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COALESCENCE OF IRON IN THE CENTRE OF THE SUN

C. Deutsch, M.M. Gombert and H. Minoo

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Abstract. — A matrical nodal expansion making use of temperature-dependent Kramers potentials for the Coulomb interactions, is used to compute $G = F + pV$ in the centre of the sun. Small but crucial quantum corrections in $n = \frac{n_{ee}}{\epsilon}$ and $4\pi n_{ee}^2$ are taken into account quantitatively in the weakly coupled hydrogenic phase. The $\alpha$D system $e-p-Fe$ is shown to be stable for small iron concentrations $\approx 10^{-5}$ and an ion fer completely stripped ($Z = 26$).

Introduction. — It has been recently [1] speculated that stripped nuclei in a hydrogen plasma under solar conditions, according to the classical Debye-Hückel (DH) model, would undergo phase separations for concentrations well below the cosmic abundance. The physical cause of that phase separation is simply that the potential energy is lower in the separated phases than in the mixtures because the local charge neutralization is much better satisfied in the two separated phases.

The higher concentration corrections, needed to characterize the iron-rich phase lead to enhanced solubility for a simplified model where the electrons form a uniform background.

This demixtion suggests that highly ionised iron would precipitate out in the solar interior and coalesce in the centre. Should this occur and lead to a reduction of the iron concentration near the thermonuclear burn region by an order of magnitude below the cosmic abundance value ($2.5 \times 10^{-5}$ ionic mole fraction), the existing neutrino dilemma might be resolved. It is estimated that such a concentration reduction would lower the opacity, and hence the temperature, sufficiently to significantly reduce the probability of fusion of $\alpha$ particles that generates the nuclei whose neutrino emission is being monitored.

This paper presents a self-consistent Debye formalism for the iron-hydrogen phase diagram which takes into account quantitatively the small but nonnegligible quantum (diffraction and symmetry) effects. Moreover, our formalism is able to retain the remnant amount (if any) of bound states.

Pollock and Alder [1] got the iron-hydrogen phase separation through a computation of the thermodynamics of the iron-rich phase. They employed a corrected version of the DH theory. Here, we prefer to work out a consistent multicomponent DH scheme in the iron-poor and weakly coupled phase. Therefore the initial plasma we first consider is mostly classical, but includes some quantum corrections in agreement with L1J

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Pollock and Alder [1] got the iron-hydrogen phase separation through a computation of the thermodynamics of the iron-rich phase. They employed a corrected version of the DH theory. Here, we prefer to work out a consistent multicomponent DH scheme in the iron-poor and weakly coupled phase. Therefore the initial plasma we first consider is mostly classical, but includes some quantum corrections in agreement with [1]:

\[ 2.3 \leq \frac{\lambda_D}{\epsilon_p} \leq 4.3 \]  

2. Multicomponent nodal expansion with pseudopotentials. — The three-component plasma of interest is characterized by the dimensionless classical parameter

\[ \Lambda = \frac{e^2}{k_B T_{\text{Debye}}} = \lambda_D \left( \frac{Z^2 + x(Z-1) - 2(Z+2)}{2x(Z-1)} \right)^{1/2} \]  

where $\lambda_D$ denotes the Debye length $\lambda_D = \sqrt{4\pi \beta e^2 (C_{e} + C_{p} + Z^2 C_{Fe})^{-1/2}}$ with $\beta = \sum_{i=1}^{3} N_i/\epsilon_p$ and $C_i = N_i/\epsilon_p$.

Ni. $\Lambda'$ is the parameter in a pure hydrogen phase with $\lambda_{D} = \left( 4\pi \beta e^2 (C_{e} + C_{p})^{-1/2} \right)$. $Z$ is the average...
charge of the iron ions, taken as points. In the iron-
poor phase, we consider, the iron concentration \( x = \frac{C_{Fe}}{C_{Fe} + C_{\text{ee}}} < 0.001 \) small enough to allow the approx-
imation \( A \approx A' \).

Recall that the interior of the sun is characterized
by \( \Lambda = \Lambda_{\text{ee}} / \sqrt{2 \pi} \) with \( \Lambda_{\text{ee}} = \sqrt{\pi \epsilon B} T / 2 \).

\[ A' = 0.055, \quad \eta' = \frac{\Lambda}{\Lambda_{\text{d}}} \approx 0.21 \text{ and } 4 \pi \rho \chi^3 \approx 0.18. (3) \]

Our weakly coupled approach make use of two remarka-
table tools \([2,3,4]\): temperature-dependent pseudo-
potentials instead of bare Coulomb interactions and a
matricial formulation of the Debye resummation of the
Coulomb tails. In \( k \)-space, the latter is expressed
under the form

\[ \nabla \phi(k) = \nu(k) \left[ I + \nabla \phi(k) \right]^{-1} \left[ \nu + \nabla \phi(k) \right], \]

with

\[ \nu = \left( \begin{array}{ccccc} \rho B_1 & O & \cdots & O \\
O & \rho B_2 & \cdots & O \\
\vdots & \vdots & \ddots & \vdots \\
O & O & \cdots & \rho B_n \end{array} \right) \]

\( I \) is the identity matrix of order \( n \) = number of
components (= 3 in the present case), while \( U(k) \) denotes
the symmetric matrix of modified Coulomb interact-
ions, with the corresponding simplified entries

\[ U_{\text{ee}}(r) = \frac{e^2}{r} (1 - e^{-r/\Lambda}) + k_B T \log 2 \epsilon \log 2 \sqrt{\pi}, \]

\[ U_{\text{pp}}(r) = -\frac{e^2}{r} (1 - e^{-\sqrt{2}r/\lambda}), \]

\[ U_{\text{Fe}}(r) = -\frac{e^2}{r} (1 - e^{-\sqrt{2}r/\lambda}) \]

\[ U_{\text{Fe}}(r) = \frac{e^2}{r} U_{\text{Fe}}(r) = \frac{e^2}{r} U_{\text{Fe}}(r) = \frac{2 e^2}{r} \]

\( U_{\text{ee}}(r) \) takes into account both the \( s = 0 \) contribu-
tions of the electron-electron interaction, as well
as the moderate corrections due to the Pauli principle.

Moreover, we introduce the drastic assumption of a
hydrogenic and point-like iron ion. This point could
be discussed further in view of the average temperature
\( k_B T \approx 1.5 \text{ KEV} \) in the centre of the sun. Strict-
ly speaking, the latter does not permit to remove the
last two electrons bound within a helium like struc-
ture \([5]\). Nevertheless, we do not expect this simplifi-
cation to interfere with the phase separation process,
because it affects mostly the \( r = 0 \) limit of the Cou-
lomb interactions. Moreover, the iron charge \( Z \) can be
varied as a free parameter.

The inclusion of the mostly classical electron back-
ground is one of the attractive feature of the pre-
ent approach.

3. Debye Thermodynamics.- To compute the Gibbs free
energy \( G = F + PV \), we need \( \beta = (k_B T)^{-1} \) the virial
expressions

\[ P = \frac{2}{\beta k_B T} \sum_{k=1}^{3} \frac{\Omega_{kk}(r)}{\gamma_k} \]

\[ E = \frac{3}{2} N k_B T + \frac{N_0}{2} \sum_{k=1}^{3} \left( \frac{3}{2} \beta U_{kk}(r) \right), \]

where \( g_{kk}(r) \) denotes the 2-point canonical pair dis-
tribution function for the particles \( k \) and \( l \). In the
present high temperature limit, one can introduce the
linearization \( g_{kk}(r) = 1 - \beta V_{kk}(r) \) in Eqs. (9) and
(10). A much more detailed account of this for-
malism will be given later on.

Restricting to first order in \( A' \) and second order in \( \eta' \), one gets after lengthy manipulations, analytic approximations for \( P \) and \( E \). The B.O.S. thus reads \([6]\)

\[ P \frac{N k_B T}{2} = 1 + \frac{3}{2} \sum_{k=1}^{3} \left( \frac{1+x_1}{2} \right)^2 \frac{1+x}{2+x} \]

\[ + \left( \frac{(1+x_1)(2+x_2) - (2+2x)(2+x_2)}{2+2x} \right)^2 \frac{1+x}{2+x} \]

\[ + \left( \frac{(1+x)(3+x)}{2} \right)^2 \frac{1+x}{2+x} \]

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\[ + \left[ \frac{1}{2} \frac{1+x}{2} \frac{1+x}{2} \frac{1+x}{2} \frac{1+x}{2} \right] \]

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\[ + \left[ \frac{1}{2} \frac{1+x}{2} \frac{1+x}{2} \frac{1+x}{2} \right] \]
\[ a = l / x + l / x (z + 1) \].

The operation \( 2x^2 + 2x \) applied to \( E \), yields the Helmoltz free energy

\[ 3 \left( \frac{1}{2} \right)^{x} \left( \frac{1}{a} \right) + \frac{1}{\sqrt{2a}} \left( \sum_{a=1}^{\infty} \frac{1}{a} \right)^{x} \left( \frac{1}{a} + \frac{1}{a+1} \right) \]

where \( a = l / x + l / x (z + 1) \). The operation \( \int_{0}^{B} d\beta \delta(\beta) \)

applied to \( E \), yields the Helmoltz free energy

\[ \frac{F}{Nk_B T} = \frac{3}{2} + \frac{3}{2} \left( \frac{1}{2} \right)^{x} (\log 2)^{5/2}, \quad (12) \]

\[ - \Lambda' \rho x^{3} \pi^{2} \left( \log 2 \right)^{5/2} \]

\[ \left( \frac{1+x}{2+x} \right)^{2} \left( 2+2x^{2} \right)^{2} \]

\[ \left( 2+2x^{2} \right)^{2} \left( 1+x(z+1) \right)^{1/2} \]

\[ + \left( \frac{1}{2} \right)^{x} \left( \frac{1}{a} \right) + \frac{1}{\sqrt{2a}} \left( \sum_{a=1}^{\infty} \frac{1}{a} \right)^{x} \left( \frac{1}{a} + \frac{1}{a+1} \right) \]

\[ + \frac{1}{2} \left( 2+x(z+2) \right)^{2} \left( 1+x(z+1) \right)^{1/2} \]

\[ + \frac{1}{\sqrt{2a}} \left( \sum_{a=1}^{\infty} \frac{1}{a} \right)^{x} \left( \frac{1}{a} + \frac{1}{a+1} \right) \]

\[ + \Lambda' \left( \frac{1+x}{2+x} \right)^{3} \pi \left( \log 2 \right)^{5/2} \]

\[ \left[ \frac{1}{2} \sqrt{2+x(z+2)} \right] + \frac{1}{6} \left( \frac{1+x}{2+x} \right)^{2} \left( 1+x(z+1) \right) \]

\[ + \left[ \frac{1}{2} \left( 2+x(z+2) \right)^{2} \left( 1+x(z+1) \right) \right] \]

\[ - \left( \frac{1+x}{2+x} \right)^{x} \left( \log 2 \right)^{5/2} \left( \frac{1}{2} \right)^{x} \]
From Eqs. (11) and (12) we compute the excess mixing free energy $\Delta G = G(x) - xG(1) - (1 - x)G(0) + x \log(x)$
+ $(1 - x)\log(1 - x)$.

The results given in Tables 1 for $Z = 26$ show that for small values of $\eta'$, there is no phase separation.

We thank D.J. Stevenson for useful advices.

Table 1. Similar results hold for $x < 10^{-1}$. Mixing free energy $\frac{\Delta G(x)}{Nk_B}$ for $Z = 26$ and $\rho k^3 = 0.16$, $\lambda' = 0.055$, $\eta' = 0.21$.

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