to time, and the comma in the subscripts denotes the partial derivative with respect to the spatial coordinates \( x_i \).

Solutions to this coupled systems of partial differential equations are sought in the form of steady harmonic waves moving in the \( x \)-direction. In this case, \( h \), \( y \) and \( \theta \) must be functions of \( x \) and \( t \) in the form

\[
\begin{align*}
\{ h_k, u_k, \theta \} = \{ h_k^*, u_k^*, \theta^* \} \exp \left[ i(k \cdot x - \omega t) \right],
\end{align*}
\]

where \( h_k^* \), \( u_k^* \) and \( \theta^* \) are constant complex amplitudes, and \( k \) is the wave vector, \( \omega \) the angular frequency and \( i = \sqrt{-1} \). In view of (1) and (2), seven algebraic equations of the form

\[
\Lambda_{\alpha\beta} \psi_\beta^* = 0 \quad (\alpha, \beta = 1, 2, \ldots, 7)
\]

are obtained, where \( \Lambda_{\alpha\beta} \) is the 7x7 coefficient matrix, \( \psi_\alpha^* \) is the column vector defined by

\[
\psi_\alpha = || h_k^*, u_k^*, \theta^* ||^t.
\]

Thus the equations for MTVE are considerably simplified because of the assumptions for the plane waves, and (3) implies the condition
det \( \text{det } I( A (1 s 1 A(1) = 0 \) (5)

for the propagation of the wave. The condition (5) gives, in general, a seventh order polynomial equation in \( k^2 \), therefore, seven modes of waves for an arbitrary direction of the PMF may arise.

Now, relatively general equation (5) may be simplified in the case when the planes of constant phase are also the planes of constant amplitude, and the PMF has only two components, i.e.,

\( k_i = (k, 0, 0); \quad \vec{H}_i = H_o (\cos \phi, \sin \phi, 0), \) (6)

where \( H_o \) is the magnitude of \( \vec{H} \), and \( \phi \) is the angle between \( \vec{k} \) and \( \vec{H} \). After some lengthy elementary row operations, the matrix \( \text{det } I( A(1 s 1 A(1) \) is put into the upper triangular form so that (5) can be deduced easily. It is interesting to notice that (5) leads to \( \hbar = 0 \) since \( \lambda_{ij} \neq 0 \) for the wave. Thus \( \hbar \) has only transverse components, and the number of the independent eigenvectors satisfying (3) are at most six. Then all the waves of small amplitudes may be considered as a linear combination of such monochromatic waves.

3. Phase Velocities and Attenuation Constants.- To carry on the further calculations, (5) is rearranged in more elegant form by introducing some dimensionless quantities. Let \( \xi \) and \( \kappa \) be, respectively, the dimensionless wave number and the frequency defined by

\( \xi = \left( \frac{c_o}{\omega_o} \right) k; \quad \kappa = \frac{\omega}{\omega_o} \) (7)

where \( \omega_o \) is the characteristic frequency, and \( c_o \) is the speed of light in magnetizable material with \( \varepsilon \) and the electric susceptibility is zero.

With the use of (6) and (7), and introducing some dimensionless quantities, (5) is expressed in the form

\( \Delta_1 (\Delta_2 + \Delta_3) = 0, \) (8)

where the first factor is given by

\( \Delta_1 = \Delta_1^{(M)} (\lambda_{ij}^{(S) - i\eta_1 \kappa} - i\eta_2 \kappa \xi^2). \) (9)

After lengthy calculations, \( \Delta_2 \) and \( \Delta_3 \) can be expressed in the same fashion. The quantities in (9) are defined in terms of the dimensionless material constants \( \nu_a (a = 1, 2, ...) \) and parameters \( \eta_a (a = 1, 2, ...) \) which are related by the PMF. They are given as
\[ \Delta(M) = \xi^2 - i \nu_2 \kappa \quad ; \quad \Delta(S) = r^2(1 - i \nu_1 \kappa) \xi^2 - \kappa^2, \]

\[ \eta_1 = \frac{2}{(\xi+1)} v_2 R_H \quad ; \quad \eta_2 = \frac{(\xi-1)(2\xi-1)}{2(\xi+1)} v_2 R_H \sin 2\phi, \]

where

\[ v_1 = \frac{i}{v_0} \omega_0 \quad ; \quad v_2 = \frac{\sigma_0}{\omega_0} \frac{c^2}{\omega_0} \quad ; \quad r = \frac{1}{c_0} \left( \frac{\sigma_0}{\omega_0} \right)^{1/2} \quad ; \quad R_H = \frac{\omega_0 R^2}{pc^2}, \]

and the subscripts M and S stand for the EM- and mechanical S-waves respectively.

Thus, concerning only the factor \( \Delta \) in (8), (9) implies that there arise the coupled and modified mechanical S- and EM-waves because of the PMF, the phase velocities and the attenuation constants of which are determined by solving the biquadratic equation

\[ a_0 \xi^4 - a_2 \xi^2 + a_4 \kappa^2 = 0, \]

where

\[ a_0 = r^2(1 + v_1^2 \kappa^2) \quad ; \quad a_2 = \left[ 1 - v_1(n_1 + n_2) \right] + i(v_2 a_0 - v_1 \kappa^2 + v_1 + n_2), \]

\[ a_4 = -v_2\kappa(v_1 \kappa + \frac{n_1}{\kappa}) + i(v_1n_1 - 1). \]

It follows from (12) that \( \xi \) has the complex roots \( \bar{\xi}_1 \) and \( \bar{\xi}_2 \) from which we can easily determine the phase velocity \( V(H) \) and the attenuation constant (rate of damping) \( \alpha(0) \). Upon using the phase velocity \( V(0) \) and the attenuation constant \( \alpha(0) \) for the S-wave in the material not subjected to the PMF, i.e.,

\[ V(0) = rK_0 \left( \frac{2}{1 + K_0} \right)^{1/2} \quad ; \quad \alpha(0) = \kappa \frac{K_0 - 1}{rK_0} \left( \frac{2}{1 + K_0} \right)^{1/2}, \]

in the resulting equations, we obtain

\[ \frac{V_1(H)}{V(0)} = \frac{1}{K_1} \cdot \frac{(K_0 + 1)^{1/2}}{1 + K_2} \quad ; \quad \frac{\alpha_1(H)}{\alpha(0)} = K_1 \frac{(1 + K_3)K_4}{(K_0 - 1)^{1/2}} \]

for the modified S-wave; and

\[ \frac{V_2(H)}{V(0)} = \frac{1}{K_1} \cdot \frac{(K_0 + 1)^{1/2}}{1 - K_2} \quad ; \quad \frac{\alpha_2(H)}{\alpha(0)} = K_1 \frac{(1 - K_3)K_4}{(K_0 - 1)^{1/2}} \]

for the modified EM-wave, where \( K_0, K_1, K_2, K_3 \) and \( K_4 \) are the quantities depending upon the material constants (e.g. \( \delta, \varepsilon, \nu_1, \ldots \)), the domain of the magnetic
parameters given by (10), and upon the characteristic frequency of the wave. For example, $K_0^2 = 1 + v_1 K^2$.

It is at once apparent from (15) and (16) that the illustration of $V_{1,2}(H)$ and $\alpha_{1,2}(H)$ would be a heavy task unless some of the parameters are specified, and then some numerical computations are done or some idealized materials are concerned.

Before considering some approximate values of $V_{1,2}(H)$ and $\alpha_{1,2}(H)$, it is interesting to note that the modification of the waves disappear if the angle $\phi = (n-1)\pi/2$, $n = 1, 2, \ldots$ (i.e., the PMF is either longitudinal or transverse) while the coupling do arise. However, for the nonvanishing magnetic field and suitable material constants, the waves are, in general, coupled and they consist of predominantly mechanical, EM- or thermal- waves. Thus the rate of dampings of these waves deviate from those of the uncoupled waves, and the modified waves continuously interact and exchange energy.

Now, for a clear idea about the frequency dependence and the effect of the PMF on $V_{1,2}(H)$ and $\alpha_{1,2}(H)$, (15) and (16) can be simplified reasonably for some materials. Of all many possible values of material constants and the magnitude of the PMF, two types of waves are, however, of particular importance: those are sound waves (SW) and ultrasound waves (USW). Depending upon the characteristic frequencies of SW and USW and the relaxation times, if the parameters in (10) and (11) are computed, then

$$\lim_{K \to 0} V_{1,2}(H) = \frac{L_1}{(1 - \epsilon_1)^{3/2} (1 + \epsilon_1)^{3/2}}; \quad \lim_{K \to 0} \alpha_{1,2}(H) = 0,$$

$$\lim_{K \to \infty} V_{1,2}(H) = \frac{L_2}{(1 - \epsilon_2)^{3/2} (1 + \epsilon_2)^{3/2}}; \quad \lim_{K \to \infty} \alpha_{1,2}(H) \to \infty,$$

are obtained, where

$$L_1 = 2r \left( \frac{2}{v_2^2 r^2 + n_1 + n_2} \right)^{3/2}; \quad L_2 = 4r \left( \frac{v_1}{1 + v_1 v_2 r^2} \right)^{3/2},$$

$$\epsilon_1 = \frac{2r}{v_2^2 r^2 + n_1 + n_2} \left( n_1 v_2 \right)^{3/2}; \quad \epsilon_2 = \frac{2r (v_1 v_2)^{3/2}}{1 + v_1 v_2 r^2}.$$

From (17), it can be concluded, strictly speaking, that the modified waves propagate without damping if the dimensionless frequency $K$ approaches zero, and they may significantly attenuate at high frequencies. While the effect of the PMF on $V_{1,2}(H)$ is, in the low scale frequencies, significant, it also becomes disappear in the high scale frequencies. Therefore, the modified waves at the low frequencies suffer the applied magnetic field more than the waves at the high frequencies.
On the other hand, for the appropriate material constants and very small (or large) frequencies if the terms involving the factor \( \kappa^2 \) (or \( \zeta^4 \)) in (12) is neglected compared with the others, the roots of the resulting equation are then computed easily. The computation implies that the wave is a predominatly S (EM)-wave affected by the PMF if the frequencies are very small (large). Furthermore if there is no externally applied magnetic field, or if the magnetic field is sufficiently weak such that \( n \leq n_1 \leq n_2 > 0 \), (15) and (16) become respectively

\[
\lim_{n \to 0} \frac{V_1(H)}{V(0)} = \lim_{n \to 0} \frac{\alpha_1(H)}{\alpha(0)} = 1; \quad \lim_{n \to 0} \frac{V_2(H)}{V_M(0)} = \lim_{n \to 0} \frac{\alpha_2(H)}{\alpha_M(0)} = 1
\]

agreement, as would be expected, with the values occur in the uncoupled waves. Here the subscript \( M \) stands for the uncoupled EM-wave. In particular, if the material is an electrically insulator, then the effect of the PMF diminishes. The cases concerning the other modes and some idealized materials can be discussed similarly.

References