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NUCLEI AT HIGH SPINS: A SUMMARY

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I. Introduction

During this conference I have tried to find one strong common motive for many studies reported here. To explain what I found let me step back in time. Originally, the research in the field of rotational states was concerned with the collective properties of those states. Up to the late 60's rotational bands were parametrized in terms of the collective model. Deviations from simple rotational behaviour, in particular the slow increase of the rotational moment of inertia with spin I at small angular momenta, were ascribed to such things as rotation-vibration interaction. Many people were at that time concerned with systematizing β- and γ-bands in nuclei and with deducing rotation-vibration coupling parameters from experiment.

The discovery of backbending in 1970 then changed the direction of research in this field completely: Although there were attempts made at first to explain backbending in terms of the collective CAP effect it soon became clear that the explanation of this effect in terms of a crossing of the groundstate band with an aligned quasi-particle band was the correct one. From this point on tremendous effort has gone into systematizing the presence (or absence) of backbending phenomena and we have heard here at this conference that now - once this has been achieved - the experimental interest has turned to the search and investigation of second backbends etc.

An example for the complexity of nuclear spectra that cannot be explained simply in terms of rotational and vibrational β- and γ-bands is given by the case of $^{84}$Sr.

Figure 1a (top), b (bottom) (see fig. 1a) presented at this conference by a group from Cologne $^1$. Also here the concept of alignment of single particle states, on which rotational bands can be built, helps considerably to interpret this spectrum (fig. 1b).

The concept of angular momentum alignment
has turned out to be the most fruitful in the whole field. It has brought as a consequence a tremendous development of microscopic theoretical descriptions of rotational states; something, that very few people ten years ago thought to be possible. As a consequence it has unified the theory of nuclear structure since now phenomena at high spins can be related to standard models, like e.g. the Nilsson model, that were originally set up to describe ground state properties. As a byproduct of these theoretical studies it has also turned out that the slow rise of the moment of inertia is due to the gradual breakdown of pairing, the CAP-effect, and that therefore all descriptions of this effect in terms of the collective potential energy surface and such things like rotation-vibration interactions are misleading.

Because of this extremely fruitful character of the concept of alignment I have chosen it as the central theme of my summary. After a short review of the population and deexcitation mechanism of highly excited nuclei I plan to discuss various manifestations of alignment.

II. Population Mechanisms

There are three methods to populate high spin states in nuclei: The traditional way by Coulomb excitation has gained new life with the increasing availability of high-Z, in particular $^{208}$Pb, projectiles. Certainly the most widely used method is that of (HI,xn) reactions, pioneered 17 years ago by Morinaga and Gugelot\(^2\). And finally, there is the population of high spin states through deep-inelastic reactions.

Coulomb-excitation allows one to study very selectively high spin states in nuclei that show a collective structure. As the excitation process proceeds along a path of enhanced $B(E2)$ values, it favours population of states within one band. The $B(E2)$ values can then rather directly be obtained through the standard coupled channel analysis. The method of Coulomb-excitation is thus ideally suited to the investigation of electromagnetic properties of high spin states. Highest spins obtained with this method are around 24 in rare earth nuclei and around 30 in the actinides where Coulomb-excitation may be the only method to get information on high spin states because of the dominance of fission in all other reaction channels. It should also be noted that Coulomb-excitation is ideal for the study of neutron-rich nuclei whereas all other processes (see below) lead to neutron deficient isotopes.

Of the two nuclear processes, (HI,xn) and deep-inelastic reactions, for the excitation of high spin states, the DI reactions are connected with the highest partial waves. The (HI,xn) method, on the other hand, depends on the fusion process that is connected with the low l-waves. It is well known that for heavy systems the angular momentum cut-off for fusion falls well below the grazing angular momentum. The dominant process at higher $l$ is then that of deep inelastic reactions. It is thus tempting to use this process for the population of high spin states. This is particularly so because DI reactions lead to very broad isotope distributions so that with this process isotopes can be reached that may be inaccessible by any other method. This advantage has, of course, to be paid for by the small flux going into any individual channel.

A typical feature of deep inelastic collisions is the large alignment of the angular momentum obtained in such reactions. This has been found for very heavy nuclei in fragment angular anisotropies of fission following DI reactions\(^3\). The alignment here is typically as large as 0.8. By measuring the out-of-plane anisotropies of discrete $\gamma$-rays in the final products Vandenbosch and coworkers\(^4\) have recently also been able to show that this alignment survives even multiple particle emission.

In deep inelastic reactions between lighter nuclei where fission does not occur, light particle emission competes favorably already
at rather low spins with γ-decay. Thus the standard conversion from γ-multiplicity to angular momentum does not work any more and correspondingly the γ-multiplicities are not a measure for the total spin\(^5\). Instead particle multiplicities, and in particular the ratio \(M_\alpha/M_p\), are sensitive functions of the total angular momentum which can be quantitatively analyzed. This is the central point of the talk by Guerreau\(^6\).

The investigation of emitted α-particles has furthermore the advantage that the particles can be assigned to a unique source in the DI collision, in contrast to γ-rays, simply by means of kinematical conditions. In addition, the energy-distributions of the emitted particles serve as a reliable indication that the emission took place from an equilibrated compound nucleus. From simple statistical arguments it is to be expected that the out-of-plane distribution of particles emitted from high spin states will show an anisotropy:

The rotational energy of a particle at the distance \(d\) from the rotational axis is:

\[
E_{\text{rot}} = \frac{m}{2} \omega^2 d^2 = \frac{m}{2} \omega^2 R^2 \sin^2 \gamma
\]

where \(R\) is the nuclear radius, \(\omega\) its angular velocity and \(\gamma\) the angle between the rotational axis and the nucleon. Thus, at a temperature \(T\) the emitted particles will have a distribution of Boltzmann-character:

\[
W(\gamma) \propto \exp\left( + \frac{m \omega^2 R^2}{2T} \sin^2 \gamma \right)
\]

\[
\propto \exp\left( + \frac{I_\alpha^2}{2J_\alpha T} \sin^2 \gamma \right)
\]

where \(I_\alpha\) and \(J_\alpha\) are angular momentum and moment of inertia of the evaporating particle. Since one has:

\[
\omega = \frac{I_\alpha}{I} = \frac{J_\alpha}{J}
\]

where \(I\) and \(J\) are the corresponding total quantities one finally ends up with a distribution:

\[
W(\gamma) \propto \exp\left( + \frac{I_\alpha^2}{2J T} \cdot \frac{J_\alpha}{J} \sin^2 \gamma \right)
\]

From the derivation it is clear that for the angular momentum \(I\) total alignment has been assumed, i.e. \(I\) is the aligned component of the angular momentum transferred which can thus be obtained from measurements of the out-of-plane anisotropy of the emitted α-particles as shown by Guerreau\(^6\) and Wurm\(^7\) in their talks.

Guerreau has shown in his talk that the \(I\) extracted from the out-of-plane anisotropies with the help of the formulae given above agrees quite well with that obtained from particle multiplicity measurements. This then proves very large alignment of the angular momentum (critical to this argument is the use of light collision partners with a corresponding narrow range of partial waves, and thus minimal \(l\)-fluctuation, in the entrance channel).

I personally would like to see the spin-determinations by means of particle multiplicity measurements extended to much lighter nuclei. I would just like to recall that the limitations observed in fusion reactions between light nuclei (such as \(^{12}\text{C} + ^{12}\text{C} \rightarrow ^{24}\text{Mg}\)) at high energies can possibly be ascribed to the effects of the yrast line of the compound nucleus formed\(^8,9\). For a clear-cut decision on this point it would, e.g. be extremely interesting to know the position of the \(14^+\) state in \(^{24}\text{Mg}\) which is only 4 units of angular momentum higher than the last experimentally known state.

Most of the data reported at this conference have been obtained by means of the \((\text{HI},xn)\) reaction. Here the fusion process is used to produce a highly excited compound nucleus that subsequently decays through particle evaporation. Different neutron numbers emitted lead to quite different regions in the \(E^*\) vs \(I\) plane. Since here the population of high spin states comes from above, states up to the fission limit can be populated.

Measurements of the γ-ray multiplicity and the width of the multiplicity distribution have allowed to reconstruct the angular momentum distribution in the decay product
after 4 or 5 neutrons have been emitted\textsuperscript{10}). From this entrance line then the nucleus cools down towards the yrast line through the emission of statistical $\gamma$-rays (see fig. 2). A group at Canberra (Sie et al, ref. 11) has shown that the average multiplicity of these statistical $\gamma$-rays is proportional to the total $\gamma$-ray multiplicity. This increase could be understood if one assumes that with increasing bombarding energy (angular momentum) the thermal excitation energy above the yrast line also increases.

In good rotational nuclei the decay then proceeds through collective E2-transitions in bands parallel to the yrast line until it finally reaches the latter at spins $I \approx 30h$ from there on down giving rise to discrete lines. This is the reason why in good rotors no individual lines can be resolved above $I \approx 30h$ where the decay strength is still distributed over several bands.

For the nuclei at the beginning of the rare earth region that are spherical or nearly oblate in their ground state T.L.Khoo has shown\textsuperscript{12}) that at high entrance angular momenta these nuclei behave quite similarly to the good rotors. This can be concluded from the analysis of populations of individual states at $I \approx 30h$ that do not increase with increasing input angular momentum. Thus also here the decay follows collective bands close to the yrast line although it reaches it at higher spins than in good rotors. A similar transition to rotational behavior at high spins had already been observed in measurements of continuum $\gamma$-ray spectra for nuclei that are spherical in their groundstate\textsuperscript{13}). The nature of these collective rotations is still unknown.

III. Rotational Structure at low spins

At this point I think it is appropriate to summarize briefly our present understanding of rotational structure of nuclei. For this purpose it is more convenient to start with considering first the structure at low spin and then to work our way up.

The most conspicuous feature of the rotational bands in the rare earths is certainly the gradual rise of the moment of inertia with spin, followed by a backbend, upbend (or the absence of both features) at spins between $I \approx 10h$ and $20h$. There is now no doubt that the gradual rise is due to a continuous break-down of the pair field whereas the backbending phenomenon is due to the mechanism proposed by Stephens and Simon in 1971, i.e. to a crossing of the gs band with an aligned quasi-particle rotational band\textsuperscript{14}).

The explanation of the backbending behavior in terms of crossing bands has gained further support from the observation of second anomalies in the spectra of $^{158}$Er and $^{160}$Yb at spins $I \approx 28h$ (ref. 15). In a contribution to this conference Byrski et al report a new strong second backbend in $^{156}$Er at $I \approx 24h$.

Whereas the first backbends in the rare earths are all due to $h_{13/2}$ neutron alignments, the second backbends in the isotones $^{158}$Er and $^{160}$Yb have been successfully related to the alignment of an $h_{11/2}$ proton pair\textsuperscript{16}). This explanation has received experimental support from the results obtained by Gaardhoje et al\textsuperscript{17}).
These authors report a band crossing in $^{161}$Yb at the same frequency ($\hbar \omega = 0.42$) as the known higher crossings in $^{158}$Er and $^{160}$Yb. In $^{161}$Yb the odd neutron blocks many of the otherwise available qp-excitations for the neutrons so that only an aligned $h_{11/2}$ proton band remains a viable explanation. That the cranking model is indeed able to account for the absence of a second backbending in $^{156}$Dy, the second upbend in $^{158}$Er and the strong second backbend in $^{160}$Yb has been convincingly demonstrated by Faessler in his talk at this meeting (fig. 3).

Stephens and Simon in their original paper\textsuperscript{14} had anticipated that the backbending phenomenon was restricted to the beginning of the rare earth region because with increasing neutron number one was further and further moving away from the strongly aligning $[606]_{1/2}^1$ Nilsson state. In his talk at this meeting, however, Bengtsson showed that due to deformation changes the distance between the Fermi-surface and this particular level remains practically constant (important for this effect is a $Y_4$-type deformation). It then depends only on the interaction between yrast and yrare band whether there is backbending, upbending or no anomaly. It was earlier realized that this interaction strength was an oscillating function of the location of the Fermi-surface\textsuperscript{18} - and again the deformation (see fig. 4) thus explaining presence or absence of anomalies in particular nuclei.

![Figure 3](image)

![Figure 4](image)

The oscillating behavior of the coupling strength between ground state and aligned band has been explained by Faessler as being due to an effective change of the rotating core\textsuperscript{16} and by Bengtsson et al as an interference effect between the two bands\textsuperscript{18}. Faessler has made the physical mechanism for this effect clear in his talk\textsuperscript{16}: By exciting the strongly aligning $i_{13/2}$ neutrons from the $[606]_{1/2}^1$ state up into core states above the Fermi-surface the nucleus can save energy relative to an excitation within the $i_{13/2}$ valence shell. Since thus the ground state and the aligned band differ by 2 qp in the valence shell there is an angular momentum mismatch in the wavefunction that leads to a disappearing interaction at a certain $I_o$. If the crossing of the bands just happens at this $I_o$ then a sharp backbend will appear whereas otherwise the bandcrossing is smoothened out by the interaction. A typical example for the fact that the $i_{13/2}$ neutrons still determine the rotational properties at the heavy side of the rare earth region is given by the study of band crossings in Os-isotopes reported by Dracoulis\textsuperscript{19} and Lieder et al\textsuperscript{20}.

Whereas for the rare earth region the
systematic behavior of the backbending phenomenon have been explored in quite some detail there are much less data available on the actinide region. Since in these nuclei fission predominates at higher excitation energies Coulomb excitation may be the only method to get information on these states. Indeed Elze has presented at this meeting results on even-even actinides\(^{21}\). Since Coulomb-excitation always tends to follow the gs-band up over the crossing, except in the case of strong interactions, all these bands look very regular. What is intriguing is the very constant moment of inertia for nuclei with quite different gs-deformations.

Bengtsson has shown in his talk\(^{22}\) first results for the actinides that seem to indicate that the frequency for the first crossing is the same here for protons and neutrons. R. S. Simon has mentioned the intriguing idea\(^{23}\) to study odd nuclei in this mass range since in them the odd particle blocks one crossing so that more structure should be expected in these results and a more unique assignment of the effects observed to protons or neutrons can be made.

Let me now come to an analysis of all these results. If the total angular momentum \(I\) is made up of a collective, rotational component (\(\hat{R}\)) and an aligned one (\(\hat{J}\)):

\[
\hat{I} = \hat{R} + \hat{J}
\]

then it is clear that collective rotational properties cannot be related to \(I\), but only to \(R\). The collective rotational energy within each of the intersecting bands, that make up the yrast line, is then

\[
E = E_j + \frac{(I-j)^2\hbar^2}{2\Theta_{\text{coll}}} = E_j + \frac{(I-j)^2\hbar^2}{2\Theta_{\text{coll}}}
\]

From this expression one immediately sees that the rotational frequency \(\omega\), defined by:

\[
\omega = \frac{dE}{d\Omega} = \frac{(I-j)\hbar}{\Theta_{\text{coll}}} = \frac{\hbar}{\Theta_{\text{coll}}} \approx \frac{1}{2}(E(I)-E(I-2))
\]

is the quantity that governs the collective rotation and can directly be obtained from the data. The dominant role of this fre-

The frequency appears naturally in the cranking model where the energy in the rotating frame is minimized

\[
\hat{\epsilon} \langle H - \omega J_x \rangle = 0
\]

This variation has been performed selfconsistently in cranked Hartree-Fock-Bogoliubov calculations with realistic effective ground state interactions\(^{24}\) and with more schematic forces\(^{25}\). These calculations give quite satisfactory descriptions of the band structure in heavy nuclei: It is, however, hard to get a feeling for the band structure to be expected from these calculations without actually doing them.

An ingeniously simple method to achieve just this has been devised by Bengtsson and Frauendorf\(^{26}\). These authors replace \(H\) by that of a deformed shell model (e.g. the Nilsson model) and decouple the pairing field from the rotation by working with a constant (\(I\)-independent) pair potential \(\Delta\). The plot of quasiparticle energies versus the cranking frequency \(\omega\) then allows immediate identification of all possible band configurations and crossings. These quasiparticle energy-schemes (see fig. 5) are in
my mind the natural extension of the Nilsson-model to rotating states. They are as transparent and simple to use as the former and provide a convenient classification scheme for rotational bands just in the same way as the standard Nilsson-model can be used to classify ground state configurations of deformed nuclei.

A very convincing example for the success of this model is given by the case of $^{160}$Yb. Here Riedinger et al.\textsuperscript{15} have found that three negative parity sidebands all cross at the same frequency between the first and second yrast backbends. This fact can be explained with the help of the mentioned diagrams: all three sidebands contain the same pair of quasiparticles. It is expected that such multiple band crossings will persist to higher frequencies (transition energies) and may be responsible for the bridges observed in the $E_\gamma^2$-$E_\gamma$ correlation diagrams where many transitions are collected at a particular transition energy (see section IV).

The examples given before, in particular the observed multiple band crossing in $^{160}$Yb, show clearly that the backbending phenomenon can be quantitatively understood in terms of a single particle model that also allows to predict excited bands and their behavior at high spin.

In order to test the whole concept of alignment further it is now interesting to see what the possible effects on electromagnetic properties, and thus on the wavefunctions, are. In particular, the magnetic moments play here a central role since they distinguish between protons and neutrons. Thus, these quantities could be a sensitive experimental test on the prediction that neutrons align.

Let me, however, first start with a discussion of the effects of alignment, contained in the $B(E2)$ values for transitions between rotational states. A group at GSI (ref. 27) has measured the $B(E2)$-values for $^{158}$Dy and $^{164}$Dy up to spins $I = 20 - 22$ (fig. 6,7). Whereas the former isotope shows an upbend at $I \approx 14 - 18$ the latter has no irregularity in its yrast band. This seems to be reflected in the measured $B(E2)$ values. For $^{164}$Dy these are in reasonable agreement with the rotational model whereas in $^{158}$Dy they are significantly reduced above $I \approx 14$, i.e. in the backbending region. This reduction corresponds to the smaller intrinsic quadrupole moment of $^{156}$Dy and points to the alignment of the two $1_{13/2}$ neutrons which decouple from the collective rotation so that here - in the beginning of the rare earth region - the core deformation and thus the collective $B(E2)$ value shrinks.
That the first backbend is really due to neutron alignment cannot be proven directly by any of the considerations discussed so far. The measurement of magnetic $g$-factors for high spin states, however, could give a unique direct experimental proof, as already mentioned before.

Diebel et al.\textsuperscript{28)} have calculated the effects on $g$ factors to be expected in a cranked HFB calculation in which the pairfield was treated fully selfconsistently whereas the average potential was approximated by a deformed Nilsson model. These calculations thus contain the CAP as well as the alignment effects. The authors predict that for $^{158}$Dy the $g$-factor should drop considerably with spin due to an alignment effect of the $i_{13/2}$ neutrons (fig. 8).

This behavior can easily be understood in terms of an alignment of neutrons: When the neutrons align the total angular momentum is increased rapidly just by the aligned angular momentum. Thus, at a given $I$ the protons contribute relatively little to it and, therefore, the $g$-factor drops. At higher spins, once the neutrons are fully aligned, the protons also pick up rotational speed and $g$ increases again. In $^{164}$Dy (fig. 9) the alignment is weak and, therefore, the $g$-factors show no effect.

Measurements for the two systems given above are presently underway at GSI. Ward in his talk\textsuperscript{29)} has also shown a number of very interesting experimental results, for Sm, Dy and Yb isotopes. Although all his results are compatible with no effect in the $g$-factors - and thus rather disturbing - one has to be careful to draw any final conclusions from this result since the actual backbending region has not been reached in these experiments. Since the cranking model on which all predictions are based - normally gives too strong band mixing and too early backbends probably only the presence - or absence - of a strong drop of $g$ at the backbend can reliably be predicted. Indeed, a group from the Weizmann Institute (Goldberg et al, ref. 30) reports the measurement of the $g$-factor for a $10^+$ state in the backbend of $^{134}$Ce. They obtain the strongly negative value of $g(10^+) = -0.2$. Since negative values unambiguously imply decoupled neutron pairs in $j = 1 + \frac{1}{2}$ orbits this result can be taken as the first experimental proof of the neutron alignment (here $h_{11/2}$) in the backbend.

IV. Nuclei at very high spins

The preceding discussions have shown that up to $I \lesssim 30h$ there is compelling evidence for the picture that the yrast line is made up of crossing, aligned bands
(fig. 10). If this feature persists to higher spins then successively more and more particles are expected to decouple from the collective rotation. These are typical single particle effects. Their actual presence would imply that a nucleus at high rotational frequencies does not simply reduce to a rotating liquid drop.

Impressive advances in the techniques to study the γ-ray continuum have made it indeed possible to address this problem. Mainly due to the work of the Berkeley-Copenhagen collaboration\(^3\) we have witnessed a remarkable progress in these methods that have employed better and better filters for the high spin events.

First, the measurement of γ-ray spectra and later of multiplicities as a function of γ-energy have given the most important information about the spin region \(I \approx 30-60\hbar\), namely the effective moment of inertia, defined by:

\[
\omega = \frac{3h}{2I} = \frac{I}{\theta_{\text{eff}}} = \frac{1}{2}(E(I) - E(I-2)) = \frac{I-I-1}{\theta_{\text{coll}}} \hbar
\]

This effective moment of inertia determines the envelope of all the individual bands that constitute the yrast line at high spins. As can be seen from this definition \(\theta_{\text{eff}}\) is determined by the energies of the collective γ-rays. It thus contains information on the rotational frequency but not on how many nucleons actually participate in the collective rotation.

It is to be expected that the effective moment of inertia - being an average of many individual configurations - shows that behavior predicted by the rotating liquid drop model. This is indeed the case as has been quite nicely demonstrated (fig. 11). It has even been possible to show\(^3\) that certain nearly magic nuclei like \(^{148}\)Sm, produced in a \(^{48}\)Ca + \(^{100}\)Mo reaction, exhibit rotational behavior only at high bombarding energies, corresponding to population of states with angular momentum \(\approx 50\hbar\).

The most advanced of the techniques employed for the study of the continuum region is clearly that of exploiting the correlation between the transition energies of more than one γ-ray\(^3\)\(^3\). This method relies on the fact that in a good rotor the transition energies are proportional to the total angular momentum \(I\) whereas the differences between the energies of two adjacent transitions are constant within a band.

The spectrum in a band that contains a cer-
tain, fixed aligned angular momentum \( j \) and a moment of inertia \( \Theta_{\text{coll}} \) is given by:

\[
E = E_j + \frac{(I-j)^2}{2\Theta_{\text{coll}}} \tag{1}
\]

where \( E_j \) is the energy of the band head. The transition energy for an E2-transition within a band is then:

\[
E_{E2} = \frac{4(I-j)^2}{2\Theta_{\text{coll}}} \tag{2}
\]

and the difference between two consecutive \( \gamma \)-energies is given by:

\[
\Delta E_{E2} = \frac{\Delta \Theta_{\text{coll}}}{2\Theta_{\text{coll}}} \tag{3}
\]

\( \Delta E \) thus gives directly the collective moment of inertia whereas the transition energy by itself is not sufficient to extract the collective moment of inertia because \( j \) has also to be known for that. A simple analysis of \( E_{E2} \) in terms of \( E_j \) yields only the effective moment of inertia, i.e. the moment averaged over many bands.

The considerations given above assume that \( j \) and \( \Theta_{\text{coll}} \) are independent of \( I \). Guided by them one may define the moments of inertia for the general case by

\[
\frac{dE}{dI} = \frac{\hbar^2}{\Theta_{\text{eff}}} \tag{4}
\]

\[
\frac{d^2E}{dI^2} = \frac{\hbar^2}{\Theta_{\text{coll}}} \tag{5}
\]

The \( E_{E2} - E_j \) correlations obtained so far contain a phantastic amount of information. Although a multiplicity filter has been employed in the experiments to enhance the selectivity for high spin states it is assuring that the phenomena known from low spin states; like known backbendings, do show up in these results\(^{33} \).

Results with this method have been obtained for the Barium region\(^{34} \) as well as for rotational Er nuclei\(^{35,36} \). All the measurements give a decrease of \( \Theta_{\text{coll}} \) at high spins. If the equations leading to the definitions of \( \Theta_{\text{coll}} \) and \( \Theta_{\text{eff}} \) are indeed correct (constant \( \Theta \) and \( j \)) then these two moments of inertia have to be related by:

\[
\Theta_{\text{eff}} = \frac{I}{I-j} \Theta_{\text{coll}} \tag{6}
\]

where \( j \) is the aligned single particle angular momentum. The results reported by Vivien et al\(^{35} \) show that in \(^{156}\)Er and \(^{160}\)Yb at \( I \gtrsim 4\hbar \) one has \( j \gtrsim 15-17\hbar \) which agrees with the alignment obtained for the yrast line after the second backbend. This result is in line with a similar analysis of Deleplanque et al\(^{36} \) who find for rotational Er nuclei at \( I \gtrsim 40\hbar \) a decrease of the collective moment of inertia by about 40% corresponding to an aligned angular momentum of \( \gtrsim 15\hbar \).

The results thus confirm the physical picture of individual bands that make up the yrast line in which more and more particles are decoupled from the collectively rotating core. The core moment of inertia \( \Theta_{\text{coll}} \) thus becomes smaller and smaller and this decrease is due to the very same physical effects that lead to the phenomena of band crossing, reduced B(E2)-values and g-factors at lower spins.

The phenomenon of a decreasing collective moment of inertia can be reproduced by the cranking model. I wish to show you here the results of a selfconsistent cranking model calculation performed for \(^{162}\)Yb (ref. 37, 24). Figure 12a shows several families of bands that make up the yrast line at high spins. Whereas all bands A are more or less axially symmetric, the bands B are triaxial (there are probably many more parallel bands between A and B than shown here). Figure 12b shows \( I(\hbar) \) for these bands. The slope:

\[
\frac{dI}{d\hbar} = \left( \frac{dE}{dI} \right)^{-1} = \left( \frac{d^2E}{dI^2} \right)^{-1} = \frac{\Theta_{\text{coll}}}{\hbar^2} \tag{7}
\]

is directly the collective moment of inertia. The figure clearly shows a decrease of the collective moment of inertia by about 50% at spin \( I = 60\hbar \).

It is also noteworthy that the band B has
in general smaller collective moments of inertia. Since this band is triaxial it corresponds to a situation where particles are partly aligned and thus bring density into the equatorial bulge.

These authors correspond to a situation where particles find quite considerable discrepancies are partly aligned and thus bring density between the results obtained from both models; in particular, they obtain at small frequencies a faster alignment in the particle-rotor model. This difference is attributed to the shortcomings of the cranking model. In the particle-rotor model the physical quantity "spin" is constant and \( \omega \) fluctuates whereas the cranking model keeps the classical variable \( \omega \) constant at the expense of large fluctuations in the spin.

Although, of course, only I is an observable it is remarkable that calculations in the framework of the particle-rotor model in general need an attenuation of the Coriolis coupling term by a factor \( 0.5-0.8 \) for a satisfactory description of rotational bands in odd nuclei. This attenuation, that is generally not necessary at high rotational frequencies\(^{39} \), has to be introduced in order to reduce the alignment at small \( \omega \). The cranking model, on the hand, leading to smaller alignments at small \( \omega \), describes the experimental situation quite well.

One is thus faced with the puzzling consequence that a "wrong" model (cranking) leads to the right answer whereas the quantum mechanically cleaner model fails in a description of empirical data.

Several possible explanations for this problem have been explored, the cause for the Coriolis attenuation, however, is not yet known\(^{40} \). It can be connected with the fundamental approximation of the core-particle model, namely the separation of the nucleus into a core and a valence part. Couplings leading out of the valence space cannot be treated in this model. Therefore, for example, effects like the gradual decrease of the collective moment of inertia with spin due to successive decouplings of the individual nucleons from the core, that has been observed in the \( E_\gamma - E_\gamma \) correlation experiments, are beyond the domain of this model. On the other hand these couplings are contained in the cranking model - as we have seen above - that can treat

\[ \begin{align*}
V. \text{ Cranking vs. Particle-Rotor Model} \\
\end{align*} \]

The success of the cranking model that manifests itself in all these analyses is quite surprising. After all, it is quite well known by now that the cranking model leads to broad I-distributions in its wavefunctions\(^{38} \) and that points on bands that cross at a given, unique value of \( \omega \) can have quite different values of nuclear spin.

The problem of spin-distributions in the cranking wavefunctions can be overcome by using a particle rotor model for the description of the rotational structure of nuclei. Calculations along this line do yield wavefunctions with good spin by construction and should, therefore, be free of any of the theoretical problems mentioned above.

A comparison between the particle-rotor model and the cranking model has been performed by Almberger et al.\(^{39} \). These authors find quite considerable discrepancies between the results obtained from both models; in particular, they obtain at small frequencies a faster alignment in the particle-rotor model. This difference is attributed to the shortcomings of the cranking model. In the particle-rotor model the physical quantity "spin" is constant and \( \omega \) fluctuates whereas the cranking model keeps the classical variable \( \omega \) constant at the expense of large fluctuations in the spin.
all nucleons simultaneously thus taking into account all possible polarization effects between valence and core nucleons. It should be realized that such couplings do not only originate in the Coriolis and recoil terms but may also enter through the Hamiltonian describing the (quasi-) particles in the rotating frame. Bohr and Mottelson\textsuperscript{41} have pointed out that additional coupling terms of a structure similar to that of the Coriolis coupling are created by the rotation. In particular a $Y_{21}$ pair field is expected to become effective in rotating nuclei. Indeed it had been found earlier that inclusion of this pairing type can raise the cranking moment of inertia. However, it has been shown by Faessler and Wakai\textsuperscript{42} that $Y_{20}$ quadrupole pairing and hexadecapole pairing taken together counteract these effects to a large extent. In a contribution to this conference Almberger et al report that also in the particle-rotor model the $Y_{21}$ pairing has an effect too small to explain the necessary attenuation\textsuperscript{43}.

Following along these same lines of thought Neergard\textsuperscript{44} has recently brought up the interesting possibility that the quasiparticle-Hamiltonian could contain a term $\sim R^3 J^3$. Such a term could not only lead to the desired attenuation but would also decrease the strength of the recoil term simultaneously. Empirical strengths of the attenuation factor of 0.5-0.8 would lead to a decrease of the recoil term by the factor 0-0.4. Although terms of the desired structure indeed exist in selfconsistent cranking calculations\textsuperscript{24} it is at present not clear whether their magnitude is large enough for a quantitative description. Also, these terms would lead to an attenuation at all spins and not just at low rotational frequencies as seems to be required by experiment\textsuperscript{39}.

Whatever the solution to the attenuation problem may be, the particle rotor model will clearly always be restricted to relatively small spins. The interesting effects seen at high to very high spins, like the decrease of the collective moment of inertia, are certainly beyond the applicability of this model. On the other hand, these effects are naturally included in the cranking model.

VI. Yrast traps

Originally one of the main motivations for studying high spin states was the possibility that for certain high angular momenta yrast traps might appear when the single particle potential becomes axially symmetric around the angular momentum direction\textsuperscript{45}. These traps were expected at spins between $I \approx 30h$ and $I \approx 60h$ and excitation energies above $\% 15$ MeV. Following these suggestions extensive searches were undertaken\textsuperscript{46}. However, in the experiments done so far no evidence of such a major structural change in rotational nuclei was observed\textsuperscript{47}. The reason for this absence is clearly the stability of shell effects under rotation that tend to keep the nucleus prolate. Thus, the oblate axis is never reached and, therefore, collective rotations with their enhanced transition probabilities are still present at high spins and prohibit the existence of isomers.

That, however, the physical considerations underlying the prediction of a transition to the oblate shape at high spin are correct is indicated by the evidence that we have from the measurements of the collective moments of inertia discussed before. These results show that in a good rotor more and more particles decouple from the collectively rotating core, align their angular momenta along the axis of rotation, and thus produce a triaxial density distribution so that the nucleus actually changes from a prolate into a triaxial configuration (see fig. 11).

Even though the searches for yrast traps at very high spins in rotational nuclei have been unsuccessful so far, these studies have turned up an island of isomeric states in the well defined region $64 \leq Z \leq 71$. 
and $82 \leq N \leq 88$. These isomers have spin values up to $30\hbar$ and they appear in nuclei that are more or less spherical in their groundstates. The isomers are probably due to the alignment of a small number of particles outside the "magic" $^{146}$Gd core. Thus one has a typical shell model test ground here where $(HI,xn)$ reactions are used for an investigation of this newly discovered shell model island. A very analogous situation exists around $N = 126$ and $Z \approx 84$.

The alignment of particles will, of course, cause an oblate, axially symmetric density distribution and this will have an important feedback on the single particle potential and on the core. This is seen quite clearly in two papers, reported at this conference, on high spin yrast isomers in $^{147}$Gd. The Chalk-River group of Mahnke et al. report measurements of static quadrupole moments for these states that have spins between $13/2$ and $59/2$. The deformation of these states, extracted from the measured $Q$ values, range from $\beta = -0.05$ to $\beta = -0.22$ at the highest spins. If the isomers are explained in terms of spherical shell model configurations, unrealistically large effective charges are needed to explain the experimental $Q$-values. If, however, the same configurations are constructed in an oblate deformed shell model with pairing correlations being taken into account then an excellent description of the measured values is obtained as Dissing and Neergard have demonstrated.

The shell model character of these nuclei also explains the observability of high spin states up to $I \gtrsim 3\hbar$ in $^{152}$Dy through discrete-line spectroscopy. Since the collective side bands do not extend so far down here, the nuclei cool faster from the entrance line down to the yrast line thus reaching it at higher spin. Measurements of the linear polarizations and angular distributions of continuum $\gamma$-rays in $^{152}$Dy have led to the speculation that these nuclei may undergo a shape change and rotate collectively at spins $I \gtrsim 40$. What is not clear, however, is whether these collective rotations are really connected with a shape change from weakly oblate to strongly prolate deformations. The alternative is that they take place around an axis perpendicular to the nuclear symmetry axis with the yrast single particle states as band heads.

The speculations about collective rotations in $^{152}$Dy at high spins have received considerable support from the results on the results on the population of individual levels presented by T. L. Khoo at this meeting. These show that the degree of population of a level with spin $I \gtrsim 3\hbar$ is practically independent of the input angular momentum so that obviously also in this nucleus the deexcitation proceeds through collective side bands.

VII. Final remarks

Finally I would like to discuss with you a nucleus that shows all the effects that we have heard so much about at this conference, namely backbending, even a giant backbend, and a gradual alignment of the valence nucleons until actually all are aligned so that the collective moment of inertia is zero and the band ends.

In the first picture (fig. 13) I show you the experimental backbending plots for two...
isotopes that differ by two neutron numbers\textsuperscript{52}). Since the number of particles is larger it takes a higher spin, although nearly the same frequency, to backbend in the heavier isotope. This backbend here is due to the transition to the oblate axis of all valence nucleons as is predicted by cranking model calculations (fig. 14). Thus here indeed the shape transition in a rotational nucleus from prolate to oblate shapes has been found.

The lifetimes of the highest states have been measured and the available data for $B(E2)$ values do reflect the transition from prolate through triaxial to oblate shapes\textsuperscript{53}).

<table>
<thead>
<tr>
<th>Transition</th>
<th>$B(E2)$ in $e^2 fm^4$</th>
<th>$B(E2)_{\text{Exp}}$; $B(E2;2\rightarrow0)$</th>
<th>$Q_{\text{o}}$ const.</th>
<th>$I_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2$\rightarrow$0</td>
<td>57±8</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4$\rightarrow$2</td>
<td>70±7</td>
<td>1.2 ± 0.15</td>
<td>1.43</td>
<td>1.26</td>
</tr>
<tr>
<td>6$\rightarrow$4</td>
<td>66±8</td>
<td>1.15±0.15</td>
<td>1.57</td>
<td>1.06</td>
</tr>
<tr>
<td>8$\rightarrow$6</td>
<td>24±8</td>
<td>0.4±0.15</td>
<td>1.65</td>
<td>0.64</td>
</tr>
<tr>
<td>10$\rightarrow$8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This nucleus is also quite interesting from a theoretical point of view. It is well known that the core in this case does not carry any angular momentum so that obviously the particle rotor model cannot be applied to it.

At this point probably most of you have realized what nucleus I am speaking about: $^{20}$Ne. This example of a very light nucleus that has been studied for decades and for which very reliable theoretical shell model calculations are available illustrates that the same physical phenomena that we deal with in high spin states in heavy nuclei have been around for quite some while.

This nucleus serves also as a nice exemplary illustration for the alignment-effects that were the central theme of my talk here: the gradual change of deformation towards completely oblate shapes, connected with a giant backbend, the hindrance of $B(E2)$ values and the decrease of the collective moment of inertia.

In spite of this long history of the alignment, however, there are enough problems left for further study: I personally would like to see much more data on electromagnetic properties of high spin states since these give a more stringent test of the wavefunctions than the energies do. At the same time, it is clear that at present the calculation of transition moments presents a challenge to theory because the most successful model, i.e. cranking, leads to spin-mixed wavefunctions that cannot directly be used. The particle-rotor model, on the other hand, that is free of this shortcoming, suffers from the attenuation problem.

Finally then, the investigation of very
high states promises to be the more exciting the more deviations from a simple rotating liquid drop behavior we see. These studies clearly depend on better and better microscopes for the high spin region. The experimental methods starting at $\gamma$-energy spectra, through the measurement of multiplicities as a function of $\gamma$-energy to the energy-sum method and the $E_\gamma - E_Y$ correlation have become more and more selective. The crystal balls that are presently being built will thus be a great step forward to an even higher selectivity for the high spin region.

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