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Two Dimensional Electron Localization

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Abstract.—This paper gives a short review of recent experimental work on weak localization in silicon inversion layers. It is shown that both this effect and effects arising from the electron-electron interaction are present. These mechanisms can be distinguished by their response to a magnetic field, which enables a confirmation of the principal theoretical predictions. The localization is cut off by a magnetic field which leads to the return of metallic conduction, and, it is suggested, a minimum metallic conductance.

Introduction.—Two types of two dimensional system have been of use in localization studies, these are thin metal films (1) and the inversion, or accumulation, layer at a semiconductor surface (2). The principal difference between these systems is that the carrier concentration in the metals (gated structures) can be altered by only a small proportion. On the other hand the semiconductor systems possess a lower carrier concentration (a maximum of about $2 \times 10^{18} \text{cm}^{-2}$) but this can be decreased continuously to a negligibly small value.

A considerable amount of work has been performed on the nature of localization in silicon inversion layers (2, 3, 4). The main points which have been established are as follows.

1. A mobility edge, $E_C$, separates extended states from localized states in the two dimensional band tail. Modifications to the concept of the 2D mobility edge will be discussed later.

2. The temperature dependence of the conductance, $\sigma$, is in agreement with Mott's prediction that, when states of the Fermi energy are localized, conduction is by excitation to, $E_C$, passing into variable range hopping ($\sigma = \exp \left( \frac{E_F}{T} \right)^1$ as the temperature is reduced.

3. When the Fermi energy, $E_F$, is at $E_C$ the conductance is not less than $0.1 \text{e}^2/\hbar \cdot (3.10^{-5} \Omega^{-1})$, the minimum metallic conductance, although values greater than this have been obtained. Metallic conduction at values of $\sigma$ below $\sigma_{\text{min}}$ for a particular specimen are not found. This increase in the value of $\sigma_{\text{min}}$ has been discussed in terms of an increase in the length of the potential fluctuations (2).

4. The location of $E_C$ increases as the localized states are occupied, implying, that, as in the semiconductor impurity band, both disorder and the electron-electron interaction are responsible for the localization.

Recently, theoretical arguments have been advanced suggesting that all states are localized in two dimensions (Abrahams et al. (5), Gorkov et al. (6), Houghton et al. (7), Haydock (8), Kaveh and Mott (9). On these arguments the mobility edge is a localization edge separating two different types of localized states. These suggestions, and the experiments which they stimulated, will now

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be discussed.

Weak localization and interaction effects in 2D. - Abrahams and co-workers (5) first suggested that all states in 2D are localized with the mobility edge being a localization edge separating strongly and weakly localized states. They find that the conductance of a 2D system is

\[ \sigma = \sigma_0 - \frac{e^2}{2\pi h} \ln \left( \frac{L}{\xi} \right) \]  

(this formula was subsequently obtained elsewhere (6, 8)). \( \sigma_0 \) is the normal conductance, \( ne^2/2m \), and the logarithmic term arises from the localization. At zero Kelvin, \( L \) is the specimen length, but at finite temperatures \( L \) is the inelastic diffusion length \( (\xi^2 L)^{1/2}, \xi \) and \( L \) are the inelastic and elastic mean free paths respectively. The pre-logarithmic term is strictly valid for small changes, \( \alpha \) is a constant possessing values 1 or 3 depending on the strength of spin flip scattering. As \( \xi^2 \) varies as \( T \) and \( L \) varies as \( T^\frac{3}{2} \) the temperature dependence of the change of \( \sigma \), \( \delta \sigma \), is

\[ \delta \sigma = - \text{const.} \ln T. \]

Kaveh and Mott have suggested that at zero Kelvin the localization edge separates exponential band tail states from states which are power law localized, \( \psi \sim 1/r \exp(ikr) \). Alternative forms of localization have been proposed by Pichard and Sarma (10) and Haydock (8). However, at finite temperatures it is suggested that the power law localization is converted into exponential localization and here the decay length is the inelastic diffusion length.

Recently Hodges (11) has explored the connection between these quantum treatments and the earlier classical theorem of Polya (12) that true two dimensional diffusion does not occur.

Another theory which predicts the same temperature dependence has been proposed by Altshuler, Aronov and Lee (13) and is based on the three dimensional work of Altshuler and Aronov (14). Here, interference between elastic scattering and the electron-electron interaction leads to a singularity in the density of states at \( E_F \). This results in a correction to the conductance given by

\[ \delta \sigma = \frac{e^2}{2\pi h^2} (2-2\pi) \ln T. \] 

(2)

Where the factor \( F \) arises from electron screening and tends to zero only at the highest values of carrier concentration, \( n \). For low values of \( n \) it is approximately given by,

\[ F \sim 1 - 4 \frac{k_F}{K} \]

where \( K \) is the 2D screening length.

Kaveh and Mott (18) have obtained the result of Altshuler et al. by a non-diagrammatic technique. They point out that the two corrections should add, the localization affecting the diffusivity and the interaction affecting the density of states. From the temperature dependence of conductance it is only possible to separate the localization and interaction mechanisms if \( F \) and the inelastic scattering mechanism are known precisely. However, the magneto-resistance and Hall effect offer a more precise method of distinguishing between the two (16, 17).

Logarithmic corrections, magneto-resistance and Hall effect. -

a) Magneto-resistance

In the presence of a magnetic field, \( B \), the cyclotron length, \( L_c \), becomes a length scale affecting the conductance. The appropriate correction is

\[ \delta \sigma \left\{ \frac{e^2}{2\pi h^2} \left[ \frac{1}{2} \left( \frac{\hbar}{eB_{\text{in}}} \right)^2 + \frac{5}{2} \ln \left( \frac{4eB_{\text{in}}^2}{\hbar^2} \right) \right] \right\}. \] 

(3)
Kaveh et al. (18) have shown that to a good approximation the correction in a magnetic field can be written as

$$\delta \sigma = \frac{\varepsilon^2}{\pi^2 \hbar} \ln \left( \frac{L_D}{\xi} \right)$$

(4)

where $L_D$, the new length scale is given by

$$\frac{1}{L_D} = \sqrt{\frac{1}{\xi^2} + \frac{1}{L_C^2}}.$$  \hspace{1cm} (5)

This approximation agrees well with equation 3 except when $L_C \gg L$ where it fails to capture the $B^2$ dependence of the conductance correction.

Thus, as $B$ increases $L_D$ becomes smaller and a negative magneto-resistance is found. Even if other mechanisms dominate the temperature dependence of resistance, the negative magneto-resistance will be found if the other scattering processes are not sensitive to small values of $B$.

Recently it has been suggested by Lee and Ramakrishnan (Private communication, to be published) that a magnetic field affects the interaction mechanism when $g \alpha B \gg kT$, the factor $2 - 2F$ being turned into $2 - F$. This increased the magnitude of the logarithmic correction particularly at low values of $n$. As the localization is affected through orbital motion, the negative magneto-resistance will be dependent on the component of $B$ normal to the plane of conduction (19). On the other hand, the enhancement of the interaction correction is a spin effect and so is only dependent on the magnitude of $B$, not the direction.

b) Hall effect

In a similar manner the Hall constant, $R_H$, ($R_H = \sigma_{xy}/\sigma_{xx}$) offers a means of distinguishing between the two regimes. Fukuyama (20) showed in the localization case the correction in $\sigma_{xy}$ is twice that of $\sigma_{xx}$, thus there is no correction in $R_H$. However, Altshuler et al. (13) suggest that for the interaction regime there is no change in $\sigma_{xy}$ only $\sigma_{xx}$, consequently the change in $R_H$ is related to the change in resistance $R$ by

$$\delta R_H / R_H = 2\delta R / R.$$  \hspace{1cm} (6)

Or, alternatively, for localization the Hall mobility, $\mu_H$, decreases logarithmically with decreasing temperature, whereas in the interaction regime $\mu_H$ increases with the logarithm of decreasing temperature.

Experimental investigations.- The first observation of the logarithmic correction was by Dolan and Osheroff (1) who investigated transport in thin metal films. Bishop, Tsui and Dynes (21) showed that the correction was present in inversion layers, this being the explanation of the weak temperature dependence in the metallic regime present in earlier work but not investigated, i.e. Adkins, Pollitt and Pepper (22) Figure 3. As mentioned previously, in the absence of definite knowledge of the inelastic scattering mechanism and the value of $F$, conductance measurements cannot distinguish between the localization and interaction regimes. However, it was shown by Uren, Davies and Pepper (23) that a combination of measurements as a function of electric and magnetic fields could distinguish between the mechanisms, and that both were present. This work and that of these authors with Kaveh will now be discussed.

Anderson et al. (24) initially pointed out that as the electron temperature $T_e$ varied with electric field, $F$, as $F^x$, information on the electron-phonon coupling can be obtained by a combination of electric field and temperature. This topic will not be discussed here but the measurement of the logarithmic correction as a function of electric field is a very convenient experimental technique. Figure 1 shows the logarithmic gradient found by Uren, Davies and Pepper as a function of magnetic field, $B$. Here the gradient is expressed as the change of conductance per decade change of electric field (due to two length scales being important in the presence of a
magnetic field (equations 4 and 5) the dependence of $\delta_0$ on electric field is "quasi-logarithmic". It is seen that the gradient initially decreases with increasing $B$ and then increases, finally staying fairly constant. The magneto-resistance in the two regimes showed different behaviour, being negative when the gradient decreased with $B$ and positive when it increased with $B$. It thus appeared that the magnetic field was distinguishing between the two mechanisms. This interpretation was enhanced by the fact that the second (interaction) mechanism was less stable against increasing electric field, enabling extraction of the gradients for the two processes at a particular value of magnetic field. From analysis of the dependence on electron temperature it appeared as if the condition for the magnetic field enhancement of the interaction process was $g\mu_B B > kT_e$, where $g$ is the g value and $\mu_B$ is the Bohr magneton. Uren, Davies and Pepper present results illustrating the decreased stability of the interaction mechanism against an increase in temperature.

Confirmation that the magnetic field produced the interaction regime was provided by measurements of the Hall effect. The ratio $\delta R_H/R_H = 2dR/R$ was found in the interaction (second) regime, this result was obtained by Bishop, Tsui and Dynes, (25) although they did not separate the two mechanisms. The gradual increase in the ratio towards 2 as $kT_e$ increases is shown in Figure 2. These results on the role of interactions were for values of carrier concentration such that $F$ was $\sim 0.85$. However, the magnitude of the pre-logarithmic factor was considerably greater than $2-2F$ and was consistent with $2-F$, in agreement with the recent suggestion of Lee and Ramakrishnan. Confirmation that the magnetic enhancement of the interaction regime is a spin effect, and not related to orbital motion, was provided by Davies, Uren and Pepper (26). These authors found little dependence in the magnitude of the interaction correction on the direction of the magnetic field.
The ratio $\delta R_H / \delta R_H$ is plotted against $k_F e$ for two inversion layers, the specimen resistance is shown at the top. The circular points were taken using a sample with $\ell = 360$ nm, the closed circles and open circles are with electron temperature determined by electric field and ambient temperature respectively. The crosses were obtained with the specimen used for Figure 1, from Uren, Davies and Pepper (23).

The work of Davies et al. also showed that it was possible to obtain metallic behaviour at 50 mK by completely separating the interaction and localization corrections. This was achieved by using specimens with a longer elastic mean free path than Uren et al. The dependence of resistance on electric field for various values of magnetic field is illustrated in Figure 3. It is seen that the magnetic field initially reduces and then eliminates the logarithmic correction, and metallic behaviour is found for values of electron temperature down to 50 mK. (At high values of electron temperature, $\sim 1$ K, the resistance again increases with increasing electric field. This is due to a change in screening with electron temperature and has no relevance to this work.) Further increase in $B$ results in a positive magneto-resistance and the return of the logarithmic correction, although here it is due to interactions rather than localization. The complete separation of the mechanisms arises from the increased elastic mean free path $\ell$ and a correspondingly enhanced diffusion length $L$. A smaller value of $B$ is required for the cyclotron length $L_C$ to become considerably smaller than $\ell$, so causing the disappearance of the logarithmic dependence on electron temperature before $B$ is sufficiently high to induce the interaction mechanism. However, the negative resistance is still obtained, this only disappears when $L_C$ is near $\ell$. The greater stability of the localization mechanism is apparent as a negative magneto-resistance is observed when the electric field is increased sufficiently to erase the interaction correction.

Another manifestation of the decreased localization length scale is that, as the magnetic field increases, a higher value of electric field is required to induce the logarithmic behaviour. Increasing $B$, hence decreasing $L_C$, necessitates a smaller value of $L$, and greater value of electric field, for this to determine the length scale.

Kaveh et al. (18) have shown that the logarithmic correction is determined by the shortest length available, regardless of its origin. The length can be the cyclotron length or the inelastic length, depending on the values of magnetic field and electron temperature as is shown in Figure 4.
Figure 3. Sheet resistance is plotted against the logarithm of electric field $L = 360 \text{ nm}$, lattice temperature $= 50 \text{ mK}$, carrier concentration $= 3.8 \times 10^{11} \text{ cm}^{-2}$. The values of magnetic field in Tesla are $A = 0$, $B = 0.008$, $C = 0.021$, $D = 0.047$, $E = 0.074$, $F = 0.1$, $G = 0.15$, $H = 0.26$, $I = 0.32$, $J = 0.52$, from Davies, Uren and Pepper (26).

Extraction of the inelastic length from the magneto-resistance has allowed investigation of the temperature dependence of the inelastic scattering rate (27, 28). It has been found that the $T^2$ law is modified by the elastic scattering and a component varying with a lower power of temperature is also present, Figure 5. This is in qualitative agreement with theoretical considerations suggesting a $T$ component due to the blurring of electron momentum when $kF\ell$ is near one.

Finally, very recently Poole, Pepper and Glew (29) have investigated the situation where the screening factor $F$ is near 0.5, and both interaction and localization effects are present in the absence of a magnetic field. The localization is suppressed by the application of a magnetic field, and whilst the resistance decreases with $B$ the Hall constant does not vary - in accordance with the suggestion of Fukuyama (20). However, when the localization is suppressed, a logarithmic correction is still found as a function of temperature. This is due to the interaction mechanism and now the Hall ratio of 2 is found. These results confirm the suggestion of Kaveh and Mott (8, 15) that the interaction and localization corrections are additive.
Conclusion.- Good agreement now exists between experiment and the predictions of the localization and interaction theories. By the application of a magnetic field, or a combination of magnetic and electric fields, the two mechanisms can be separated and clearly distinguished by the behaviour of the Hall effect and magneto-resistance. It is possible to obtain metallic behaviour at temperatures as low as 50 mK, although it is not clear if this situation is maintained down to absolute zero. Eventually $kT$ becomes smaller than $g\beta$ and the interaction correction will become apparent. However, if this mechanism is only a perturbation metallic behaviour will be obtained at 0 K and the minimum metallic conductance will be observed. The role of the magnetic field in restoring metallic condition will result in a field induced metal-insulator transition.

In the presence of intense magnetic fields these mechanisms will not have significance for the quantized Hall resistance (30), here $\sigma_{xy}$ is negligibly small and $\sigma_{xx}$ is unaffected by interactions. However, as the Fermi energy is passed through the Landau levels a whole series of metal insulator transitions will be obtained. As described in earlier work (31) the cyclotron orbit will delocalize electrons when the length scale of the potential fluctuations is long.

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Figure 5. \((\tau_{ee}T)^{-1}\) is plotted against \(T\), \(\tau_{ee}\) is the electron-electron scattering time and \(T\) is the temperature. The presence of a component varying at a lower power than 2 is signified, from Uren, Davies Kaveh and Pepper (22).