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PARTICLE OCTUPOLE EXCHANGE COUPLING IN THE YRAST LINES OF TERBIUM AND DYSPROSIUM NUCLEI

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An analysis of high-spin particle-hole excitations\(^1\) in the N=82 nucleus \(^{146}\)Gd has indicated that there is a large gap in the single particle spectrum at Z=64, and in several recent investigations\(^2,3\) we could show that the yраст spectra of few-valence-particle nuclei around \(^{146}\)Gd can be well described within the spherical shell model using empirical one- and two-nucleon interaction energies. Similar techniques have earlier\(^4\) been successfully applied to analyze the high spin states of nuclei close to the doubly closed \(^{208}\)Pb. One thus finds that the - fully or near fully aligned - few-particle yраст configurations have rather similar characteristics in the \(^{208}\)Pb and \(^{146}\)Gd regions. However, the properties of the phonons differ strongly in the two core nuclei. The 3\(^+\) as well as the 2\(^+\) state in \(^{146}\)Gd occur at much lower excitation than in \(^{208}\)Pb. Furthermore, in contrast to \(^{208}\)Pb, where many proton and neutron particle-hole excitations contribute\(^5\) to the 2.6 MeV 3\(^+\) state, the \(^{146}\)Gd octupole has a dominant \(\pi h_{11/2}d_{5/2}^{-1}\) component; a preliminary RPA calculation\(^6\) gives \(\approx 0.75\) for the \(\pi h_{11/2}d_{5/2}^{-1}\) amplitude of the 1.6 MeV 3\(^+\) state. One therefore expects that the coupling modes of \(h_{11/2}\) valence protons to the \(^{146}\)Gd 3\(^+\) state will be strongly influenced by the Pauli principle.

We have recently in an \((a,8n)\) experiment investigated\(^2\) the one-particle nucleus \(^{147}\)Tb, which has an \(h_{11/2}\) proton in its ground state (fig. 1). The \(^{147}\)Tb \(\rightarrow 2p1h\) yrast states in the 2.5 to 3.5 MeV region provide one example for the applicability of the spherical shell model for the nuclei around \(^{146}\)Gd. Rather good agreement with experiment is found for the energies calculated with empirical two-nucleon interactions taken from the known\(^1,7\) level spectra of \(^{146}\)Gd and \(^{148}\)Dy.

The measured E3 transition rate of 31 ± 6 W.u. for the 1266 keV ground state transition characterizes the 15/2\(^+\) state as an octupole excitation, and we interpret the 2038 keV level as the 17/2\(^+\) member of the \(\pi h_{11/2} \times 3^-\) septuplet expected in the neighbourhood of 1.6 MeV. The unexpectedly large .77 MeV energy split is due to the dominant \(\pi h_{11/2}d_{5/2}^{-1}\) contribution of the \(^{146}\)Gd core octupole. (The analogous energy separation of the 13/2\(^+\) and 15/2\(^+\) members of the \(n_{9/2} \times 3^-\) septuplet in \(^{208}\)Pb is 140 keV).

![Fig. 1: Yrast levels of \(^{147}\)Tb observed in \((a,8n)\) and calculated shell model yраст states (ref. 2).](image)

mamamoto\(^5\) has considered the problem of the coupling of a single particle (or hole) to the octupole vibration in \(^{208}\)Pb and has presented the lowest-order diagrams which contribute to the energy shifts in a particle plus phonon multiplet. The diagram appropriate to the case of the \(^{147}\)Tb \(h_{11/2} \times 3^-\) coupling, with two \(h_{11/2}\) particles and one \(d_{5/2}\) hole in the intermediate state is shown in fig. 3, bottom left. When the \(h_{11/2}\) valence proton is added to the 3\(^-\) state of the core, the large \(\pi h_{11/2}d_{5/2}^{-1}\) component of the octupole state will be effectively blocked out by the Pauli principle in some coupled states, and their energies will consequently be much higher than the unperturbed 3\(^-\) energy. In second order perturbation theory the energy shifts are given by:

\[
\delta E = 7 \langle W(3 11/2 11/2 3; 5/2 1) \times \langle h_{11/2}d_{5/2}^{-1} | H | 3^- \rangle^2 \times E(h_{11/2}) - E(d_{5/2}) - E_{3^-} \rangle (1)
\]
where $W$ is a Racah coefficient. For the state with maximum angular momentum, $I = 17/2$, the geometric coefficient $7W$ is $+17/2$, i.e. positive and large, and the state is pushed up in energy. For the $I = 15/2$ state the coefficient is $-7/2$. The negative sign implies that the Pauli principle enhances rather than reduces the $h_{11/2}^2$ component in the $3^{-}$ phonon, because the two $h_{11/2}$ protons occur more in an antisymmetric than a symmetric arrangement. In the $^{147}$Tb level spectrum the $17/2^{+}$ and $15/2^{+}$ levels are shifted from 1.58 MeV in the expected directions, and the ratio of the energy shifts is 1.45 compared to the theoretical ratio of 22/12 (cf. fig. 3). This approximate agreement must be considered satisfactory, since the situation is characterized by strong coupling, where higher than second-order effects may be important. Nevertheless, the $^{147}$Tb level energies provide an experimental number for the exchange coupling strength. From the observed 722 keV 15/2 to 17/2 splitting the energy factor of eq. (1) becomes

$$\frac{\langle |H| \rangle^2}{\Delta E} = 856 \text{ keV}$$

which we will use later to calculate energy shifts of multiplet members in $^{148}$Dy.

The high spin level spectrum of the two-proton nucleus $^{148}$Dy studied$^{7,8}$ in $(\alpha,4n)$ and $(160,4n)$ in-beam experiments is presented in fig. 2. Below 3 MeV the excitations are of two-proton character. In the yrast decay the complete $s_{11/2}^2$ multiplet is populated, together with $5^{-}$ and $7^{-}$ two-proton states which involve $s_{1/2}$ and $d_{3/2}$ valence particles.

At $I = 10$ the $^{148}$Dy valence spin is exhausted and higher yrast states must involve breaking of the $^{146}$Gd core. Since the $3^{-}$ octupole is the lowest core excitation, the yrast line can be expected to continue by excitations of the type $10^{+} \times 3^{-}$. In the experiment, the strong 1061 keV E1 transition is found$^{8}$ to populate the $^{148}$Dy $10^{+}$ isomer from a 3980 keV $11^{-}$ level, which we interpret as the lowest member of the octupole multiplet built on the $s_{11/2}^2$ $10^{+}$ state.

In the $^{148}$Dy $10^{+}$ state at 2919 keV, two $h_{11/2}$ protons are aligned and in this case one can expect the lowest member of the octupole multiplet to be two units in spin less than the maximally aligned $13^{-}$ member. The particle-phonon exchange diagram appropriate to the $^{148}$Dy $s_{11/2}^2 \times 3^{-}$ coupling, with three $h_{11/2}$ particles and one $d_{5/2}$ hole in the intermediate state, is shown in fig. 3 to the right. In second order perturbation theory, the energy shifts for members of the octupole multiplet in $^{148}$Dy are given by

$$\Delta E(h_{11/2}^2/1) = 14(2I'+1) \times$$

$$\times X \left( \frac{11}{2}, \frac{11}{2}, 1' \right; \frac{5}{2}, \frac{5}{2}, 3; \{ I' \} \) \times$$

$$\times \frac{\langle h_{11/2}^2 d_{5/2}^2 \rangle^2}{E(h_{11/2}^2)-E(d_{5/2})-E_{3^{-}}}$$

where $X$ is a $9j$ symbol, and $I'$ specifies the coupling of the two $h_{11/2}$ protons in the initial state. The crucial point here is that the energy factor $\langle |H| \rangle^2/\Delta E$ containing the interaction matrix element is the same as in the $^{147}$Tb case, and therefore one can use the empirical energy factor (2) from the one-particle phonon coupling to describe the exchange interaction of two particles with the phonon. With the value of 856 keV for that energy factor, derived above from the observed 772 keV splitting of the $15/2^{+}$ and $17/2^{+}$ levels in $^{147}$Tb, we have calculated the expected energy shifts for the four highest spin members of the $h_{11/2}^2 \times 3^{-}$ multiplet in $^{148}$Dy. The $I' = 10$ and 8 couplings both contribute to the $11^{-}$ and $10^{-}$ states, and the theoretical energies shown are the lower energy solutions obtained by diagonalizing the interaction in this two-dimensional basis). In fig. 3 the calculated level energies are compared with the experimental results. The good agreement with the observed $11^{-}$, $12^{-}$ energies provides strong support for the interpretation of these states as octupole multiplet members. In future experiments it may be possible to locate additional members of the $h_{11/2}^2 \times 3^{-}$ multiplet, particularly the $13^{-}$ member which should be an yrast state.

The fact that the excitation energy of the $3^{-}$ octupole state is higher in $^{148}$Dy than in $^{146}$Gd can also be understood as a Pauli interference effect. In this case, the geometrical blocking coefficient $14(2I' + 1)X$ for $I' = 0$ equals 2/12 assuming that two of the twelve $h_{11/2}$ protons are present in the $^{146}$Gd $0^{+}$ ground state. With the same empirical matrix element, the calculated energy shift is $\Delta E = +143$ keV, which is close to the experimental number of

$$E_{3^{-}}(^{148}\text{Dy}) - E_{3^{-}}(^{146}\text{Gd}) = 109 \text{ keV}.$$
This slightly smaller increase of the 3- energy in $^{148}$Dy is not unexpected since the proton pair in the $^{148}$Dy ground state also partially occupies the $s_{1/2}$ and $d_{3/2}$ orbitals.

As far as we know, this type of particle phonon exchange coupling involving two particles has not been observed before; it is encouraging that the empirical coupling strength derived from the one-particle case describes the more complex situation so well.

It is noteworthy that octupole yrast states with $\Delta I = 3$ systematically occur in the nuclei above $^{146}$Gd. In the Tb isotopes with N>82 low lying octupole states are found, which due to Pauli interference have $\Delta I = 2$, analogous to the 1266 MeV excitation in $^{147}$Tb discussed above. Such states have been identified in $^{148}$Tb$_{83}$ ($E_x = 1006$ keV, $I^\pi = 11^-$, Ref. 2), $^{149}$Tb$_{84}$ (1094 keV, 15/2+$^-$, Ref. 9), $^{150}$Tb$_{85}$ (874 keV, 11$^-$, Ref. 10), and $^{151}$Tb$_{86}$ (1097 keV, 15/2+$^-$, Ref. 11).

Octupole transitions with $\Delta I = 1$ are found in the yrast cascades of the Dy isotopes above 3 MeV excitation, where the two $h_{11/2}$ valence protons are aligned. For example, in the three particle nucleus $^{149}$Dy$_{83}$ recent experiments$^{14}$ identified a 984 keV $E1$ transition feeding the state, suggesting a $\left(h_{11/2} \otimes f_{9/2}\right)_{27/2^+}$ configuration for the 3645 keV level. In $^{150}$Dy$_{84}$ a 742 keV $E1$ transition deexcites$^{12}$ the 5813 keV 19$^-$ octupole state to the $\left(h_{11/2} \otimes f_{9/2}\right)_{3/2^+}$ level, and in $^{151}$Dy the $\left(h_{11/2} \otimes f_{9/2}\right)_{11/2^+}$ state is fed$^{13}$ by an 839 keV $E1$ transition from the $\left(\pi l_{1/2} \otimes \pi l_{1/2}\right)_{43/2^+}$ yrast state at 5743 keV. In all cases these octupole excitations are built on yrast states in which all available $h_{11/2}$, $f_{9/2}$, and $v_{1/2}h_{gf2}$ valence spins are fully aligned. The fact that octupole core excitation competes successfully with lifting of a neutron into the $\pi l_{11/2}$ orbital reemphasizes the rather high single particle energy of that orbital noted in earlier$^{14}$ spectroscopy studies.

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