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ANALYTICAL MODELLING OF THE INFLUENCE OF THE NOZZLE GEOMETRY AND STAGNATION CONDITIONS ON PARAMETERS OF A CW CO₂ 18.4 µm GASDYNAMIC LASER.

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Résumé.- Un modèle théorique de laser gaz-dynamique CO₂/Ar produisant la radiation de longueur d'onde égale à 18.4 µm a été présenté dans cette communication. Les considérations présentées ici concernent l'influence des conditions initiales et des paramètres géométriques de la base sur les performances de laser en cause, quand la relaxation de l'énergie vibratoire de CO₂ ne peut pas être néglignée.

Abstract.- The analytical model of CW CO₂/Ar gasdynamic lasers operating at 18.4 µm wavelength is extended to present the influence of the nozzle geometry and stagnation conditions on their parameters in the cases in which the relaxation of the vibrational energy stored in the coupled symmetric stretching and doubly degenerate bending modes can not be neglected.

The results presented in Refs 1 - 5 indicate the possibility of designing a CW CO₂/Ar gasdynamic laser operating on the Q - branch of the 0310 → 1000 transition characterized by a high net output (of the order of 80 kW/kg [1,4,5]) and sufficiently high overall efficiency (of the order of 2% [1,5]). The relaxation of the E₂v₂ energy occurring during a real expansion causes the pertinent decrease of its value available at the cavity entrance for a possible conversion into the laser field energy. The hereby presented analysis is based on the following assumptions:

1. The population of each v₂ level can be calculated basing upon the Treanor type distribution function

\[ N_q \propto \exp \left( \frac{E_q v_2}{kT} - \frac{\Delta E_q v_2}{kT} \right) \]  

where rotational constant \( B \approx 0.39 \text{ cm}^{-1} \).

2. There is an equilibrium between the rotational and translational motion of CO₂ molecules

\[ N_q \propto \exp \left( \frac{B_q J(J+1)}{kT} \right) \]  

where rotational constant \( B \approx 0.39 \text{ cm}^{-1} \).

3. Laser action takes place on the Q - branch \( \Delta J = 0 \) of the transition \( (0310 , J) \rightarrow (1000 , J) \) for \( J \) corresponding to the maximum gain value

\[ J \approx \frac{1}{2} \left( \frac{kT}{2B} \right) - \frac{1}{2} \]  

4. In the subsonic part of the nozzle the thermal equilibrium is fully conserved.

5. In the supersonic part of the nozzle the gasdynamic parameters change according to a step - by - step adiabatic approximation. The length of each numerical step varies to ensure that the number of collisions responsible for the violation of the local adiabatic state remains much lower than one.

Under the above assumptions parameters of the CO₂/Ar CW gasdynamic laser can be determined from the following gasdynamic and vibrational energy relaxation equations [2,6]

\[ p = \rho T \]  

\[ \frac{d}{dx} (\rho v^2) = 0 \]  

\[ \rho v \frac{dv}{dx} + \frac{dp}{dx} = 0 \]
where $E_{v_1}$ and $E_{v_2}$ indicate the asymmetric stretching and bending mode vibrational energy per single CO$_2$ molecule.

\[ \frac{d}{dX} \left( h + \frac{v^2}{2} \right) = 0 \]  

(7)

\[ \nu \frac{d}{dX} \left( E_{v_1+v_2} - E_{v_2} \right) = -p \sum \gamma_M k_{v_T}^M \times \]  

\[ \times \left( E_{v_1+v_2} - E_{v_2} \right) \]  

(8)

\[ \frac{d}{dX} E_{v_3} = \frac{1}{\beta} p k_{\text{eff}} \left[ \frac{\beta^2 E_{v_3}^3 (t_{v_3})}{\beta v_2^3} - E_{v_3} (2h_{v_3}) \right] \]  

(9)

where $m$ and $f$ indicate the asymmetric stretching and bending mode vibrational energy per single CO$_2$ molecule. $\gamma_M$ denotes the mole of the gas mixture components. The enthalpy $h$ comprises also the vibrational enthalpy of CO$_2$. The respective vibrational energy relaxation and transfer rate constants $k_{v_T}^M$ and $k_{\text{eff}}$ are taken directly from Ref.2. For each pre-established set of the stagnation conditions ($p_0$, $T_0$) and the composition $X_{CO_2}$, $X_{Ar}$, $X_{H_2O}$ and for an assumed nozzle geometry the solution of Eqs 4 - 9 makes possible calculation of the value of $E_{v_1+v_2}$

\[ E_{v_1+v_2} = \sum \frac{Q'}{m_{v_1}^0} \frac{E_{v_1+v_2}}{T_v} \exp \left[ \frac{\beta v_1 v_2}{kT_v} \right] \left( \frac{\Delta \nu_{v_1} \gamma_{v_1}}{r} \right) \]  

(10)

together with values of the population inversion

\[ \Delta \nu = N_0 Q_{v_2} Q'_{v_1} \left( 2h_{v_2} + 1 \right) \exp \left[ \frac{2\beta v_1 v_2}{kT_v} - \frac{hC(3+1)}{kT_v} \right] \]  

\[ \times \exp \left[ \left( \frac{\beta v_1 v_2}{kT_v} \right) - \frac{\Delta \nu_{v_1} \gamma_{v_1}}{r} \right] \exp \left( - \frac{\Delta \nu_{v_2} \gamma_{v_2}}{r} \right) \]  

(11)

and gain

\[ \alpha = \frac{A_m}{\gamma_{m}^0} \frac{C^2}{\beta} \Delta \nu_{v_1} \left( \nu_{m}, \nu \right) \]  

(12)

In Eq. (12) $A_m$ is the Einstein coefficient of spontaneous emission, $\nu_{m}$ stands for the central frequency of (0310,0) transition, and $\gamma_{m}^0 (\nu_{m}, \nu)$ is the line shape function.

The numerical results presented in this paper concern a nozzle of logarithmic shape [6]

\[ q(x) = 2q \ln \left( ax + c \right) \]  

(13)
increases (see Figs 2, 5, 7, 8, 10 and 11) and when the mixture composition and stagnation temperature changes (see Figs 6, 7, 8 and 9). Summing up the following three conclusions seem be worth to become emphasized:

1) For other conditions pre-established there exists an optimum value of the stagnation temperature $T_0$. This optimum value of $T_0$ moves down with the increase in CO$_2$ concentration. It seems clear that this value is related to changes in the velocity of the $E_{\nu_1}$ energy relaxation.

2) There exists the optimum value of X$_{CO_2}$ concentration. With increase of X$_{CO_2}$ gain and inversion goes initially up but for too high X$_{CO_2}$ they begin to fall down as at these values the expansion can not be sufficiently deep to ensure that the time of $E_{\nu_1}$ relaxation remains much longer then the CO$_2$ molecule residence time in the nozzle.

3) Addition of H$_2$O to the working gas mixture is extremely harmful for the laser action on 0$^3$0 $\approx$ 10$^0$ transition. This fact quite clearly results from the known effectiveness of H$_2$O in relaxing the $\nu_3$ and $\nu_2$ modes of CO$_2$ and indicates that the purity of gases used in 4$\mu$m gas dynamic lasers is of paramount importance.

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Fig. 1 Distribution of the laser parameters along the flow axis for: $X_{CO_2}$=0.3, $X_{Ar}$=0.7, $T_0$=1000$^0$K, $p_0$=7 atm, $\theta$/2=20$^0$, $h^2/2$=0.02 cm.

Fig. 2 Relation between $E_{\nu_1}$ and the stagnation pressure for $\theta$/2=15$^0$.

Fig. 3 Relation between $E_{\nu_1}$ and the stagnation pressure for $\theta$/2=20$^0$. 
Fig. 4 Gain as a function of the stagnation pressure for $\theta/2 = 15^\circ$.

Fig. 5 Gain as a function of the stagnation pressure for $\theta/2 = 20^\circ$.

Fig. 6 Energy $E_{\nu_1+\nu_2}$ and population inversion as function of the concentration of CO$_2$:
1. $T_0 = 1000$ K, $p_0 = 7$ atm, $h^{X/2} = 0.02$ cm, $\theta/2 = 20^\circ$;
2. $T_0 = 1000$ K, $p_0 = 15$ atm, $h^{X/2} = 0.1$ cm, $\theta/2 = 45^\circ$. 
Fig. 7 Gain as function of the stagnation temperature.

Fig. 8 Gain as a function of CO$_2$ concentration for $X/h^x=200$. 
Fig. 9 Influence of water concentration on the gain regarded as a function of CO\textsubscript{2} concentration.

Fig. 10 Influence of the nozzle divergence angle on the gain and vibrational energy.

Fig. 11 Influence of the characteristic parameter $p_0 h^x$ on the vibrational energy and gain.

References


