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VECTOR INTERPRETATION OF DEFORMABLE MIRROR C.O.A.T. SYSTEM *

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Abstract.—This study concerns a self-adaptive optics fitted with two deformable mirrors, each of them subjected to N actuators. The first mirror corrects the beam variations that result from either the deformations of the emitted wave-front or the variations of refractive index of the transmitting medium the beam passes through to reach the target. The second mirror's only role is to introduce the recurrent disturbances of the optical path that are necessary to produce the phase error signals which are to control the actuators of the first mirror. We deal with the stationary state of the servo-system, that is the residual phase errors are small. It is then possible to replace all the surfaces that are involved in the calculations by an equivalent vectorial system and to obtain very simply from it the main characteristics of this system.

I - INTRODUCTION (FIGURE 1)

The goal is to deliver from a laser system a maximal power on a remote target, but the shape of the laser emitted wave-front may vary, and moreover the optical path between the system aperture and the target is subject to deformations due to atmospheric perturbations. We must introduce in the optical system two deformable mirrors M₁ and M₂. The first one has to compensate the optical path variations and its shape is adjusted by N actuators. The second one is submitted through N other actuators to very small prescribed amplitude vibrations at various frequencies and permits to measure the residual phase errors. An optical receiver, followed by a lock-in amplifier, gives the phase error signals to a control system which feeds the M₁ actuators.

We call E (m) the amplitude of the optical field at the point m of the aperture, the total area of which is S, and ' (m) the optical length of the ray going through m from the laser wave-front to the target.

II - RELATIVE POWER LOSS

We assume that the phase variations do not modify the amplitude distribution on S and we take as quality criterion the ratio:

\[ \frac{P}{P_{\text{MAX}}} = \left| \frac{\int_{S} E e^{i' \rho} dS}{\int_{S} E dS} \right|^2 \]

where P_{\text{MAX}} would be obtained if ' would be a...
constant for every m.

We write:

\[ \int_S E e^{j\Phi} dS = R e^{j\gamma} \]  
(R real)

and

\[ R = \int_S E e^{j(\Psi - \gamma)} dS \]

then

\[ \frac{P}{E_{\text{MAX}}} = \left( \frac{R^2}{\left( \int_S E dS \right)^2} \right)^{1/2} \]

We assume now that the system is correctly performing, so the difference between \( \Psi \) and \( \gamma \) is small everywhere on \( S \). We write:

\[ \frac{1}{E} \int_E dS \quad \text{then} \quad \frac{dS}{S} = \delta_0 \]

and we obtain:

\[ \Psi \neq \int_0^\sigma e_r \Psi d\sigma \quad \text{and} \quad \frac{P}{E_{\text{MAX}}} = 1 - q \]

with

\[ q = \int_0^\sigma e_r (\Psi - \gamma)^2 d\sigma \]

which is the relative power loss (R.P.L.). For example, if \( q = .25 \), the loss is 1 dB.

III - SURFACE VECTOR SPACE

The last relation shows that we shall have to consider expressions like \( \sqrt{e_r} \left( \Psi(m) - \gamma \right) \) where \( \Psi \) and \( \gamma \) result from the addition of several similar functions representing wave-front, or mirror deformations. We shall call weighted surfaces or "surfaces" expressions like \( \sqrt{e_r} \cdot \Psi(m) \cdot \Psi(m) \). It is very easy to verify that these surfaces satisfy the vector spaces axioms, and we write

\[ \sqrt{e_r} \cdot \Psi(m) \cdot \Psi(m) \]

We define the inner (or dot) product of two vectors \( \psi_1 \) and \( \psi_2 \) as:

\[ \psi_1 \cdot \psi_2 = \int_0^\sigma e_r \psi_1 \psi_2 d\sigma \]

\[ \psi^2 = \int_0^\sigma e_r \psi^2 d\sigma \]

If \( \psi^2 = 1 \), \( \psi \) is of norm 1. Every vector \( \psi \) may be considered as the product of a scalar number with a vector \( \psi' \) of norm one: \( \psi = \sqrt{e_r} \cdot \psi' \)

If \( \psi_1 \cdot \psi_2 = \int_0^\sigma e_r \psi_1 \psi_2 d\sigma = 0 \), \( \psi_1 \) and \( \psi_2 \) are "orthogonal" (which does not mean that the corresponding wave-fronts are perpendicular). A set of surfaces \( \psi_i \) such that:

\[ \psi_i \cdot \psi_j = \delta_{ij} \]

constitutes an orthonormal basis and a surface can be represented by:

\[ \psi = \Sigma g_i \psi_i \quad \text{with} \quad g_i = \hat{\psi} \cdot \hat{\psi}_i \]

\[ = \int_0^\sigma e_r \phi \psi d\sigma \]

particularly:

\[ \gamma = \int_0^\sigma e_r \psi d\sigma \]

\[ \gamma = \int_0^\sigma (\sqrt{e_r} \psi \cdot 1) (\sqrt{e_r} \psi) d\sigma = \hat{I} \cdot \hat{\psi} \]

.../...
\( \gamma \) is the weighted mean value of \( \psi \). If \( \gamma = 0 \), the surface \( \psi \) is centered. We shall assume in the following that every surface is centered, thus:

\[
q = \bar{\gamma}^2
\]

IV - MINIMAL POWER LOSS

The phase \( \psi (m) \) results from the addition of several terms:

\[
\psi = \bar{\gamma} - \Sigma_i a_i \bar{a}_i + \Sigma_j a_j \bar{b}_j
\]

\( \bar{\gamma} \) is the shape of a distortion of the wave-front created by the laser, or the atmosphere. \( c \) is its amplitude. We admit that the small distortions created by the actuators add linearly. \( \Sigma_i a_i \bar{a}_i \) corresponds to the shape of the correcting mirror \( M_1 \) and results from the addition of the deformations caused by the actuators. \( \bar{a}_i \) is the shape of the deformation created by the actuator \( i \), with amplitude \( a_i \). \( \Sigma_j a_j \bar{b}_j \) corresponds to the effect of \( M_2 \).

\( \bar{\gamma}, \bar{a}_i, \bar{b}_j \) are centered and of norm 1. \( c_i, a_i, b_i \) are time dependent.

\[
q = \bar{\gamma}^2 = (c\bar{\gamma} - \Sigma_i a_i \bar{a}_i)^2 + 2 \Sigma_j b_j (c\bar{\gamma} - \Sigma_i a_i \bar{a}_i) \cdot \bar{b}_j + \ldots
\]

The terms \( b_i \) are sinusoidally time dependent and small, so we neglect the term containing products \( b_i b_j \). The second term is only a small contribution to the mean value of \( q \), but it contains the phase error \( c\bar{\gamma} - \Sigma_i a_i \bar{a}_i \), projected on each of the \( \bar{b}_j \) and will be useful later.

So, to minimize \( q \), it is necessary to minimize \( (c\bar{\gamma} - \Sigma_i a_i \bar{a}_i) \) which is equal to zero only if \( c\bar{\gamma} = \Sigma_i a_i \bar{a}_i \), which means that \( \bar{\gamma} \) is in the subspace defined by the \( \bar{a}_i \) as a basis. If not, we write:

\[
(c\bar{\gamma} - \Sigma_i a_i \bar{a}_i)^2 = c^2 \bar{\gamma}^2 - 2 c \Sigma_i a_i \bar{a}_i \bar{\gamma} + \Sigma_i a_i \bar{a}_i \bar{a}_i \]

which introduces the vector of components:

\[
C_{\gamma i} = \bar{\gamma} \cdot \bar{a}_i = \int_0 \epsilon \bar{a}_i d\sigma
\]

and the matrix of element

\[
C_{ik} = \bar{a}_i \bar{a}_k = \int_0 \epsilon \bar{a}_i \bar{a}_k d\sigma
\]

This matrix converts the contravariant components of the vector space defined by the \( \bar{a}_i \) into covariant coordinates. It is easy, by differentiation, to determine the set of the values \( a_i \), such that \( q \) is minimal. It is the vector \( \bar{a} \) given by:

\[
\bar{a} = c\bar{\gamma} \left( C_{ik} \right)^{-1} C_{\gamma i}
\]

The components \( a_i \) of this vector \( \bar{a} \) are the contravariant coordinates of the orthogonal projection of the vector \( c\bar{\gamma} \) on the subspace defined by the vectors \( a_i \) (fig. 2).

The minimal loss is:

\[
q_{\min} = c^2 \left[ 1 - \frac{1}{|C_{ij}|} \right] \left( \frac{|C_{ij}|}{|C_{ij}|} \right)
\]

It is the squared distance between the point \( c\bar{\gamma} \) and the subspace \( a \), as shown on figure 2.
V - ERROR SIGNALS - SUBOPTIMAL LOSS

The frequency-locked detection of the terms containing \( b_j \) enables us to obtain the error signals:

\[
E_j = (c\hat{\theta} - \Sigma_i a_i \hat{\alpha}_i) \hat{\beta}_j
\]

The servo loop system tends to cancel these errors, which will occur if \( c\hat{\theta} = \Sigma_i a_i \hat{\alpha}_i \) (\( \hat{\theta} \) in the subspace \( \hat{a} \)), or if the orthogonal projection of the phase error on each \( \hat{\beta} \) is zero, which means that \( c\hat{\theta} \) and \( \Sigma_i a_i \hat{\alpha}_i \) have the same projection on \( \hat{\beta} \).

This is shown by Figure 3, where the number of dimensions of the vector space, which could be infinite, is reduced to two. The best choice for \( a_i \) corresponds to \( A_i \), the orthogonal projection of \( T \) on \( \hat{a} \), but the servo system will choose \( A' \), such that vector \( \hat{A}' \) is orthogonal to \( \hat{\beta} \).

So we see that it is advisable that \( \hat{\alpha} \) and \( \hat{\beta} \) systems be about the same, that is, that the distortions impressed on mirrors \( M_1 \) and \( M_2 \) by the actuators have similar shapes. This constraint may be released if \( \hat{\theta} \) is close to the subspace \( \hat{a} \).

To compute \( q \), we must introduce the matrix

\[
D_{ij} = \hat{\beta}_j \hat{\alpha}_i
\]

and the vector

\[
F_{ij} = \hat{\theta} \cdot \hat{\beta}_j
\]

The best values of \( a_i \) are given by:

\[
q = c^2 \left[ 1 - 2 F_{ij}^T |D_{ij}|^{-1} F_{ij}^{\perp} + F_{ij}^{\perp T} |D_{ij}|^{-1} |C_{ij}| |D_{ij}|^{-1} F_{ij}^{\perp} \right]
\]

which may be obtained by application of Pythagorus Theorem.

VI - SERVO SYSTEMS

With the error signal vector:

\[
E = c \frac{F_{ij}}{|D_{ij}|} - |D_{ij}| \tilde{a}
\]

it is possible to compute a new error vector:

\[
E' = |D_{ij}|^{-1} E = c |D_{ij}|^{-1} F_{ij}^{\perp} - \tilde{a}
\]

and to apply it to the actuators of \( M_1 \) with a gain \( G(p) \) which makes:

\[
\tilde{a} = G(p) E' \text{ and gives:}
\]

\[
\tilde{a} = c \frac{G(p)}{1 + G(p)} |D_{ij}|^{-1} F_{ij}^{\perp}
\]

and we see that \( \tilde{a} \) assumes the suboptimal value when \( G \) tends to infinity. To attain stability requires that the real parts of the roots of \( 1 + G(p) \) be negative. It is possible to dispense with circuitry needed for implementing \( |D_{ij}|^{-1} \), by applying directly \( E \) to \( \tilde{a} \) with a gain \( G(p) \):

\[
\tilde{a} = G \left( c \frac{F_{ij}}{|D_{ij}|} - D_{ij} a \right)
\]

or

\[
\tilde{a} = c G |D_{ij}| \tilde{a} = c \frac{F_{ij}}{|D_{ij}|}
\]

( \( \Phi = \) unity matrix), we see that \( \tilde{a} \) takes also the suboptimal value for infinite \( G \), but to be stable, the system must stay away from values of \( G \) such that \( -\frac{1}{G} \) is an eigen value of \( |D_{ij}| \), which limits the system performance.

\[
\ldots \ldots
\]
VII - CONCLUSION

This vector space interpretation of a COAT system implies several simplifying hypotheses and is valuable only for small phase tracking errors. It must be considered as a first order theory and may be useful for a simple understanding of the behaviour of such a system and to its primary design.

![Diagram of COAT system](image)

**FIGURE 1 - C.O.A.T. SCHEMATIC**

**FIGURE 2 - MINIMAL POWER LOSS**

**FIGURE 3 - SUBOPTIMAL LOSS**