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THE DEGREE OF PHASE COMPENSATION OF LASER BEAMS USING GAS JETS

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Résumé. -- Lors de la propagation d'un faisceau laser dans l'atmosphère, des distorsions de phase peuvent apparaître, soit de façon passive à cause de la turbulence, soit de façon active à cause de la défocalisation thermique. Il a été prouvé qu'il est possible de compenser partiellement ces écarts de phase en modifiant physiquement l'optique de transmission. L'utilisation de jets de gaz avec des gradients de densité adéquats, convenablement interposés dans le faisceau, est présentée ici comme une autre méthode d'introduction de facteurs correctifs. Le degré de compensation de phase susceptible d'être atteint en principe a été étudié pour des jets idéaux. On montre que l'écart quadratique moyen peut être réduit considérablement dans le cas de la défocalisation thermique.

Abstract. -- When a laser beam propagates in the atmosphere, its phase can be distorted either passively, due to turbulence, or actively, due to thermal blooming. It has been demonstrated that it is possible to partially compensate for these phase errors by physically distorting the transmitting optics. The use of gas jets, with controllable density variation, that are suitably interposed in the beam is investigated as another method of adding the correcting elements. The degree of phase compensation, which may be achieved in principle, has been studied using ideal jets. It is shown that for the thermal blooming case the mean square phase error can be substantially reduced.

1. Introduction. -- Active optics refers to optical components whose characteristics are controlled during actual operation, in order to modify wavefronts. While the concept of active optics has been known for some time, only recently is the technology emerging that will influence the design of high performance laser systems [1,2]. Phase variations produced by internal cavity disturbances, atmospheric turbulence, or thermal blooming impose severe limitations on delivering a high irradiance laser beam at a distance. One of the main applications of active optics is the compensation of these wavefront distortions in order to enhance the intensity of a laser beam on a distant target. It is possible to correct to a substantial degree for these distortions by passing the beam in question through an array of elements which impose controlled phase variations which compensate for the distortion. This involves the measurement and control of wavefronts often in real time in order to concentrate the energy on to a detector or target. Usually this is done by physically distorting mirrors in the transmitting optics [2] and thus varying the optical paths of the various rays of the beam. Good improvements have been achieved with low-power visible laser propagation in the atmosphere. Correction for high-power infrared lasers is more difficult because of the need for larger amplitude mirror deflections and heating of the active mirror surfaces.

A new method of phase compensation using gaseous optics is described here as an alternative method for phase front control. Gaseous optics offers a potentially efficient technique for phase control of the beam because of small light losses, tolerance of extremely high power levels, and a characteristic rapid response. If many gas jets with different optical properties and with sufficient optical depth are placed in the path of the laser, then the gas jets produce phase shifts in the beam itself. The geometry required to bring

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about localized phase compensation in the laser beam is possible by using independent jets of gas. Actively changing the gas index of refraction using flow will permit the control necessary to achieve phase compensation.

In order to illustrate the concept, consider a uniformly illuminated laser beam propagating in the z direction as shown in Fig. 1. Fig. 1a shows three views of an array of five rectangular phase-shifting elements assembled to form an aperture through which a laser beam is passed. Each phase-shifting element can be independently controlled to produce any desired phase shift (up to a certain maximum value). The phase-shifting elements are gas jets with variable refractive indices. Fig. 1(b) shows the array in operation with the phase-shifting elements producing linearly decreasing phase delays from element 1 to element 5. The approximate result of operating the array in this manner would be to tilt the output beam as shown. The figure illustrates one particular type of laser beam control, chosen as an introductory example because of its simplicity. However, the same technique can be applied using crossed jet arrays to adjust the phase fronts of a laser beam differently.

Suppose now that the phase distribution over the laser beam is originally \( \phi(x,y) \). The reduction in light intensity on the centerline of the undisturbed beam in the far field is given by [3]

\[
I = I_o [1 - \langle \Delta \phi \rangle^2] ,
\]

where \( I_o \) is the intensity for a perfect (unaberrated) beam and \( \langle \Delta \phi \rangle \) is the mean square error of phase over the beam cross section. In deriving Eq. (1), it has been assumed that the aberrations are small, say \( \Delta \phi^2 \leq (\pi/5)^2 \). In order to reduce this phase distortion (and thus increase the light intensity), gas jets are passed through the beam in two orthogonal directions, as shown in Fig. 2. If the density variation of an ideal jet (defined as a jet in which diffusion and entrainment are absent) in the y direction is denoted by \( \rho(x,z) \), then the relative phase change of the laser beam traversing the jet is given by [4]

\[
f(x) = -\frac{2\pi \rho}{\lambda} \int_0^t [\rho(x,z) - \rho_p] \, dz ,
\]

where \( \lambda \) is the wavelength of the laser light, \( t \) is the thickness of the jet, \( \rho_p \) is some reference density and \( \beta \) is the Gladstone-Dale constant. Similarly, the phase change due to the jet in the x direction is:

\[
f(y) = -\frac{2\pi \rho}{\lambda} \int_0^t [\rho(y,z) - \rho_p] \, dz ,
\]

The figure illustrates one particular type of laser beam control, chosen as an introductory example because of its simplicity. However, the same technique can be applied using crossed jet arrays to adjust the phase fronts of a laser beam differently.
direction is denoted by $g(y)$. Since the phase changes due to the two jets are additive, the phase of the laser beam after passing the two crossed jets is

$$
\psi(x,y) = \varphi(x,y) - f(x) - g(y).
$$

The main problem addressed in this paper is the determination of the functions $f(x)$ and $g(y)$ such that the mean square of the new phase function, is minimized. For purposes of simplicity, the jets are assumed to be inviscid for these calculations. Both discrete and continuous jet cases have been studied. It is recognized that real jets will have an effect on the performance of the fluid adaptive optics scheme since jets can cause optical aberrations of their own. Thus, what is analyzed here is the best phase compensation which may be achieved using ideal jets.

2. A Simple Minimization. -- In this section the cross section of the laser beam is assumed to be a rectangle $D$. The phase correction is performed by $M$ constant density jets in the $x$ direction and $N$ constant density jets in the $y$ direction (see Fig. 3). The intersection of the boundaries of these jets as viewed by the laser subdivide $D$ into $M \times N$ smaller rectangles in each of which the phase $\varphi_{ij}$ is assumed constant. The problem is to determine $f_i$ and $g_j$ so that

$$
\psi_{ij} = \varphi_{ij} - f_i - g_j, \quad i=1,2,\ldots,M, \quad j=1,2,\ldots,N,
$$

minimizes $J$, where

$$
J = \sum_{i,j} A_{ij} (\varphi_{ij} - f_i - g_j)^2,
$$

The $A_{ij}$ are the areas of the rectangles.

To minimize $J$ we must have

$$
\frac{\partial J}{\partial f_i} = \frac{\partial J}{\partial g_j} = 0,
$$

where all $A_{ij}$ were assumed to be equal. Thus, the analysis is restricted to equal width jets. Jets of different widths are a possibility, but have not been considered in this or in subsequent calculations.

The $M+N$ equations (6,7) for the unknowns $f_i$ and $g_j$ have a non-unique solution, since if $f_i$ and $g_j$ is a solution so is $f_i + C, g_j - C$, where $C$ is an arbitrary constant. In order to make the solution unique we require the minimization of the function

$$
p(f_i, g_j) = \sum_{i=1}^{M} f_i^2 + \sum_{j=1}^{N} g_j^2
$$

subject to the constraints given by eqs. (6) and (7).

This problem is solved by the method of Lagrange multipliers by defining

$$
S = \sum_i f_i^2 + \sum_j g_j^2 + \sum_i (N f_i + \sum_j g_j - \sum_{ij} \lambda_{ij}) + \sum_j \nu_j (M g_j + \sum_i f_i - \sum_{ij} \nu_{ij}),
$$

where $\lambda_{ij}$ and $\nu_j$ are the Lagrange multipliers. Hence

$$
\frac{\partial S}{\partial f_i} = 0 = \frac{\partial S}{\partial g_j},
$$

Solving eqs. (6), (7) and (10) for $f_i$, $g_j$, $\lambda_{ij}$ and $\nu_j$, one obtains

$$
f_i = \frac{1}{N} \left( \sum_{j=1}^{N} \varphi_{ij} - \overline{\varphi} \right), \quad i=1,2,\ldots,M,
$$

where $
\overline{\varphi} = \frac{1}{M+N} \sum_{j=1}^{N} \sum_{i=1}^{M} \varphi_{ij} = \frac{1}{M} \sum_{i=1}^{M} f_i = \frac{1}{N} \sum_{j=1}^{N} g_j.
$$
3. **A Continuous Case.**-- Here the cross section of the laser beam is assumed to be a convex domain D in which its phase error $\phi(x,y)$ is continuously given. This phase error is to be decreased by two crossed jets of variable index of refraction in the x and y directions, as described in the introduction, so that

$$J = \int \left[ \phi(x,y) - f(x) - g(y) \right]^2 dxdy = \text{min.}$$

(14)

Equation (14) is solved by standard variational methods [5]. One obtains two integral equations which are analogous to eqs. (6) and (7) for the finite differences scheme [6].

Again, the solution of these two integral equations is not unique, and to make it so we require the minimization of the functional

$$p(f,g) = \int_a^b [f(x)]^2 dx + \int_c^d [g(y)]^2 dy.$$  

(15)

As in the previous section, the problem is solved by using Lagrange multipliers $\lambda(x)$ and $\mu(y)$. After some manipulations one can show that

$$\int_a^b f(x)dx = \int_c^d g(y)dy = \int \phi(x,y)dxdy = \hat{\phi}.$$  

(16)

where $a, b, c, d$ are the extreme limits of the convex domain. Additional details require the specification of the geometry of the domain.

For the special case where the domain is a rectangle, the equation can be solved explicitly to yield

$$f(x) = \frac{1}{d-c} \left( \int_a^b \phi(x,y)dy - \hat{\phi} \right),$$  

(17)

$$g(y) = \frac{1}{b-a} \left( \int_c^d \phi(x,y)dx - \hat{\phi} \right),$$  

(18)

where

$$\hat{\phi} = \frac{(b-a)+(d-c)}{bc} \int_{ca}^d \int_a^b \phi(x,y)dxdy,$$  

(19)

which is similar to the solution obtained previously in eqs. (11), (12), and (13).

Unfortunately, it is not possible to write an explicit solution for the general case, where the domain is not a rectangle. One way to proceed is to solve the equation iteratively [6]. This method has been used successfully for the case of the circular aperture and the results will be described in the next section.

4. **A Numerical Example**

Recently, Bushnell and Skogh [7] have computed the phase compensation which can be achieved for a thermally bloomed laser beam by mirror deformation. A similar problem has been solved, except that now the compensation is achieved by means of discrete gas jets, as described above. Here we will give results which are applicable to a square array of free jets.

The phase distortion of a thermally bloomed beam is given in polar coordinates [8] by

$$\phi(r,\theta) = \frac{3}{\pi} \sum_{n=0}^{\infty} f_n(r) \cos n(\theta - \theta_0)$$

(20)

where $r = \sqrt{x^2 + y^2} \leq 1$, $\theta = \tan^{-1}\left(\frac{x}{y}\right)$ and $\theta_0$ is the direction of the wind. The functions $f_n(r)$ are given by

$$f_0 = C_1[A_{20}z(4) + A_{40}z(11) + A_{60}z(22)]$$

(21)

$$f_1 = C_1[A_{11}z(2) + A_{31}z(7) + A_{51}z(16)]$$

$$f_2 = C_1[A_{22}z(5) + A_{42}z(12)]$$

where $C_1$ is a non-dimensional coefficient equal to 3/4 in the calculations. This coefficient is equal to the constant C (eq. (11) of [7]) divided by the wave number for $CO_2$ laser radiation. The $z(i)$ are Zernike polynomials and are defined in [3], [7] or [8]. Finally, the coefficients $A_{ij}$ in eq. (21) that were used in the calculation are given in [7] as

$$A_{20} = -0.053, \quad A_{40} = -0.050, \quad A_{60} = +0.021$$

$$A_{11} = +0.440, \quad A_{31} = +0.072, \quad A_{51} = -0.061$$

$$A_{22} = +0.101, \quad A_{42} = +0.096, \quad A_{33} = -0.075.$$  

The first exercise considered was to reduce the phase error $\phi(r,\theta)$ of a circular laser beam by $N$ equal width jets in the x direction and $N$ equal
width jets in the y direction in an optimal manner (see Fig. 4). The solution obtained in Sec. 2 cannot be applied in a straightforward manner since the domain $D$ is now circular rather than rectangular, and, secondly, the phase is not constant over the sub-rectangles generated by the intersection of the jets. The problem was solved by rewriting the functional $J$ in eq. (14) as

$$J_N(f, g_j) = \frac{1}{N_1} \int \left[ \frac{1}{A_1} \int \left( \phi(x, y) - f - g_j \right)^2 dx dy \right] \frac{N}{A}$$

where $\phi(x, y)$ is the cartesian equivalent of eq. (20). It should be noted that $\delta\phi^2$ used in eq. (1) is equal to $(3/4)^2 \int_{\Delta} \phi^2 dx dy$ divided by the area. Hence, $\delta\phi^2_N = (9/16)J_N/A$, where the area of a circular beam of normalized radius is $\pi$. The minimization of $J_N(f_1, g_j)$ was carried out by using the general purpose minimization program MIN written by Professor Carl E. Pearson of the University of Washington. The results of these calculations for $\theta_0 = 0$ and $N = 1$,

<table>
<thead>
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<th>$f$</th>
<th>$g_1$</th>
<th>$g_2$</th>
<th>$g_3$</th>
<th>$g_4$</th>
<th>$g_5$</th>
<th>$g_6$</th>
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</thead>
<tbody>
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Table 1. Exact Numerical Results for $N \times N$ Jets with $\theta_0 = 0$. $N$ varies from 1 to 7.

Table 2. Exact Numerical Results for $N \times N$ Jets with $\theta_0 = \pi/4$. $N$ varies from 1 to 5.

$2, \ldots, 7$ are shown in Table 1 and for $\theta_0 = \pi/4$ and $N = 1, 2, \ldots, 5$ in Table 2. As was noted in Sections 2 and 3, the solution of this problem is not unique, and the data presented in Tables 1 and 2 is obtained by shifting the computed data so that eq. (13) is satisfied. The main conclusion to be drawn from these calculations is about the rate of decrease of $J_N$ with $N$, where $J_N$ is a measure of the loss in the maximum intensity, which can be achieved by focusing the laser beam. It is seen that most of the compensation is achieved by using a small matrix, say of $2 \times 2$ or $3 \times 3$ jets. The variation of $J_N/J_1$ as function of $N$ is shown in Fig. 5. The asymptotic value of $J_N/J_1 \approx 0.040$ was computed by substituting the solutions $f(x)$ and $g(y)$ of the continuous case in eq. (14).

The phase distortion has been expressed in terms of Zernike circle polynomials (eq. (20)), each...
term of which corresponds to individual aberrations such as tilt, focus, coma, and so forth [7,8]. In order to compare the results of those of [7], the functional J has been recomputed for the case of no tilt aberration (\(A_{11}\) set equal to zero in eq.(21)). Actually, a practical adaptive optics system may be one in which tilt (and focus too) can be corrected by the transmitter itself. In practice, this may be done by simply pointing the laser in a slightly different direction. Since the polynomials are orthonormal over the range of interest, \(0 \leq r \leq 1, 0 \leq \Theta \leq 2\pi\), this does not affect the other higher order aberrations and is equivalent to using a plane mirror to correct for tilt. These calculations were carried out and the results for \(J_N\) are given in Table 3 for \(\Theta_0 = 0\). The value of the rms phase error \(\sqrt{\Delta \phi^2} = 3/4 \sqrt{\sum \phi^2}\), versus N is shown in Fig. 6 for \(\Theta_0 = 0\). It is seen that the relative improvements, while substantial, are not as good as those which include tilt (Fig. 5). In fact, for a small matrix of 2x2 jets, the mean square of the error is not decreased at all. This is because only a tilt correction is possible with a small number of jets.

Figure 6 may be used to compare with Fig. 9 in [7], where a similar effect of reducing the phase error is achieved by deforming the mirror. By multiplying the rms phase error by \(1/2\alpha = 10.6\mu/2\pi\), the ordinates of the two figures are made identical. While a comparison of the relative improvement between [7] and this work is difficult due to the different methods of approach, it should be remembered that the number of actuators with the free jet system is 2N. The rms phase error is reduced by 28% for N = 7, but the results of [7] show a larger improvement for fourteen actuators. Increasing the number of jets further is not too effective as the limiting value of \(\sqrt{\Delta \phi^2}\) is 0.035.

If further reduction in phase error is desired, additional jets in different directions should substantially reduce the error. In an effort to assess this possibility, a few approximate calculations were carried out involving a double set of crossed jets, all of equal width and number. That is, the first set of 2N crossed jets was in turn crossed with a second set of 2N jets placed at 45° with respect to the first, making a total of 4N independent jets. In order to use the preceding optimization routine, the residual error from the optimization with 2N jets was used with a least squares fit to the form of eq. (20) to obtain a new phase error distribution. As the resulting \(\Delta \phi^2\) of the fitted distribution was slightly smaller than the actual mean square error, the coefficients were adjusted proportionally so that they matched. This adjusted error distribution was in turn minimized by using the second crossed array at 45° (i.e., \(\Theta_0 = 45^\circ\)) to the original. The value of \(J_N\) from this calculation is a measure of the overall phase error.

The results, labeled double array, are shown in Fig. 6 for tilt corrected (i.e., \(A_{11} = 0\)) cases.
5. Improvement of Individual Aberration Errors

In this section individual aberrations are examined to see how each of them is minimized by a single crossed jet array. A series of calculations was carried out in which the phase error was taken to be a single classical aberration. That is, only one $A_{ij}$ coefficient in eq. (20) was taken to be non-zero and then the optimization scheme was applied for different numbers of jets. The relative improvement in $\gamma_4/N^{1/2}$ was noted and is plotted in Fig. 7 as a function of the aberration and $N$ for $\theta_0 = 0$. The numbers for each line refer to the subscript on the $A_{ij}$ factors. The lowest order aberrations are 11 (tilt), 20 (focus), 22 (astigmatism), 40 (spherical), and 31 (coma), for example. Except for some minor irregularities at low values of $N$, all aberrations showed improvement with the crossed jet principle. The most dramatic improvement was in tilt, as expected, although all the lowest order aberrations are improved greatly. There is a systematic progression in loss of ability to remove modal errors as the order of the radial coordinate $(r)$ increases. That is, simple functions can be minimized easily with small values of $N$ whereas complex functions having many maxima and minima across the beam require larger values of $N$ to reduce the phase error. Somewhat similar results exist for the case of $\theta_0 = 45^\circ$.

The case of tilt about the $y$ (or $x$) axis for a square laser beam is easy to examine in detail. The phase error is given as

$$\varphi_0 = 2CA_{11}x \cos \theta_0 = CA_{11}x.$$  \hspace{1cm} (24)

The phase corrections are discontinuous changes in $\varphi_0$, each of which is $2CA_{11}/N$ different from its neighbor and set at the local mean value of $\varphi_0$. A sketch of the idea is given in Fig. 8. Equation (23) becomes for this case

$$J_N = 2N \int_0^{1/2N} \int_0^{1/2N} (2CA_{11}x)^2 dx dy.$$  \hspace{1cm} (25)

Obviously, then, $\sqrt{J_N/J_i} = 1/N$. While the computation is somewhat more complicated for a circular geometry, the result for $\sqrt{J_N/J_i}$ should be the same. The preceding result closely describes the result for the reduction in tilt error in Fig. 7.
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