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ACOUSTO-OPTIC INTERACTION OF SECOND HARMONICS IN LAMB WAVES

N.G. Brower and W.G. Mayer

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1. Introduction. - Finite amplitude mechanical vibrations in an elastic solid are nonlinear in nature which may give rise to a number of interesting interactions. One such interaction is the generation of harmonics as a finite amplitude acoustic wave is propagating in an elastic solid.

Acoustic bulk second harmonic generation was observed by Gedroits and Krasil'nikov. When an initially monochromatic longitudinal ultrasonic bulk wave was launched in an elastic medium, a secondary longitudinal wave was observed. The second harmonic amplitude increased linearly with the interaction length, i.e., with the distance of travel.

Harmonic generation of acoustic surface waves in crystals was observed by Løpen. His experimental results agreed with the theoretical analysis. As in the case of bulk waves, the amplitude of the second harmonic surface wave varied linearly with the interaction length and quadratically with the fundamental surface wave amplitude.

The possibility of harmonic generation in elastic isotropic plates is investigated in the present experiment. As an introduction, general features of second harmonic generation are presented as a theoretical background. Next, conditions for generation are discussed. The detection scheme utilizing a laser optical probe is outlined. A brief description of the experimental design is given. Finally, results of the experiment are presented.

2. Theoretical Background. - Using a perturbative approximation, the nonlinear equations of motion for an isotropic elastic solid bulk medium reduce to

\[ \frac{\partial^2 U}{\partial t^2} - \frac{C_v^2}{C_L^2} \nabla \cdot (\nabla \cdot U) + \frac{C_T^2}{C_L^2} \nabla \times \times \times \hat{U} = \hat{Q}, \]  

where \( \hat{U} \) is the particle displacement amplitude and \( C_L \) and \( C_T \) are the longitudinal and transverse bulk wave velocities. The term on the righthand side of eq. (1), \( \hat{Q} \), is the source or forcing term. In the case of harmonic generation, \( \hat{Q} \) consists of terms quadratic in the fundamental plane wave amplitude. An iterative solution to the nonlinear equation of motion yields

\[ U_2 = C U_1^2 x. \]  

The amplitude of the second harmonic, \( U_2 \), is proportional to the square of the fundamental wave amplitude, \( U_1 \), and also to the interaction length, \( x \), with \( C \) a proportionality constant. This result is also obtained by Løpen for surface waves in which case \( U_1 \) is the fundamental surface wave amplitude. Similarly, for plate mode interactions, eq. (2) is valid, where \( U_2 \) is the Lamb mode fundamental amplitude. The characteristics implied by eq. (2) will be used to identify a generated harmonic.

In order that a harmonic be generated, the velocities of the fundamental and the harmonic must be equal, that is

A. Basic Theory.

The optical detection system is based on the fact that a light beam (laser) will be diffracted upon reflection from a periodic surface corrugation. This corrugation may be an acoustic surface wave (a Rayleigh wave) or a plate vibrational mode (a Lamb wave). A theoretical treatment of this acousto-optic interaction for a Rayleigh wave has been given by Mayer, Lamers, and Auth /6/. This treatment can be extended to include Lamb waves /7/. Fig. 2 is a schematic diagram of the laser-surface wave interaction. The light is incident into the surface at an angle $\theta$, $\theta$ is the angular spacing of the first light diffraction order, and $x$ denotes the direction of travel of the Lamb wave on the plate.

The two cases of possible resonance are experimentally investigated here. Table I lists the fundamentals used and the possible harmonics generated.

**TABLE I. Fundamental Lamb Modes and Harmonics**

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Fig. 1. - Velocity dispersion curves for a free brass plate as a function of product frequency times plate thickness ($fd$ in units of MHz.mm). Selected points for harmonic generation are indicated by numbers 1 to 4 - see text.

Fig. 2. - Schematic diagram for acousto-optic interaction caused by Lamb waves. $K$ refers to the wave vector of the light incident at an angle $\theta$, $\theta$ is the angular spacing of the first light diffraction order, and $x$ denotes the direction of travel of the Lamb wave on the plate.

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frequency, contains a second harmonic component of amplitude $U_2$. In this case the particle displacement will no longer be pure sinusoidal and a distorted Lamb wave will interact with the incident light beam. The resulting diffraction pattern will be asymmetric with respect to the central order. The amount of asymmetry will depend on the magnitude of $U_2$ relative to $U_1$.

This process is similar to that described by Neighbors and Mayer /8/ for nonsinusoidal Rayleigh waves. Using the same approach for nonsinusoidal Lamb waves and assuming that only the fundamental and a second harmonic component are present in the wave, the light intensities in the positive and negative first orders are given by

$$I_{+1} = \left[ - J_3(y_1) J_2(y_2) - J_1(y_1) J_0(y_2) + J_1(y_1) J_0(y_2) \right]^2,$$  \hspace{1cm}  \text{(5)}

$$I_{-1} = \left[ - J_3(y_1) J_2(y_2) - J_1(y_1) J_0(y_2) - J_1(y_1) J_1(y_2) \right]^2,$$  \hspace{1cm}  \text{(6)}

where $\gamma_2 = 2KU_2 \cos \theta$.

The series of products of Bessel functions in above equations can safely be truncated to the terms shown since both $y_1$ and $y_2$ are usually small so that insignificant errors in the values of $I_{+1}$ and $I_{-1}$ are introduced by neglecting terms in $J_r J_s$ with $r, s > 3$.

Equations (5-6) are the bases in the optical detection of the presence of second harmonics. If $U_2 \neq 0$, one finds that $I_{-1} > I_{+1}$, giving rise to an asymmetric diffraction pattern. It should be noted that eqs. (5-6) reduce to eq. (4) if $U_2 = 0$. In this case all terms vanish except $J_1(y_1) J_0(y_2)$ which equals $J_1(y_1)$.

This analysis provides a method of determining the presence or absence of second harmonics in a vibrating solid plate. Probing the surface with a light beam and measuring the intensity distribution in the diffraction pattern yields information about the magnitude of $U_2$ compared to $U_1$. Moreover, probing the surface at different distances from the source of the Lamb wave generation, one can measure the change in harmonic content as a function of distance.

Due to the method of generating the fundamental Lamb mode, i.e., by means of the "liquid wedge" method /9/, the possibility exists that second harmonic components are generated in the liquid prior to incidence on the plate surface. However, if such components should be present and should continue to propagate along the plate they would at best decrease in amplitude as a function of distance traveled on the plate. Their amplitude could not increase as a function of distance on the plate unless harmonic generation on the plate is possible. Thus eq. (2) will be satisfied only if harmonic generation on the plate itself occurs. If harmonics propagate on the plate, asymmetry in the diffraction pattern will be observed; to obtain numerical values, a new parameter, $I_p$, is introduced, defined as

$$I_p = \frac{I_{-1} - I_{+1}}{I_{-1}} \times 100$$  \hspace{1cm}  \text{(7)}

Using eq. (2), it may be shown that $I_p$ is a linear function of distance from the source of the Lamb wave /7/ if the plate is considered to be a dissipationless medium. Therefore, $I_p$ can be measured as a function of distance.

![Fig. 3. - Experimental setup for optic probing of harmonic generation in Lamb waves.](image)
B. Experimental Design and Results.

The design for the optical probing system is shown in Fig. 3. Light from a He-Ne laser is expanded and collimated (details of the necessary optical components not shown in Fig. 3.) The collimated light beam is reflected from a highly polished brass plate which supports the Lamb wave. The diffracted-reflected light is focussed approximately 8 m from the plate. This distance is sufficient to properly separate the diffraction orders so that intensity measurements of $I_{+1}$ and $I_{-1}$ can be made with a photodiode.

Measurements of $I_p$ were made at various distances $x$ on the plate for the different fundamental frequencies and modes listed in Table I. Excitation of the modes was accomplished by adjusting the quartz transducer to the required angular position for the generation of the desired Lamb mode.

The polished brass plate, excited in the $A_1$ mode at 7 MHz, was probed with the He-Ne laser. A diffraction pattern was observed and the asymmetry between the first diffraction orders was measured. At a relative interaction length of 0.8 cm, the percent asymmetry was approximately 15%. The laser probe was then moved along the plate in the positive $x$-direction and the asymmetry increased. At a distance of 1.2 cm, a 29% asymmetry was observed. Likewise, for the plate in the fundamental $A_1$ mode, excited at 5 MHz, asymmetry was observed. In this case the asymmetry also increased with interaction distance. The results are listed in Table II.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$I_p$</th>
<th>$A_1$ mode at 7 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8 cm</td>
<td>15%</td>
<td></td>
</tr>
<tr>
<td>1.2 cm</td>
<td>29%</td>
<td></td>
</tr>
<tr>
<td>0.9 cm</td>
<td>14%</td>
<td>$A_1$ mode at 5 MHz</td>
</tr>
<tr>
<td>1.3 cm</td>
<td>39%</td>
<td></td>
</tr>
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</table>

As a control, the $S_0$ mode at 3 MHz was excited (point 5 in Fig. 1.). As can be seen in Fig. 1., there is no second harmonic which satisfies Eq. (3) because the required location (point 6) does not fall on a curve. Therefore, there should be no second harmonic content in the detected mode. The measured diffraction pattern showed this to be true: no detectable asymmetry in the first orders could be found.

In conclusion, second harmonic generation in Lamb modes was detected. Harmonics were observed for both a locally dispersionless mode and also for the mode coupling case, provided the appropriate conditions for harmonic generation in plates were satisfied.

Acknowledgment. - This work was supported by the Office of Naval Research, U.S. Navy.

REFERENCES


Asymmetry will be influenced or be absent under certain conditions of the phase relationship between the fundamental and the second harmonic, as pointed out by ALIPPI A., et al. J. Phys. 48 (1977) 2182.