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DISSIPATIVE COLLISION AND COMPOUND-NUCLEUS FORMATION

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Abstract.—The successes, limitations and generalizations of transport theories for dissipative heavy-ion collisions are reviewed. The evolution of the collision complex can be divided roughly into three stages: mutual approach of the nuclei, local equilibration and slow relaxation of macroscopic degrees of freedom. In particular, the last stage has been thoroughly investigated within the framework of transport theories. The main steps in the derivation of transport equations is discussed and applications of Fokker-Planck equations are presented. Recent developments try to bridge the gap between the initial stage and the third stage of the process. An important feature of the mutual approach is due to the initial correlations imposed by the ground-state configurations (doorway) of the colliding nuclei. These correlations give rise to an explicit time-dependent dynamical potential in the relative motion. The resulting modified Fokker-Planck equation is applied to compound-nucleus formation and dissipative collisions. The possible existence and some characteristic features of a slow component of dissipative collisions (besides the familiar fast component) are studied.

1. Introduction

We consider collisions between heavy nuclei with energies of typically a few MeV per nucleon above the interaction barrier (Coulomb barrier). An important feature of the interacting composite system are the large angular momenta which are involved in the bombardment with heavy nuclei. Depending essentially on the initial value of the relative angular momentum the colliding nuclei may fuse and form a compound nucleus or break apart without going through the intermediate stage of a compound nucleus. The latter case is referred to as dissipative heavy-ion collision and is characterized by a partial memory to the incident channel. The most exciting discoveries in these collisions are the continuous loss of relative kinetic energy and the broadening of the fragment-mass distribution with increasing kinetic-energy loss [1-4]. Within the classical equation of motion the loss of kinetic energy has been ascribed to friction forces acting between the nuclei [5,6]. The mass distribution was subsequently interpreted as a result of a statistical transfer (diffusion) of nucleons between the interacting nuclei and hence, has been described by master equations and Fokker-Planck equations [7-9]. The dissipative character is common to the typical phenomena observed in these processes. Here, the word dissipation is not restricted to the dissipation of relative kinetic energy and angular momentum. For example, also the diffusion of nucleons between the nuclei is a dissipative process. Dissipative collisions cover the range between direct reactions and compound-nucleus formation. Whereas only a few degrees of freedom participate in direct reactions, all degrees of freedom are involved in compound-nucleus formation. Therefore, dissipative collisions carry information about the dissipative processes which lead from simple nuclear states to more complex configurations.

Dissipative heavy-ion collisions show the following well-established characteristic features [10-14]:

1. Projectile and target are mostly rather heavy nuclei with mass numbers A \textless 40.

2. The incident energy at the Coulomb barrier is typically 1 to 3 MeV/u (MeV per nucleon) which
correspond to maximal angular momenta of a few hundred h. 

(3) The identity of projectile and target is essentially preserved throughout, although

(4) a considerable amount of mass (ΔA ≈ 30) can be transferred during the collision.

(5) The angular distribution is strongly non-isotropic which means that the time $t_{\text{int}}$ of the nuclear interaction is significantly smaller ($t_{\text{int}} = 10^{-22}\text{s} \ldots 10^{-20}\text{s}$) than the time needed for a complete rotation of the composite system ($t_{\text{rot}} > 10^{-20}\text{s}$).

(6) A large amount of relative kinetic energy (typically more than 100 MeV) and relative angular momentum (up to $\approx 50$ units of h) is dissipated, i.e. transferred from relative motion into intrinsic excitation. The excitation energy is distributed on the fragments proportional to the fragment masses.

(7) Dissipative collisions cover the total range between direct reactions and compound-nucleus formation. Their share in the total cross-section increases with increasing bombarding energy and masses of the colliding nuclei. For heavy nuclei typical values for the dissipative cross-section are 1 to 2 barns.

(8) The measured cross-sections are inclusive cross-sections because not all reaction products can be observed. Usually only scattering angle, energy and charge of one fragment are detected. Since the excitation energies are large, the channels cannot be resolved and therefore, only averaged quantities are observed.

Several reviews on theories of dissipative collisions can be found in the literature [11,13-15]. An attractive approach is the time-dependent Hartree-Fock theory (TDHF) which treats the single-particle motion in the time-dependent mean field generated by the nucleons [16]. With respect to the collective degrees, however, the results of TDHF calculations show many classical features which are imposed by the mean field. This is particularly apparent for the relative motion. For a given impact parameter the TDHF collision leads to a definite deflection angle. Therefore, whereas mean quantities like mean scattering angle, mean mass split and mean total kinetic energy come out reasonably well, the corresponding fluctuations are severely underestimated in TDHF calculations. The occurrence of large fluctuations even for small interaction times ($=10^{-21}\text{s}$) indicate that more complex configurations than those described by a Slater determinant are excited already early in the collision process.

As illustrated in fig. 1, the total process can be roughly divided into three stages.

(1) The initial stage is characterized by the mutual approach of the nuclei in their ground states. The motion of the nucleons during this first stage of the collision is expected to be governed by their self-consistent single-particle potential which evolves slowly in time. This stage should therefore be well described by the time-dependent Hartree-Fock theory.

(2) By residual interactions the Slater determinant of time-dependent single-particle states decays to more complex configurations. This decay leads to a local statistical equilibrium. At the end of this decay the system occupies the total phase space (total configuration space) which is available for fixed values of the macroscopic (collective) degrees of freedom.

(3) During the third stage the slow macroscopic (collective) variables relax towards their equilibrium distributions. The complexity of the wave function in this stage of the process suggests to take advantage of random properties of the system as im-
plied by the local statistical equilibrium. This leads naturally to the formulation of transport theories.

2.1 Derivation of transport equations

There are three essential steps in the derivation of transport equations. For simplicity we treat the relative motion classically and assume that the trajectory \( \mathbf{r}(t) \) and the coupling \( V(t) \) between excited states of the interacting nuclei are known. This semiclassical treatment [8,13,21] of relative motion has been generalized by including the relative variables explicitly in the equation of motion [19] (cf. § 2.3).

(1) By dividing the total channel space into subsets \( \mathcal{X}_u \) we define the macroscopic occupation probabilities

\[
I_u(t) = \sum_{m \in \mathcal{X}_u} \rho_{mm}(t)
\]

which denote the total occupation probabilities of the subsets \( \mathcal{X}_u \). Rewriting the Liouville equation for the density matrix \( \rho(t) \), yields the generalized master equation [8]

\[
\frac{d}{dt} f_u(t) = I_u(t, t_0) + \int_0^{t-t_0} d\tau \ K_{uv}(t, \tau) d_v f_v(t-\tau).
\]

The first term on the r.h.s. represents the contribution from those parts of the initial density matrix \( \rho(t_0) \) which are not contained in \( f_u(t) \). The second term on the r.h.s. describes the coupling between different subsets. In analogy to the Boltzmann equation it is referred to as the collision term. The collision term consists of gain terms \( (v \neq u) \) which correspond to transitions from all subsets \( v \neq u \) to the subset \( u \), and another term \( v = u \) which describes the loss from the subset \( u \). The kernel \( K_{uv}(t, \tau) \) is non-local in time. The change of the macroscopic occupation probabilities at time \( t \) depends on all earlier times \( t-\tau \). Because of this property, the kernel and its characteristic decay are referred to as the memory kernel and the memory time \( T_m \) respectively.
The second step in the derivation of a transport equation is the introduction of random properties of the coupling matrix $V(t)$. This leads to the following structure of the memory kernel,

$$K_{uv}(t,t-r) = i\tau^{2} \langle m(t) n(t-T) V(t,t-r) \rangle_{v} + \text{c.c.} \quad (3)$$

The mean correlation function $\langle m(t) n(t-T) V(t,t-r) \rangle_{v}$ decays as function of $\tau$ within a characteristic time $\tau_{\text{cor}}$ (correlation time). This correlation time is related to a correlation length $\sigma$ by $\tau_{\text{cor}} = \sigma/|\tau|$. The correlation length $\sigma$ is typically given by the mean distance between the nodes of the single-particle wave functions. For strong overlap we have $\sigma = 1\text{ fm}$, while in the nuclear surface $\sigma = (3\ldots4)\text{ fm}$ [22]. Therefore, $\tau_{\text{cor}} \approx 10^{-22}\text{ s}$. The mean propagator $G_{uv}(t,t-r)$ for a state $n \in \mathcal{H}$ and a state $m \in \mathcal{H}$ has a lifetime denoted by $\tau_{\text{dec}}^{u,v}$. The relative values of $\tau_{\text{cor}}^{u,v}$ and $\tau_{\text{dec}}^{u,v}$ determine the two limits of weak and strong coupling. The weak-coupling limit is defined by

$$\tau_{\text{dec}}^{u,v} \ll \tau_{\text{cor}}^{u,v} \quad (4)$$

and the strong-coupling limit by

$$\tau_{\text{mem}}^{u,v} \ll \tau_{\text{dec}}^{u,v} \quad (5)$$

Calculations [23] of the decay time for small overlap give values $\tau_{\text{dec}} \approx 5 \times 10^{-23}\text{ s}$. Thus, in the basis of asymptotic eigenstates $\tau_{\text{dec}} \ll \tau_{\text{cor}}$, such that the strong-coupling limit is applicable for dissipative collisions. The initial correlations $I_{uv}(t,t_{0})$ have the same structure as the memory kernel (3). It is usually assumed that the resulting transport equation is applied for $t-t_{0} \gg \tau_{\text{mem}}^{u,v}$ where $I_{uv}(t,t_{0})$ can be neglected. The memory time characterizes the time interval during which the phase correlations are lost.

As discussed at the beginning of this section, the macroscopic variables have to be chosen as the slow modes of the system. This implies that the relaxation times for the macroscopic variables satisfy $\tau_{\text{relax}} \gg \tau_{\text{mem}}^{u,v}$ and hence, justifies the Markov approximation. It consists in replacing $f_{\nu}(t-t_{0})$ by $f_{\nu}(t)$ in eq. (2) and yields the master equation for $t-t_{0} \gg \tau_{\text{mem}}^{u,v}$,

$$\frac{d}{dt} f_{\nu}(t) = \sum_{\mu} w_{\nu\mu}(t) \left[ d_{\nu\mu}(t) f_{\mu}(t) - d_{\mu\nu}(t) f_{\nu}(t) \right] \quad (6)$$

with the transition probability $w_{\nu\mu}(t) = \int d\tau K_{\nu\mu}(t,\tau)$. Such a master equation was originally introduced by Pauli [24] in order to justify the $\lambda$-theorem from the quantum-mechanical point of view. Moretto and Sventek [9] have directly applied such a master equation to the diffusion of nucleons between the colliding nuclei.

2.2 Fokker-Planck equation, transport coefficients and relaxation phenomena

A particularly convenient method of describing the solutions of the master equation (6) has been introduced by transforming the integral equation into a second-order differential equation, the Fokker Planck equation

$$\frac{df_{\nu}(t)}{dt} = -\frac{g}{3} \sum_{i=1}^{g} \frac{3}{y_{i}} \left[ v_{i}(\vec{y},t) f(\vec{y},t) \right] + \frac{g}{3} \sum_{i,j=1}^{g} \frac{2}{y_{i} y_{j}} \left[ D_{ij}(\vec{y},t) f(\vec{y},t) \right]. \quad (7)$$

Here we have replaced the discrete variables $u$ by the continuous variable $\vec{y}$ with $g$ components. The transport coefficients $v_{i}$ (drift coefficients) and $D_{ij}$ (diffusion coefficients) are completely determined by the second moments of the transition probabilities and the density of states. Drift and diffusion coefficients are related by (generalized) Einstein relations [7,13,14,21,25].

In the further evaluation of the microscopic expressions for the transport coefficients the method of spectral distributions is used which has been successfully applied to the calculation of mean spectroscopic nuclear quantities [23]. This method employs the statistical properties of shell-model calculations in a large single-particle basis. It gives an accurate prescription for the evaluation
of averages in such a shell-model space. The evaluation of the level densities $d(\gamma)$, the memory or decay time $\tau_{\text{dec}}(\gamma, \gamma')$ and the transition probabilities $w(\gamma, \gamma')$ are reduced to the calculation of such averages.

The transport coefficients have been calculated for three macroscopic variables [25]; the mass $A_1$ of one fragment (mass-asymmetry variable), the $z$-component $M$ of the total intrinsic angular momentum and the total excitation energy $E^*$. Drift and diffusion coefficients are simple analytical expressions depending on the macroscopic variables, on the total mass, on the relative angular momentum $\mathcal{I}$ and on three characteristic parameters $\Delta_0$, $\alpha$ and $\gamma$ of the interaction matrix elements. The allowed change $\Delta_0$ of angular momentum in the transfer or excitation of a single nucleon is determined by the size of the overlap region. For touching nuclei we estimate [14] $\Delta_0 = 1.5 (A_1 + A_2)^{1/6}$. The corresponding energy change is determined by $\Delta = (\mathcal{I}^2/\mathcal{I}^2_{\text{rel}}) (\mathcal{I} - M) \Delta_0$ where $\mathcal{I}_{\text{rel}}$ is the moment of inertia for the relative motion. The third parameter $\gamma$ determines the strength of the coupling matrix elements.

A considerable number of mass distributions measured in dissipative collisions have been analyzed [26,27] using a Fokker-Planck equation (7) for the single variable $A_1$. Figure 2 shows as an example the fit (solid curve) of the mass-transport coefficients $v_A$ and $D_{AA}$ to the data for $^{86}\text{Kr} (5.99 \text{ MeV/u}) + ^{166}\text{Er}$. Using the theoretical values for the transport coefficients the dashed line is obtained.

Figure 3 compares experimental and theoretical values for the diffusion constant for various reactions [14]. In calculating the theoretical values we use $\gamma = 3 \text{ MeV}$ and the dynamical values $\Delta_0$ and $\Delta$ as given above with $\mathcal{I} = (2/3) \mathcal{I}_{\text{rel}}$ and $M = M_{\text{st}}$ (sticking). Note that the fitted value of the...
Fig. 4. The total angular momentum $I_{\text{tot}}(Z_1)$ of the fragments as function of the fragment charge number and corresponding $\gamma$-multiplicity data from ref. [29] for the reaction $^{86}\text{Kr} (5.99 \text{ MeV/u}) + ^{166}\text{Er}$. The dashed curve is obtained by neglecting fluctuations. From ref. [30].

The strength parameter $\gamma$ is in agreement with estimates for the touching configuration. Comparing experimental and theoretical values we realize that the dependence of the mean diffusion coefficient on bombarding energy, total mass and mass asymmetry is well reproduced. Large effects from shell structure in the mass diffusion coefficient are not apparent from the experimental values, cf. also ref. [28]. Note that these values correspond to mean diffusion coefficients at rather high excitation energies. The good agreement with the experimental values show that predictions of mass distributions on the basis of these theoretical diffusion coefficients are rather reliable.

Figure 4 compares the calculated angular momentum of the fragments with results from $\gamma$-multiplicity measurements. The dip around the projectile charge is understood as follows: The fragments close to the projectile are predominantly produced in collisions with large $\varepsilon$-values where the interaction time and hence, the dissipated angular momentum is small. Sufficiently far away from the projectile-charge number, the dissipated angular momentum saturates. These fragments are mainly populated in collisions with $\varepsilon$-values where the interaction time is large enough to reach sticking. An important feature of angular-momentum dissipation is the occurrence of large fluctuations which affect even the mean values shown in fig. 4. These fluctuations in angular momentum together with fluctuations in the total kinetic energy loss or $Q$-value imply that there is no sharp correlation between impact angular momentum $\varepsilon$ and the $Q$-value. This becomes important in particular for small $\varepsilon$-values. Corresponding effects have been clearly demonstrated in sequential-fission measurements in bombardments of $U$ and $Pb$ on various targets by von Harrach et al. [31].

Returning to the time evolution of the collision complex let us discuss the characteristic times involved in the process (cf. fig. 1). From the analysis of angular distributions it is possible to deduce nuclear interaction times [26,27]. These interaction times range from about $10^{-22}$ s for
grazing collisions up to several $10^{-21}$ s for close collisions (with small parameters $b \ll b_{gr}$). They have been used in the analyses of mass distributions and angular momentum dissipation. From a fit to experimental energy spectra the relaxation times for the loss of radial kinetic energy ($\tau_{rad}$) and for the evolution of fragment deformations ($\tau_{def}$) have been determined [27]. In this analysis the theoretical relaxation time $\tau_{ang}$ for the dissipation of relative angular momentum has been used (cf. fig. 4).

The values for the relaxation times ($\tau_{rad} = 0.3 \cdot 10^{-21}$ s, $\tau_{ang} = 1.0 \cdot 10^{-21}$ s, $\tau_{def} = 4 \cdot 10^{-21}$ s) imply that the fast loss of radial kinetic energy is followed by the dissipation of relative angular momentum and finally by the evolution of fragment deformations. The analysis of mass distributions (cf. fig. 2) show that no equilibrium is reached in the mass-asymmetry coordinate for typical interaction times. The corresponding equilibration time ($\tau_{mass} = 2 \cdot 10^{-20}$ s) is larger by an order of magnitude and typically of the same order of magnitude as the mean time $\tau_{rot}$ necessary for a complete rotation of the collision complex.

2.3 Explicit treatment of relative motion

So far we have focused our attention to the relaxation phenomena with respect to observable properties of the fragments. In doing this we have assumed that the classical trajectory is known. This treatment has the advantage to bring out the main ingredients and features of transport theory, but of course, is not completely satisfying. General transport equations which include the explicit treatment of relative motion, have been given by Agassi et al. [18] in a stationary formulation and by Ayik and Nörenberg [19] for the time-dependent problem.

Instead of the macroscopic occupation probabilities $f_{\mu}(\vec{r}, \vec{p}; t)$ defined by eq. (1) we consider the macroscopic Wigner functions $f_{\mu}(\vec{r}, \vec{p}; \tau)$ which in the semi-classical limit are the joint probability distributions for finding the system at time $\tau$ in the subset $\mu$ at the relative distance $\vec{r}$ with the relative momentum $\vec{p}$. The general transport equation is given by

$$\frac{\partial}{\partial \tau} \frac{\vec{p}}{M_{\mu}} \cdot \vec{r} - \left[ \vec{r}, U_{\mu}(\vec{r}) \right] \cdot \vec{p} \right) f_{\mu}(\vec{r}, \vec{p}; \tau)$$

$$= d_{\mu} \int_{0}^{\infty} dt' \int d^{3} r' d^{3} p' \frac{\partial}{\partial \tau} f_{\mu}(\vec{r}', \vec{p}'; \tau-t)$$

in the semi-classical limit [19,32]. The left-hand side of (8) describes the change of the probability distribution $f_{\mu}(\vec{r}, \vec{p}; \tau)$ due to the velocity $\vec{p}/M_{\mu}$ and force $-\vec{r}/M_{\mu}$ within the subset $\mu$. The quantities $M_{\mu}$ and $U_{\mu}(\vec{r})$ denote the corresponding mean reduced mass and the mean potential, respectively. The collision term on the right-hand side of (8) describes the coupling to other subsets. Explicit expressions for the memory kernels $K_{\nu\mu}(\vec{r}, \vec{p}; \vec{r}', \vec{p}'; \tau)$ are given in ref. [19] and are not discussed here. Ko et al. [33] have studied the off-shell behaviour of the collision term within a stationary transport equation for a one-dimensional model. Their results can be interpreted within the time-dependent formulation as follows. The decay of the memory kernel is characterized by the memory time $\tau_{mem}^{\mu\nu}$. Therefore, transitions are allowed which can go off the energy shell up to the order $u_{\mu}/\tau_{mem}^{\mu\nu}$. The resulting off-shell probabilities are not allowed to survive in the asymptotic region $\tau \rightarrow \infty$, $r \rightarrow \infty$. When the nuclei start to separate, the memory time begins to increase and tends to $\infty$ asymptotically. As a consequence, all off-shell probabilities feed back into on-shell probabilities. Qualitatively, the time development of the macroscopic Wigner functions is as follows. Suppose that we have initially a narrow distribution in $E_{\mu}$ and $\vec{p}$. Due to the coupling this distribution broadens.
In addition, due to the density-of-state factor in the collision term, the mean value of the intrinsic excitation energy $E_v$ drifts towards higher densities of states. Since the off-shell transitions are limited to $\eta/\eta_{\text{mem}}$, the $\pi$-distribution has to follow (with some delay) the drag which is enforced to it by the $E_v$-distribution. According to eq. (4) the memory time in the weak-coupling limit is given by the correlation time. The energy change for on-shell transitions is also limited by the correlation time. As a result of this equality the distributions in $E_v$ and $\pi$ remain close to on-shell distributions and hence, do not change significantly by the feed-back from off-shell to on-shell probabilities when the nuclei separate. In contrast to this $\gamma_{\text{mem}}^{vu} = \gamma_{\text{dec}}^{vu}$ << $\tau_{\text{COR}}$ in the strong-coupling limit and hence, the system can go far off-shell such that the mean value of the excitation-energy distribution is far from the on-shell value. Therefore, in the strong-coupling limit the feed-back mechanism is in general an essential stage in the reaction process.

A stationary transport equation is obtained from eq. (8) by considering the solution $f_v(\vec{r},\vec{p};t)$ for an impinging plane wave. For $t \rightarrow \infty$ the time dependence induced by the wave front has died out. Thus we obtain the corresponding stationary transport equation by taking $f_v(\vec{r},\vec{p};t \rightarrow \infty) = f_v(\vec{r},\vec{p})$ to be independent of $t$. By this procedure the initial-value problem is transformed to a boundary-value problem with a plane wave of sharp energy impinging from infinity. The stationary limit of the time-dependent transport equation (8) is quite similar to the transport equation of Agassi et al. [18]. The left-hand sides are identical. The right-hand sides both contain gain and loss terms which carry information about a transition being on shell or off shell. In detail there are differences, which are due to some different approximations concerning the coupling between intrinsic and relative angular momentum and the evaluation of the intermediate propagator in the memory kernels.

Although the general transport equation (8) is considerably simpler than the Liouville or Schrödinger equation, numerical solutions of this integro-differential equation are hard to obtain. For the practical application it is therefore important to look for further approximations. These are obtained at the additional dispense of a detailed knowledge of the macroscopic Wigner functions $f_\mu(\vec{r},\vec{p};t)$ by considering the macroscopic occupation probabilities, cf. eq. (1),

$$f_v(t) \equiv \int d^3r \int d^3p \ f_v(\vec{r},\vec{p};t)$$

(9)

of the subsets $v$ and/or the mean phase-space distribution

$$f(\vec{r},\vec{p};t) \equiv \sum_v f_v(\vec{r},\vec{p};t).$$

(10)

For these quantities the coupled equations

$$\frac{\partial}{\partial t} f_v(t) = \sum_\mu w_{\nu \mu}(t) \left\{ d_v f_\mu(t) - d_\mu f_v(t) \right\},$$

(11)

$$\left\{ \frac{\partial}{\partial t} + \frac{\vec{p} \cdot \vec{\pi}}{M} - (\vec{p} \cdot \vec{U}) \cdot \vec{\pi} \right\} f(\vec{r},\vec{p};t) =$$

$$- \sum_\nu \frac{\partial}{\partial \vec{r}} \left\{ v_\nu(\vec{r},\vec{p};t) f(\vec{r},\vec{p};t) \right\} + \sum_{\nu, \nu'} \frac{\partial^2}{\partial \vec{p}_1 \partial \vec{p}_j}$$

$$\left\{ D_{\nu \nu'}(\vec{r},\vec{p};t) f(\vec{r},\vec{p};t) \right\}$$

(12)

are obtained [34]. The master equation (11) and the Fokker-Planck equation (12) are coupled through the definitions of the transition probabilities $w_{\nu \mu}(t)$ and the transport coefficients $v_\nu(\vec{r},\vec{p};t)$ and $D_{\nu \nu'}(\vec{r},\vec{p};t)$. Whereas the transition probabilities are averages taken with $f(\vec{r},\vec{p};t)$, the transport coefficients are determined from averages over $f_v(t)$. In the weak-coupling limit the transition probabilities and the transport coefficients are naturally separated into dissipative and conservative parts according to real and virtual transitions. In the strong-coupling limit the master equation (6) is
recovered if only the on-shell transitions are considered.

A numerical treatment of the transport equations discussed here has been given by Agassi et al. [35]. Whereas the dissipation of momentum is treated by a microscopic input, the mass transfer is taken into account by fitting a diffusion coefficient. The strong-coupling limit is replaced by a weak-coupling limit with an effective coupling strength. Ko [36] includes in this treatment shape deformations in a phenomenological way as introduced by Siwek-Wilczynska and Wilczynski [37]. Figure 5 shows the results for $^{136}\text{Xe} (1130 \text{ MeV}) + ^{209}\text{Bi}$. A rather good agreement with the experimental data is obtained.

2.4 Limitations and insufficiencies

We may summarize the applications of transport equations by the following statements. The microscopic theories are able to account quantitatively for various relaxation phenomena, in many cases even without fitting any parameter. We understand best the slow processes which are connected with the transfer of nucleons and the dissipation of angular momentum. Despite of this success the transport theories, as formulated up to now, are subject to four major restrictions.

(1) Restriction due to the choice of collective and macroscopic variables. For the sake of simplicity one takes into account only a few collective degrees of freedom (for example, the relative distance between the centers of the fragments) and some macroscopic variables which characterize the observable intrinsic properties of the fragments (for example, excitation energy, mass asymmetry and intrinsic angular momentum). It is clear that further collective coordinates like the deformations of the fragments, and maybe also additional macroscopic variables are necessary for a more complete understanding of the process.

(2) Restriction to local statistical equilibrium. For fixed values of collective and macroscopic variables it is assumed that all intrinsic degrees are populated according to their statistical weight. This is apparent either by the application of the method of spectral distributions [23], or by the use of random values (Gaussian distributions) for the coupling matrix elements [8,18,19], or by employing the canonical ensemble with temperature $T$ [17].

(3) Restriction due to the choice of basis. The derivations of transport equations assume complete randomness of the coupling between different basis states. This is only partly a consequence of the assumption (2) and goes beyond it because the ran-

Fig. 5. Double-differential cross-section $\frac{d^2\sigma}{dE dZ}$ (integrated over c.m. scattering angle as indicated) for the reaction $^{136}\text{Xe} (E_{\text{Lab}} = 1130 \text{ MeV}) + ^{209}\text{Bi}$. The dashed curves are the results of the calculation. From ref. [36].
domness assumption within a particular basis (for example the eigenstates of the separated nuclei) artificially eliminates correlations which might be important.

In the application to experimental data additional assumptions are made. Since the treatment of off-shell contributions is difficult, Agassi et al. \cite{35} use an effective weak-coupling limit thereby restricting all transitions to be on the energy shell. This is similar to the assumption \cite{8} that the coupling matrix elements themselves are limited to be on shell. This restriction to on-shell transitions introduces for both approaches effective strengths. Effects from off-shell contributions have been studied recently by Saloner and Weidenmüller \cite{38}.


The restriction to local statistical equilibrium limits the applicability of present formulations of transport theory essentially to the third stage of the total process (cf. fig. 1). The apparent success of transport theories in the interpretation of experimental data can be attributed to the short duration of the initial stages of the process as compared to the third stage. It is a challenging theoretical problem to extend the present formulations of transport theory towards the initial stage of the process.

As mentioned in the introduction the time-dependent Hartree-Fock theory (TDHF) seems to be particularly suited for describing the approach phase of the collision. In order to extend TDHF towards the following stages it is necessary to take the residual interactions into account \cite{39-42}. All extensions of TDHF conserve the selfconsistency and hence, lead to highly non-linear integro-differential equations. At present it is not clear if reasonable approximations can be found such that numerical treatments become possible. Note, that in order to account for the fluctuations it is necessary to consider not only the single-particle density but also the correlated part of the two-particle density.

In view of the numerical efforts which are necessary in all extended TDHF calculations, it is advantageous to follow a simpler although not so rigorous approach which is strongly related to the transport theories treated in section 2.

3.1 Initial correlations and local equilibration

During the fast approach of the nuclei the individual nucleons cannot follow the lowest possible (adiabatic) levels. The nucleonic wave functions stay essentially unchanged. Such a 'diabatic' (i.e., non-adiabatic) behaviour is well known for electrons in atom-atom collisions \cite{43}. Similar non-adiabatic processes around quasi-crossings of adiabatic levels have been considered in the fission process \cite{44-47} and in heavy-ion collisions \cite{48}. The motion of nucleons on diabatic levels gives rise to a large potential energy in addition to the adiabatic potential. This additional potential can be estimated from the inspection of two-center shell-model calculations \cite{44,49,50}. From a more schematic consideration of correlation diagrams we obtain

\[
(\Delta \epsilon)_{\text{lab}}^0 = 30 \cdot A_1^{1/3} \text{ MeV}
\]

(13)

for the additional repulsive potential at the compound-nucleus shape for $A_1 = A_2$. The occurrence of an additional repulsive potential has been recognized also from TDHF calculations \cite{51}.

The doorway configuration which is formed during the approach of the two nuclei can be considered as a highly correlated state of \textit{(n particle, n hole)}-excitations with respect to the adiabatic configuration. This correlation is effectively lost by the
decay via residual interactions. The time which is necessary to obtain local statistical equilibrium between all excited states at a given shape, is denoted by $\tau_{\text{loc}}$. Estimates for this local equilibration time can be obtained from the decay time of one of the particle-hole states or from the corresponding time in precompound reactions. These considerations lead to $\tau_{\text{loc}} \approx 10^{-21}$ s.

3.2 Correlation parameter and consequences for transport theories

For describing the effect of the doorway configuration and its decay, we introduce an order parameter $\gamma$ which measures the degree of correlation with respect to the entrance channel. It is defined to be one initially and approaches zero for $t \gg \tau_{\text{loc}}$. This parameter is an additional macroscopic variable which has to be taken into account explicitly in the transport theory [19]. Only if $\tau_{\text{loc}}$ would turn out to be much smaller than all other characteristic times of the collision process we could neglect $\gamma$. As a consequence the relative motion of the nuclei would be determined by the adiabatic potential. Since we expect $\tau_{\text{loc}}$ to be of the order of $10^{-21}$ s, such an approximation seems not to be justified.

We assume in the following that the correlation parameter $\gamma(t)$ is given by

$$\gamma(t) = \exp \left[ - \frac{1}{\tau_{\text{loc}}} \int_{t_0}^{t} f(r(t')) dt' \right]$$

where we regard $\tau_{\text{loc}}$ as an unknown parameter. The integral over the form factor $f(r)$ smoothly switches on the equilibration. The quantities $r(t)$ and $t_0$ denote respectively the mean trajectory and a time well before the collision. As compared to the transport theory formulated so far [17-20], the essential new feature is the introduction of an explicitly time-dependent dynamical potential,

$$U_{\text{dyn}}(r,t) = U_{\text{ad}}(r)[1-\gamma(t)] + U_{\text{diab}}(r) \gamma(t)$$

where $U_{\text{ad}}$ and $U_{\text{diab}}$ denote the adiabatic and diabatic potentials, respectively. The relative motion of the nuclei and the transfer of nucleons is described by five variables: position $r$ and momentum $p$ of radial motion, angle $\theta$ and angular momentum $\varepsilon$ of rotational motion, and mass asymmetry $\alpha = (A_2 - A_1)/(A_1 + A_2)$ without a corresponding momentum (cf. [7,25]). With these variables the Fokker-Planck equation reads

$$\frac{df}{dt} = \frac{\partial}{\partial r} \left( \frac{\partial f}{\partial r} \right) + \frac{\partial}{\partial p} \left( \frac{\partial f}{\partial p} \right) + \frac{\partial}{\partial \theta} \left( \frac{\partial f}{\partial \theta} \right) + \frac{\partial}{\partial \varepsilon} \left( \frac{\partial f}{\partial \varepsilon} \right)$$

with the reduced mass $\mu$ and the relative moment of inertia $I_{\text{rel}}$. The potential $U_{\text{dyn}}$ includes besides the dynamical potential $U_{\text{dyn}}$ of eq. (15) the rotational energy. The transport coefficients are calculated from the expressions of ref. [25] with modifications arising from the treatment of the radial motion. In particular, a form factor is introduced in accordance with the numerical calculations of ref. [22]. For the adiabatic potential we use the results of Möller and Nix [52]. The diabatic potential is given by adding to the adiabatic potential the central value (13) with an adequate form factor. The proximity form for the mass-asymmetry dependence is used for calculating $(\Delta U)^{0}_\text{diab}$ for different projectile-target combinations,

$$(\Delta U)^{0}_\text{diab} = \frac{1}{\lambda_1 + \lambda_2} \lambda_1^{1/3} \lambda_2^{1/3} \text{MeV.}$$

For $\text{Ar} + \text{Pb}$ this gives $130 \text{ MeV}$. Effects from the dynamical potential are present directly in the relative motion but also in the transport coefficients.
Here, the dynamical potential enters via the effective excitation energy
\[ E_{\text{eff}} = E - \tilde{U}_{\text{dyn}} - E_{\text{kin}}^{\text{rad}} \]
which is available as heat. Whereas the transport coefficients \( v_p, D_{pp}, v_a, D_{a}, D_{aa}, D_{pp} \) are only affected via the effective temperature
\[ T_{\text{eff}} = (E_{\text{eff}}/a)^{1/2} \]
with the level-density parameter \( a \), the mass-drift coefficient \( v_a \) becomes explicitly proportional to the dynamical force \(-\partial U_{\text{dyn}}/\partial a\) and hence, should directly show the local equilibration. An indication of such an effect is shown in fig. 6 where the mean values of the element distribution for \( ^{86}\text{Kr} \) (5.99 MeV/u, 8.18 MeV/u) + \( ^{166}\text{Er} \) are plotted as functions of the total kinetic energy loss \( \Delta E \) and the interaction time \( t_{\text{int}} \), respectively. At both bombarding energies a local equilibration time of the order \( 10^{-21} \) s is indicated. This retardation in the drift coefficient explains also the discrepancy of the experimental and theoretical element distributions shown in fig. 2.

3.3 Numerical results for the compound nucleus \( ^{248}\text{Fm} \)

For the numerical treatment of the Fokker-Planck equation we introduce a moment expansion. Figure 7 shows three different trajectories for \( ^{40}\text{Ar} \) (400 MeV) + \( ^{208}\text{Pb} \) in the \((r, a)\)-plane for a local equilibration time \( 0.75 \times 10^{-21} \) s. The trajectory with the smallest \( \varepsilon \)-value (\( \varepsilon = 0.51 \)) leads to the formation of a compound nucleus.

The largest \( \varepsilon \)-value (\( \varepsilon = 0.104 \)) corresponds to a fast process known as dissipative (or deeply inelastic) collision. The trajectory with \( \varepsilon = 0.102 \), although similar to that with \( \varepsilon = 0.104 \) for small times, is captured. In contrast to the trajectory with \( \varepsilon = 0.51 \) it does not lead to a compound nucleus because \( \varepsilon > \varepsilon_{\text{crit}} = 70 \) (largest \( \varepsilon \)-value for the existence of the compound nucleus \( ^{248}\text{Fm} \)). Instead, the system develops towards mass symmetry (\( \alpha = 0 \)) and is expected to split into two fragments if deformations of the fragments are allowed. This long-living (slow) component of dissipative collisions results after the system has rotated several times and hence, exhibits
essentially a \( \frac{1}{\sin \phi} \) angular distribution like the fragments from compound-nucleus fission. In contradistinction to compound-nucleus fission, the mass distribution of this long-living component of dissipative collisions is expected to be broader because it is not limited by the saddle-point shape.

We can divide the reaction cross-section schematically according to the \( \lambda \)-values. For \( 0 < \lambda < \lambda_{\text{crit}} \) we have compound-nucleus formation if the critical \( \lambda \)-value \( \lambda_{\text{cap}} \) for capture (here \( \lambda_{\text{cap}} = 103 \)) is larger than \( \lambda_{\text{crit}} \) (here \( \lambda_{\text{crit}} = 70 \)). For \( \lambda_{\text{cap}} < \lambda < \lambda_{\text{crit}} \) compound-nucleus formation is limited by \( \lambda_{\text{cap}} \). The slow component of dissipative collisions is obtained for \( \lambda_{\text{crit}} < \lambda < \lambda_{\text{cap}} \). Calculated results for the fast component for Kr (8.18 MeV/u) + Er are compared in fig. 8 with experimental data.

![Fig. 8. Angular distribution, element distribution and energy loss \( \Delta E \) vs. variance \( \sigma_z^2 \) for the fast component of Kr (8.18 MeV/u) + Er.](image1)

Since no deformations of the fragments are included in the model, energy losses \( \Delta E \) larger than \( \approx 190 \) MeV cannot be described. Because of this we probably miss some cross section for \( \theta \approx 30^\circ \). The element distribution and the correlation \( \Delta E \) vs. \( \sigma_z^2 \) are well reproduced for \( \Delta E \leq 190 \) MeV. A value \( 10^{-21} \) s is used for \( t_{\text{loc}} \). The variation of the value within a factor two does not significantly affect the results. Note that no further parameter is adjusted in these calculations.

![Fig. 9. Excitation functions for capture in Ar + Pb calculated for three different values of the local equilibration time. The quantities \( V_C \) and \( A_{\text{red}} \) denote the interaction barrier and the reduced mass, respectively.](image2)

Figure 9 illustrates the calculated excitation functions for the capture cross-section \( \sigma_{\text{cap}} \) in units of the reaction cross-section \( \sigma_R \). For \( \sigma_{\text{cap}} < \sigma(t_{\text{crit}}) \) the capture leads to compound-nucleus formation. For small bombarding energies the compound-nucleus formation is strongly suppressed. It reaches the maximal value at the crossing point with
the dashed line which corresponds to \( \varepsilon_{\text{crit}} \). Above this cross-over a long-living (slow) dissipative collision occurs. The threshold for this component is well above the interaction barrier \( V_C \). A significant dependence of the capture cross-section on the local equilibration time is obtained only for \( (E_{\text{cm}} - V_C)/A_{\text{red}} > 3 \text{ MeV} \). In this region the capture cross-section is roughly determined by an upper limit \( \varepsilon_{\text{cap}} \) of relative angular momentum which only depends on \( \tau_{10c} \). As a result of this the capture cross-section exhibits a rather pronounced maximum as function of the bombarding energy.

![Graph](image)

Fig. 10. Excitation functions for capture in the collisions \( \text{Ar + Pb, Kr + Er and Xe + Sn} \) calculated for \( \tau_{10c} = 10^{-15} \text{s.} \) From ref. [55].

Figure 10 illustrates the dependence of the capture cross-section on the projectile-target combination. The depth of the pocket in the relative potential is reduced for more symmetric projectile-target combinations and hence, the capture cross-section goes down correspondingly.

Experimental evidences for the existence of the long-living dissipative component have been reported for \( ^{238}\text{U} (5.7 \text{ MeV/u}) + ^{48}\text{Ca} \) [56], \( ^{132}\text{Xe} (5.9 \text{ MeV/u}) + ^{56}\text{Fe} \) [57], \( ^{20}\text{Ne} + \text{nat}^{\text{Re}} \) and \( ^{40}\text{Ar} + ^{166}\text{Ho} \) [58]. Thresholds somewhat above the reaction barrier as indicated in figs. 9 and 10 have been observed.

4. Concluding remarks

Dissipative collisions play a dominant role in heavy-ion reactions and reveal new nuclear properties which are connected with mass transfer, kinetic energy loss and angular momentum dissipation. They represent an interesting many-body problem at rather high excitation energies. In contrast to nuclear spectroscopy dissipative heavy-ion collisions supply information about relaxation phenomena in nuclei. Such phenomena have also been observed in precompound reactions and in fission, but only in heavy-ion collisions a rich variety of such phenomena have been discovered. In this respect the study of dissipative collisions has opened a new field of nuclear research.

Transport theories have been successfully applied in describing the dissipative features of heavy-ion collisions. Transport coefficients calculated from microscopic theories account quantitatively for the gross properties of mass transfer, angular momentum dissipation and kinetic energy loss. Still many questions concerning a detailed account of the data remain open, in particular the dynamics of fragment deformations.

Present formulations of microscopic transport theories encounter two essential problems, the strong-coupling limit and the assumption of local equilibrium. The restriction to local statistical equilibrium does not apply to the first stages of the collision process. In order to overcome these problems two ways can be considered:

(i) the extension of TDHF by including the effects...
from residual interactions and
(ii) the extension of the microscopic transport
theories by the explicit treatment of local equili-

tration. A partly phenomenological model involving
an explicitly time-dependent dynamical potential in
the relative coordinate, has been introduced as a
first step. Then the mechanism of energy loss be-

comes two-fold. Part of the kinetic energy is lost
directly by friction. Another part is first stored
as potential energy (dynamical potential) and then
transformed into heat via residual interactions.

Thus we expect the relative motion to be quite dif-

ferent from the treatment by friction forces and
the adiabatic potential.

This concept has been applied to heavy-ion col-

lisions describing both compound-nucleus formation
and dissipative collisions. A long-living (slow)
component of dissipative collisions is found to
exist besides the well established fast component.
The long-living component is characterized by
similarities to compound-nucleus fission. It differs
from compound-nucleus fission by a broader mass dis-

tribution and by a threshold in bombarding energy
which lies well above the interaction barrier. Cal-

culations of mass distributions and angular distri-

butions including a dynamical treatment of fragment
deformations are in progress [59].

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