A MODEL FOR THE EXCHANGE IN LIQUID He3
B. Castaing

To cite this version:

HAL Id: jpa-00220173
https://hal.archives-ouvertes.fr/jpa-00220173
Submitted on 1 Jan 1980

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
A MODEL FOR THE EXCHANGE IN LIQUID $^3$He

B. Castaing

Groupe de Physique des Solides de l'École Normale Supérieure, 24, rue Lhomond, 75231 Paris Cedex 05

Abstract - One remarks that normal liquid $^3$He can be described as a set of two level systems which are the spins of atoms (thus it is assumed that, for an unknown reason, + and - polarisations of each spin have different energies). The $C/\chi T$ ratio thus obtained is close to the experimental value ($\chi$: susceptibility, $C$: specific heat, proportional to the temperature $T$).

INTRODUCTION - There exists a very reliable description of the Fermi liquids at low temperature. It is given by Landau's model /1/. It tells us that the specific heat for example will be linear in $T$ at low temperature, and will depend only on the density of states at the Fermi level:

$$C = \frac{\pi^2}{3} N(\varepsilon_F) k_B^2 T$$ (1)

It tells us also that the susceptibility is constant at low temperature. In the case of an ideal gas the formula would be $\chi_{id} = \mu^2 N(\varepsilon_F)$ where $\mu$ is the magnetic momentum of the particles. In the general case a correction factor appears:

$$\chi = \frac{\chi_{id}}{1 + F_0}$$ (2)

which is called the exchange factor, by reference to the Hartree-Fock treatment of the electron gas in metals where such renormalisation of the susceptibility occurs. In fact, obviously, the whole Fermi liquid behavior is due to exchanges between particles.

The Landau model is general and gives no answer on the microscopic situation. To go further, it is necessary to go into the models which will allow us to calculate the Landau parameters such as $\varepsilon_0^A$.

For example, it is an old problem to know if liquid $^3$He must be considered as almost ferromagnetic or almost solid. The paramagnons model /2/, which considers it as nearly ferromagnetic, has had many successes. It can predict for example, without adjustable parameter, the temperature dependence of the susceptibility. However, as the pressure increases, liquid $^3$He obviously tends toward solidification. It would thus be interesting to understand if the picture of nearly localized atoms is compatible with the main results of the paramagnons model. As a first step, we will present here a model where we consider the $^3$He atoms as nearly localized.

1. The model

Let us first remark that the linear dependence of the specific heat with the temperature can be described as a glass-like behavior /3/, due to the existence of a set of two level systems. The question of what these systems are is answered by the behavior of the entropy. At the degeneracy temperature $T_p$, the liquid entropy is equal to the solid one, which is entirely due to the disorder of the spins. Moreover, the characteristic temperature for the non spin degrees of freedom (that is the Debye temperature $T_D$), is much larger than $T_p$. Thus it is natural to assume that the decrease of...
entropy at low temperature is entirely due to the progressive ordering of the spins. We will thus assume that the two-level systems we need are simply the two spin states of each atom.

The assumptions of the model are thus the following:

a) The $+\,$ and $-$ spin states of each atom have different energies $+\,E$ and $-\,E$.

b) If $f(\varepsilon)\,dc$ is the number of atoms for which $E$ is between $\varepsilon$ and $\varepsilon + dc$, $f(0)$ is different from zero.

Without the second hypothesis, as we will see, the specific heat would not be proportional to $T$. As discussed in the Introduction, we do not consider any other excitation of the liquid, neglecting their contribution to the entropy.

A similar model has been introduced by Blandin and Friedel /4/ in order to explain the properties of magnetic alloys (spin glasses). In such a mean field theory, the spin correlations are neglected. The entropy is thus the sum of the individual spin entropies in their local "field" $E$:

$$ S(T) = \int f(\varepsilon) \, S(\varepsilon + c) \, dc $$

where:

$$ S(\varepsilon) = k_B \ln (2 \cosh \varepsilon) - \varepsilon \tanh \varepsilon $n

At low temperature:

$$ S = \frac{f(0)}{k_B T^2} \int f(\varepsilon) \, \frac{\varepsilon^2}{\cosh^2 \frac{\varepsilon}{k_B T}} \, d\varepsilon = \frac{\varepsilon^2}{c} \int f(0) \, k_B^2 T = C \quad (3) $$

Within this model we can also calculate the magnetization $M$ induced by a magnetic field $H$, and thus the susceptibility. Neglecting in a first step the shift of $f(\varepsilon)$ in energies due to the magnetization, we can write:

$$ Nm = M = - \mu \int f(\varepsilon) \, \varepsilon \tanh \frac{\varepsilon + \mu H}{k_B T} \, d\varepsilon \quad (4) $$

$N$ is the number of atoms. At low temperature:

$$ \chi = 2 \mu^2 \int f(0) \, d\varepsilon $$

Putting $\frac{N}{2k_B T} f(0) = \frac{T}{T_0}$, we obtain:

$$ C = \frac{\pi^2}{12} \frac{N}{k_B^2 T_0} \quad (5); \quad \chi = \frac{N \mu^2}{k_B T_0} \quad (6) $$

By comparison between (5), (6) and (1), (2) we obtain $\frac{T}{T_0} = -0.75$, which is very close to the experimental value, near the solidification pressure. We can go further. Let us introduce a shift in energy of $f(\varepsilon)$ under magnetization $\delta$ and (6) become:

$$ M = - \mu \int f(\varepsilon) \, \varepsilon \tanh \left( \frac{\varepsilon - \mu H + C}{k_B T} \right) \quad (4') $$

$$ \chi = \frac{N \mu^2}{k_B(T + \delta)} \quad (6') $$

Thus

$$ 1 + \frac{\mu}{T} = \frac{1}{\mu} \left( 1 + \frac{\mu}{T_0} \right) $$

Note that $\delta > 0$ corresponds to a shift of antiferromagnetic type, as it is actually in the solid phase. In the figure we give the values of $\delta$ we need in order to fit the experimental data /5/,/6/, versus the molar volume $V$ in logarithmic scales.

The differences between the two series of data are due to the small number of published measurements of liquid He$^3$ specific heat /7/. Anyway, they are in agreement on the main point, which is that the $\delta$ value at high pressure goes toward the Néel temperature of the solid.

Conclusion

Normal liquid He$^3$ is thus very well represented by a "spin glass" model. Obviously, such a model is a little frustrating, because it gives no account of the main property of liquid He$^3$, which is its fluidity. It says only that, as far as energy and susceptibility are concerned, the quasi particles look very much like localized spins. This is obviously wrong if we are interested in momentum transfers.

References


Figure 1 - $\theta$ dependence versus molar volume (logarithmic scales). These values are computed from the data of ref. (5) : • and (6) : Δ. The Néel temperature of the solid is given by the full line and its extrapolation by the dashed line.