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THE AFTERCLAP OF DEGENERATE CARBON IGNITION REVISITED

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Résumé.- L'ignition du carbone et le mode de propagation de la combustion décident de manière critique du sort des étoiles qui développent des coeurs de carbone/oxygène, c'est-à-dire explosion ou implosion. Le processus le plus rapide (détonation, conduction ou convection) détermine la vitesse de propagation du front de combustion. On peut probablement éliminer la formation d'une détonation à cause de la petitesse de la surpression générée par la combustion à haute densité. Nous démontrons qu'après une courte durée de combustion par conduction, un régime de convection s'établit. Nous en tirons la conclusion que si l'ignition a lieu à densité suffisamment grande ($\rho > 5 \times 10^9$ g/cc) les captures d'électrons et les pertes de neutrino concomitantes causent la réimplosion du coeur de l'étoile.

Abstract.- Whether the degenerate C-O cores, with develop in the heart of 4-8 $M_\odot$ stars, get fully disrupted or implode into neutron stars depends critically on the results of carbon ignition and on the nature of the propagation of the burning front. The velocity of this front is determined by the fastest of several processes, namely (1) detonation, (2) conductive burning, and (3) convective burning. Detonation can probably be excluded because of the small overpressures resulting from burning at high density. Since conductive burning is estimated to be very slow, the burning front is shown to propagate by convection. We conclude that if ignition occurs at sufficiently high density ($\rho > 5 \times 10^9$ g/cm^3), electron captures and concomitant neutrino losses will then offset the effects of burning and cause the implosion of the core.

1. Introduction

Stars in the 4-8 $M_\odot$ range form electron-degenerate carbon/oxygen cores, which then grow as the double shell burning of hydrogen and helium proceeds. The fate of these stars is at present not well determined. If carbon ignition occurs, then the mode in which the burning front propagates determines the dynamic behavior of the core. The dynamic effects of an assumed supersonic burning of the core by a detonation wave have been studied extensively. Its features and problems have recently been reviewed by Mazurek and Wheeler (1979) and by Buchler et al. (1974). A detonation, if it gets formed, will totally disrupt the star leading to the ejection of large amounts of "unwanted" iron peak elements. However the formation of such a stable detonation wave is questionable. (Mazurek et al. 1977).

The evolution of C/O cores with a subsonic burning front has been studied by Buchler and Mazurek (1975), Nomoto et al. (1976) and Chechetkin et al. (1977), with widely different assumptions regarding the velocity of the front. Buchler and Mazurek have assumed a very slow propagation of the burning. The energy released in burning could thus be removed by electron captures, and a hydrostatic C/O burning phase followed ; at fuel exhaustion core collapse ensued. Nomoto et al. have assumed convective burning with eddy dimensions of a fraction of the pressure scale height and obtained total disruption of the core. Thus the fate of such cores depends crucially on the rate of burning and the rate of electron captures. A competition between these two processes determines whether the overpressures from nuclear burning can be removed before dynamic expansion of the core. It so, the core will collapse if the ignition density is sufficiently high.

Recently recomputed electron-screening rates (Jancovici, 1979) are lower than those used in past work (Dewitt et al. 1973). This indicates that thermonuclear runaway will occur at somewhat higher densities than those of Nomoto et al. We suggest that the ignition density may be sufficiently high ($\rho > 5 \times 10^9$ g/cc) to give rise to core collapse.

It should be noted that an alternative scenario for the evolution of these stars that eschews C/O ignition is also possible. If the hydrogen envelope of these stars is lost before carbon ignition can occur, then the final outcome is an inert white dwarf. Studies by Tuchman et al.(1978) indicate that such may be the fate of all stars below ~ $7M_{\odot}$. However, we feel that sufficient uncertainities are involved in such calculations that the alternative of degenerate C/O ignition warrants further study. In addition binary evolution could also lead to carbon ignition under the conditions described herein.

Subsonic burning can occur according to two mechanisms, conductive burning or convective burning.

2. Conduction regulated burn.- We first calculate the burn velocity from conduction alone. Let us suppose an infinitely small mass of fuel is instantaneously heated to the equilibrium burn temperature. If heat is conducted away faster than the heat is regenerated by burning, then obviously the burning is quenched. The necessary and sufficient condition for burn then is an adequate scale or burn front thickness ∂ such that the thermal conduction heat transport equals the heat regeneration rate by burning. Since in general nuclear burning is a highly sensitive function of temperature (because of the Coulomb barrier) and in the present case thermal conduction will be weakly temperature dependent, because of degeneracy, the structure of the burn front will be determined at the maximum burn rate and hence maximum temperature. The maximum burn rate is strongly limited by the nuclear statistical equilibrium of the product nuclei so that a natural buffered limiting temperature occurs. Thus a characteristic burn time τ_{burn} to recreate this temperature exists. This time is shorter than the fuel depletion time. The characteristic burn velocity, due to conduction v_{cond}, then becomes

$$v_{cond} = \partial/\tau_{burn}$$

where

$$\partial = [(\kappa/C_v)\tau_{burn}]^{1/2}$$

so that

$$v_{cond} = [(\kappa/C_v)/\tau_{burn}]^{1/2}$$

where κ/C_v is the diffusion coefficient, the thermal conductivity and C_v is the specific heat at constant volume.

From Van Horn (1969) we approximate the thermal conductivity as

$\kappa \approx 5 \times 10^{11}\, T_9\, \rho_9$ erg/s/deg/cm

and again because of degeneracy

$C_v \sim 3/2\ R/A = 10^7$ erg/deg/cm^3/g.

Therefore the diffusion coefficient

$\kappa/C_v = 5 \times 10^4\, T_9 \rho_9$ cm^2/s, or $2.5 \times 10^5 T_9$ cm^2/s

at our assumed ignition density of 5×10^9 g/cm^3.

From Michaud (1972) we calculate a typical energy regeneration rate at a burning equilibrium temperature of $T_9 \approx 6, \rho_9 = 5$ of $\tau_{burn}^{-1} = 4 \times 10^4$ s^{-1}. The velocity of the burning front becomes finally :

$$v_{cond} = 2.5 \times 10^5 \text{ cm/s}.$$

and

$$\partial = 6 \text{ cm}.$$

A similar result is obtained with the more accurate expression of Kondratev (1965). If the radius of the core is 5×10^7 cm for $\rho_9 = 5$, then conduction burn will take 200 seconds to traverse the core, a time long compared to convective burn.

3. Convective burn front.- In order for convection to occur we must satisfy the following conditions : (1) an unstable buoyancy gradient must exist ; (2) there must be adequate time for an interchange of two fluid elements to take place before condition (1) may be violated.

The existence of an unstable buoyancy gradient is self-evident in our case because the initial of the star is presumably stable and we have added a buoyancy increment, due to burn, of $\Delta p/p$. The time during which this instability condition exists is limited in several ways : If our fluid interchange occurs on a scale λ_{min} and the burn front passes a distance λ_{min} at a velocity greater than the convective velocity associated with λ_{min}, then this wavelength will not sensibly contribute to convection. This wavelength is a minimum wavelength because the convective velocity increases with element size. We can calculate this size by equating the convective velocity to the conductive burn velocity. The buoyancy could also be removed by sufficiently fast electron-captures, $\tau_{cap} \lesssim \tau_{growth}$.

When an element of fluid burns and generates an incremental pressure Δp then we must first inquire whether the volume element remains at constant density or whether the excess pressure causes expansion to occur within the time scale of the phenomenon. The use of the term "convection" alons implies that we have tacitly assumed quasi static processes and hence constant pressure. We have already calculated a conductive burn velocity $v_{cond} \cong 2.5 \times 10^5$ cm/s that is small compared to sound speed. At the same conditions, $\rho^9 = 5$, the sound speed is $C_s = (\gamma p/\rho)^{1/2} \approx 10^9$ cm/s, and hence all processes will be quasi static unless we find an increase in burn propagation by 10^4. (A detonation is the progression of a burn front due to sound alone). Thus in the quasi static approximation limit our increment of pressure Δp from burn will be tranlated essentially instantaneously into an increment of density $\Delta\rho$.

The available specific potential energy for an interchange of two equal volume elements with a density difference $\Delta\rho$ is $W_s = 1/2\Delta\rho g\lambda$. From Mazurek and Wheeler (1979) we have $\Delta p/p \approx \frac{4}{3} (\Delta\rho/\rho) = 16\%$, at $\rho_9 = 5$ g/cm^3 and $g \approx (4/3)\pi\rho RG$, where R is the radius of the convective interchange assumed much smaller than the scale height. In a convective plume the entrainment corresponds to roughly doubling the mass per unit length per minor rotation of the maximum size eddy. (This results in the classic $1/2\pi$ half angle of a plume (Turner 1969)). Hence the mean vertical velocity of a plume is 1/2 the velocity associated with a zero entrainment free rise. (The mass is doubled by entrainment in an elastic collision of two equal masses, and so 1/2 the relative energy shows up as heat). Our convective overturn velocity should be

$$v_{conv} = \xi\left[(\pi/3)G\Delta\rho R\lambda\right]^{1/2} \approx 5\xi(R\lambda)^{1/2} \text{ cm/s}$$

for

$$\Delta\rho = .12\rho \text{ at } \rho = 5 \times 10^9 \text{ g/cm}^3$$

where $\xi < 1$ is a factor taking into account the plume shape.

One observes that the convective velocity depends upon the displacement of ignition from the center of the star. In order for convection to exceed conduction requires $(R\lambda) \gtrsim 10^{10}/\xi$cm^2. If the burn is initiated highly symmetrically with a displacement $\Delta R \lesssim 10^5$ cm, then the burn will proceed conductively until R is large enough so that the gravitational acceleration can drive the convection which then takes over. Ignition should be governed by the sound waves in the core. The spherically symmetric modes are expected to be largest, experiencing generally the lowest damping. Since ignition will be sensitive to the central density extremum it should be highly centered, unless a strong temperature inversion exists in the core.

For convection to develop there must be a period of instability growth from an initial perturbation that leads ultimately to convection. In order for a given eddy to contribute to convection and hence to the burn velocity, it must first proceed through a period of exponential Rayleigh-Taylor growth, then a nonlinear period of growth and finally a convective eddy that must make at least one revolution before an equal mass entrainment takes place. The equal mass entrainment is a necessary condition for the burn front progression, because in turbulent entrainment, as in a plume, all scales smaller than the largest scale are fully mixed. This then is a sufficient condition for burn.

The time scale for each process will now be estimated as a function of λ. First we neglect the nonlinear growth phase of bubbles and spikes as just the early growth of our eddy overturn. The bubbles which are the incipient eddies classically rise relative to the fluid at just our eddy overturn velocity, i.e., $v_{conv} = 1/2 v_{potential}$ where

$v_{potential}$ is the potential energy velocity (Turner 1969).

Ultimately we will look to the reduction in $\Delta\rho$ from positive to negative due to electron captures in a time τ_{cap} to be the critical time constant determining λ and hence the burn velocity. If $h/v_{burn} \gtrsim \tau_{cap}$, then the star will not explode, but instead collapse. We choose h to be the radius of the shell which bounds, say 1/5 of the core mass, i.e. $h = 5 \times 10^7$cm.

The small amplitude growth rate is, under the idealized situation of a stratified incompressible medium

$$x(t) = x_o \exp(\sqrt{kAg}\ t)$$

where

$$k = \frac{\ell}{R} \equiv \frac{2\pi}{\lambda}$$

and

$$A = (\rho_1\rho_2)/(\rho_1 + \frac{\ell}{\ell+1}\rho_2)$$

is the Atwood number, and g the acceleration. In our case $A=3/8(\Delta p/p)$. If we assume that the carbon fuel is ignited at a radius ΔR from the center of the star, then our previous value of g becomes $g=(4/3)\pi\Delta RG\rho = 1.2 \times 10^3 \Delta R$ cm/s^2 for $\rho = 5 \times 10^9$ g/cm^3. Therefore if $\alpha = \ell n(\lambda/x_o)$ the number of generations of linear growth required to reach the nonlinear amplitude of $x \approx \lambda$, then $t_\ell = 0.1\alpha/\sqrt{\ell}$ s. One notes that in such a spherical system the growth time of all the modes is in first order independent of ΔR.

The initial amplitude of the perturbation depends on the degree of non sphericity of the central part of the core which could be due e.g. to non-radial modes or to some rotation (e.g. a period of say 1 hr. would give $x_o/R \sim 2 \times 10^{-9}$). We shall adopt this value of x_o/R as a conservative estimate which yields α=20 and t_2= 1.4 s. We note that the linear growth takes about the same time as the convective eddy overturn $t_{conv} = \lambda/v_{conv} = \frac{\sqrt{\pi}}{5\xi} \approx 1$ s. After that point convective entrainment much like a convective plume must dominate the expansion of the burn front.

We are now ready to address the question of when the burning front will switch from a conductive to a convective character. Conduction will stabilize those modes which are such that $v_{cond} > \lambda/\tau_{growth}$. If we allow the largest eddy $\lambda=\pi R(\ell=2)$ to contribute to convection, we find that convective burn will start at $\tau = 1.4$ s. and $R=3.5 \times 10^5$cm.

The time scale for the buoyancy of the burned material to be removed by electron captures and the overpressure to be converted into a pressure deficiency is under these conditions $\tau_{cap} \sim .25$ s. (Mazurek and Wheeler, 1979).

If on the other hand the burn is ignited off center by a distance $\Delta R > \lambda_{min}$ then the initial burn volume will be buoyant and start to rise as a buoyant plume. This velocity will be the convective velocity $v_{conv} = 5\xi(\lambda\Delta R)$ cm/s^{-1} and the burn will reach R=h in a time $\int_{\Delta R}^{h} dR/v_{convect} = 1/5\ \ell n(\Delta R/h)$s. For the bubble to rise it must proceed through an initial period of linear growth. This also takes a time of the order of 1 s depending logarithmically upon the initial perturbation. Therefore, both the linear growth as well as plume convection to a critical radius for explosion $R \doteq h$ both take comparable times provided $\Delta R/h \lesssim 1\%$. Therefore again there is sufficient time for electron captures to remove the overpressures and buoyancy.

The ensuing pressure removal and rarefaction wave, moving with the local sound velocity $v_{sound} >> v_{convect}$, will then very effectively quench the convective burning front and ultimately cause the collapse of the core.

Finally we note that the beta capture rates are a far more sensitive function of density than the conductive and convective burn velocities so that the density at ignition will be the determining factor in the question of collapse versus disruption.

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