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THE MAGNETIZATION AND DENSITY OF SPIN POLARIZED ATOMIC HYDROGEN

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Abstract.- The magnetization and density distribution of spin polarized atomic hydrogen in a magnetic field is very sensitive for the presence of magnetic field gradients. This feature is discussed in relation to realistic experimental conditions, accounting for the specific Bose-Einstein character of the gas. To a limited extent comparison with experiment is possible.

Gaseous spin polarized atomic hydrogen (H\textsubscript{+}) has been shown \cite{1, 2} to be stable against recombination in a small (1 cc) open ended container with walls covered with superfluid \textsuperscript{4}He. Several minutes after loading the cell at 270 milli-Kelvin in a magnetic field of 7 Tesla the presence of H\textsubscript{+} could be demonstrated by breaking the He-film, so that the atoms recombined and the resultant heat of recombination was observed. In these experiments a field dependent density decay was observed and attributed to thermal leakage (escape) of the atoms out of the cell.

In this paper we discuss some of the consequences of field inhomogeneities for the study of H\textsubscript{+} at low temperatures. For this purpose we will consider the geometry of the magnetic field presently being used in our experimental apparatus. The field is produced by a conventional superconducting coil with a 1% inhomogeneity over a sphere of 1 cm in diameter. The axial and radial field inhomogeneities are coupled by Laplace's equation. The magnetic field distribution has the shape of a saddle point in space which is expressed in the quadratic field approximation by:

\[ B_x(z) = B_0 - B_z^2 \]
\[ B_z(c) = B_0 + \frac{1}{2} B_z^2 \]

where \( B_z(z) \) and \( B_z(\rho) \) are the axial and radial field distributions near the center of the magnet. \( B_0 \) is the maximum axial field and \( B = B_0/\rho \) where \( \rho \) is defined by \( U_z(z = \rho) = 0 \). In our solenoidal field, when fit to the quadratic form, \( \rho = 51 \) mm. The density distribution is then easily calculated by requiring the chemical potential \( \mu \) to be a constant over space. Neglecting interactions we find for densities smaller than the Bose-Einstein condensation (BEC) density \( n < n_c \):

\[ n(B) = \frac{g}{\lambda^3} \frac{\rho}{\sqrt{\pi}} \exp \left( \frac{\mu + \mu_B B/kT}{\sqrt{\kappa}} \right) \]

where \( n(B) \) is the density at field \( B \), \( g \) is the (nuclear) degeneracy factor, \( \lambda \) is the thermal wavelength, \( \mu_B \) is the Bohr magneton and \( k \) is Boltzmann's constant. The magnetization is simply proportional to the density \( \bar{\lambda} = \mu n \), where \( |\bar{\lambda}| = \mu_B \) for complete spin polarization. For densities where \( \lambda \) is much smaller than the inter-particle spacing and using the quadratic approximation for the field, equation (2) can be written as.

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\[ n(z) = \frac{\alpha}{\lambda^3} z \exp\left(-\frac{\mu_B B_0}{kT} \left(\frac{z}{z_0}\right)^2\right) \]  
\[ n(r) = \frac{\alpha}{\lambda^3} r^2 \exp\left(\frac{\mu_B B_0}{2kT} \left(\frac{r}{r_0}\right)^2\right) \]  
\[ Z = \exp\left(\mu + \frac{\mu_B B_0}{kT}\right) \]

where \( n(z) \) and \( n(r) \) are the axial and radial density distributions near the center of the magnet.

The axial density distribution thus has a Gaussian shape with half-width
\[ \Delta z = z_0 \left(\frac{kT}{\mu_B B_0}\right)^{\frac{1}{2}} \]  

If we assume the atoms to be confined to the axis of the magnet coil by means of a cylindrical tube we can calculate the axial and radial homogeneity in the density over this tube as a function of the temperature. Some characteristic values are given in Table 1 for \( B = 10 \) Tesla and a tube of radius \( r_0 = 1.8 \) mm.

Evidently at low temperatures the \( \text{H}^+ \) is highly localized near the center of the magnet and pressed to the walls of the confinement cell.

There are two important consequences of eq. (2). In the first place the total number of atoms that can be "stored" in the field is limited, i.e. there is a saturation density which is determined by the (steady state) density in zero field.

Table 1

<table>
<thead>
<tr>
<th>( T ) (mk)</th>
<th>( \Delta z ) (mm)</th>
<th>( n(r)/n(0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>19.2</td>
<td>1.004</td>
</tr>
<tr>
<td>100</td>
<td>6.2</td>
<td>1.043</td>
</tr>
<tr>
<td>10</td>
<td>1.9</td>
<td>1.52</td>
</tr>
<tr>
<td>1</td>
<td>0.6</td>
<td>656</td>
</tr>
</tbody>
</table>

Secondly, if the zero field density is reduced to zero, the density in the field region should also decay to this value with a time constant determined by thermal leakage.

In order to describe both features it is useful to introduce the concept of the magnetic compression factor \( c \)
\[ c(B_0 - B) = n(B_0)/n(B) = \exp \left( \frac{\mu_B (B_0 - B)}{kT} \right) \]  

This factor enables us to calculate the saturation density in the center of the magnet, neglecting recombination, once the (steady state) density is known at a given field \( B \). The implications of this concept for experiments are evident. \( \text{H}^- \) atoms should be injected in a \(^4\text{He} \) covered stabilization cell at as low a field \( B_1 \) as possible in order to fully exploit magnetic compression. For \( T = 0.5 \) K and \( \Delta B = B_0 - B_1 = 10 \) T, \( c = 7 \times 10^5 \) whereas for \( \Delta B = 5 \) T we only find \( c = 830 \). For \( T \lesssim 0.1 \) K and \( \Delta B \geq 5 \) T, \( c \sim 4 \times 10^{14} \) indicating that under these conditions the saturation density is not easily reached enabling the study of more intrinsic limiting effects in an open ended geometry.

The rate of thermal leakage is basically determined by the density in the region \( z = z_r \) where recombination starts to become important (i.e. for \( z > z_r \) we assume the He-film not to be thick enough to prevent surface recombination from occurring. To a first approximation the rate of leakage
\[ \frac{dN}{dt} \sim n(z_r), \]  
where \( N \) is the total number of atoms in the cell. A more detailed analysis (given in ref. /2/) leads also to a time constant
\[ \tau = 4cV_{\text{eff}}/\nu K A \]  
where \( V_{\text{eff}} = N/n_0 \) is the effective volume of the cell, \( K \) an effective Clausius factor /3/, \( \bar{\nu} = (8kT/\pi m)^{\frac{1}{2}} \) the average velocity of the atoms and \( A \) the cross-sectional area of the confinement tube. For a tube of uniform cross-section \( V_{\text{eff}} = A z_0 (kT/\mu_B B_0)^{\frac{1}{2}} \) and
\[ \tau = \pi \sqrt{2} K^{-1} c z_0 (n/\mu_B B_0)^{\frac{3}{2}} \]  

(6)
Increasing the density to values where the inter-particle separation becomes comparable to the thermal wavelength, degeneracy effects become important and will also show up in the density profile as long as H - H interactions remain negligible. In fig. 1 we show density profiles as calculated from eq. (2) for \( n << n_c \) and \( n = n_c \). A narrowing of the distribution as the critical density is approached is characteristic of the Bose gas. Beyond \( n_c \) BEC may result in a sharp peaking up of the axial density distribution at the field maximum although for realistic densities (temperatures) interactions between the atoms are expected to largely broaden this effect. The extent of this broadening can be estimated using the calculated /4/ ground state energy per atom in the low density limit: 

\[
E_1(n) = \alpha n - \mu B_0 \quad \text{where} \quad \alpha/k = 306 \, kR^2.
\]

Requiring \( \mu = (\alpha/\pi n) (nE_1(n)) \) to be a constant over the system yields for the characteristic halfwidth due to interactions

\[
\Delta z_{\text{int}} = z_0 \left( 2 \alpha n_0 / \mu B_0 \right)^{3/2}
\]  

(7)

In order to observe the effects of the statistics on the density profile we must require \( \Delta z / \Delta z_{\text{int}} \gg 1 \) at the Bose condensation density. Using equation (4), (7) and \( n_c = 2.612/\lambda^3 \) we find \( \Delta z / \Delta z_{\text{int}} = 1.8 \, \xi^{-2} \), where \( T \) is the temperature. This means that although \( H^+ \) can be prepared as a very weakly interacting system in comparison to \( ^4\text{He} \), temperatures in the (sub)millikelvin regime are probably required to enable observation of these interesting effects.

This concept of smearing of the wave functions as a result of the interactions is well studied for \( ^4\text{He} /5/ \). Another /6/ way of stating the requirements for observable effects of BEC on the density distribution is that the characteristic healing length, defined by \( \xi = (8\pi a)^{-1/2} \), is much larger than a characteristic dimension of the wave function. Here \( a \) is the s-wave scattering length \( (a = 0.72 \, \kappa \, \text{for } H^+) \). For \( n = 1.6 \times 10^{16} \) \( (T_c = 1 \, \text{mK}) \), \( \xi = 0.18 \, \mu \text{m} \) whereas a characteristic dimension for the wave functions is \( 6.8 \, \mu \text{m} /2/ \).

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References

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