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NONLINEAR SOUND GENERATION BY HADRON SHOWERS

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Abstract.—In connection with Project DUMAND, the proposal to utilize the ocean as a giant acoustic detector of neutrinos, we have studied the applicability of a recent theory of thermoacoustic arrays (Peter J. Westervelt and Richard S. Larson, J. Acoust. Soc. Am. 54 (1973) 121). In the static case or at very low frequencies, about 10% of the coefficient of thermal expansion for water at 20°C can be attributed to Debye-like modes. Debye-like modes generate sound via the non-linear mechanism responsible for the operation of the parametric acoustic array (Peter J. Westervelt, J. Acoust. Soc. Am. 35 (1963) 535). The contribution of the Debye-like modes to the thermal expansion coefficient, and thus to the sound pressure, is essentially independent of the ambient water temperature. Hence if the Debye-like modes are not fully excited as we postulate to be the case at high frequencies, then the thermal expansion coefficient will be less than the static value by an amount that causes it to vanish at about 6°C instead of at 4°C, the temperature of maximum water density. This theory is in agreement with recent measurements of the temperature dependence of sound generated by proton deposition in water (L. Sulak et al., Proceedings of the La Jolla Workshop on Acoustic Detection of Neutrinos, 25-29 July 1977, Scripps Institute of Oceanography, U.C.L.A., San Diego, Hugh Bradner, Ed.). A further consequence of our work implies that the conventional thermoacoustic theory be modified by subtracting the parametric source terms at high frequencies.

When high energy neutrinos interact with nucleons in the ocean a jet of hadrons is produced which deposits thermal energy. This thermal energy is expected to produce a sonic pulse which will enable the energy and direction of the incident neutrino to be estimated. Initial laboratory experiments intend on investigating the feasibility of this detection scheme have been performed by L. Sulak et al. at Harvard University and this paper is devoted to an analysis of an unexpected result of these experiments.

The theory of sound production by a line distribution of thermal energy was first formulated in 1973 /1/ and experimental verification of this theory appeared in 1976 /2/. After a brief review of this theory, a modification will be suggested which enables the theory to account for a previously inexplicable experimental finding of L. Sulak et al. /3/, namely, that sound generated by depositing protons in water vanishes when the ambient water temperature is about 6°C rather than 4°C, the temperature of maximum water density.

The inhomogeneous acoustic wave equation for the sound pressure p generated by H, the thermal deposition of energy per unit volume and per unit time, is:

\[ \ddot{p} + \frac{\omega_p^2}{c^2} \dot{p} - \frac{1}{C_p} \frac{\partial^2 p}{\partial t^2} = -\frac{\beta}{C_p} \frac{\partial H}{\partial t} \]

where c is the velocity of sound, \( \beta \) is the logarithmic coefficient of thermal expansion, \( C_p \) is the specific heat per unit mass. In the example of Ref. 1 in which the heat deposition results from the energy lost from a laser beam modulated at...
frequency \( w \),
\[
H = aI_0 e^{-z - iwt},
\]
where \( z \) is the coordinate along which the laser beam propagates, \( I_0 \) is the output intensity of the laser and \( a \) is the attenuation coefficient for the laser beam.

The asymptotic solution to Eq. (1) in the limit \( r \gg \omega a^2 c \) where \( r \) is the distance from the laser is:
\[
p = \frac{1}{4\pi c r} \frac{e^{ikr - i\omega t}}{\alpha + ik \cos \theta}
\]
where \( P_0 \) is the power output of the laser, \( k = \omega / c \), and \( \theta \) is the angle between the line connecting the field point and the laser and the direction of propagation of the laser beam. This solution exhibits the well-known behavior of Cerenkov radiation in the case where the emitting particle is progressing at the speed of light, far greater than the speed of sound. Thus the predominant acoustic emission occurs in a disc perpendicular to the laser beam with an intensity half width given by:
\[
\theta_{1/2} = \pi/2 \pm \alpha/k.
\]

This theory has recently received experimental confirmation /2/ with respect to the dependence of the sound pressure on \( \theta \) (see Fig. 1), and with respect to absolute magnitude (see Fig. 2).
Note that the sound vanishes at 6°C instead of 4°C, the temperature at which the static coefficient of thermal expansion for water vanishes.

Sound can be generated by sound, a mechanism which in the static limit represents the contribution to thermal expansion of the Debye-like excitations. The wave equation for the pressure $p_s$ of the sound generated by primary waves whose pressure is $p_1$ is

$$\nabla^2 p_1 = -p_0 \frac{\partial^2}{\partial t^2},$$  \hspace{1cm} (5)

where:

$$q = \rho_0 c_0 \left( \frac{1}{2} \rho_0 c_0^2 \left( \frac{d^2 p}{dp} \right)_{p=0} \right) \frac{\partial^2}{\partial t^2}. $$  \hspace{1cm} (6)

and $\rho_0$ and $c_0$ are the ambient density and speed of sound, respectively, for water. The quantity in the square brackets in Eq. (6) has a value of 3.5 for water.

The ratio of the Debye-like source strength to the total thermal source strength can be obtained from Eqs. (1)-(6). In obtaining this ratio it is legitimate to ignore the time derivatives since they appear twice in each source type. The ratio $R$ is:

$$R = \frac{3.5 c_p \rho_1^2}{E \rho_0 c_0^2},$$  \hspace{1cm} (7)

in which $E$ is the time dependent thermal energy density of which $p_1, p_{0}^{-1} - 2$ is the Debye-like part. Introducing $c_p = 4.2 \times 10^7 \text{erg/gm}^\circ\text{C}, \beta = 3 \times 10^{-4} / \text{C}$ at 20°C and $c_0 = 1.5 \times 10^5 \text{cm/sec}$ into Eq. (7), the value of $R$ valid at 20°C is:

$$R = 20 \frac{\rho_1^2 p_0^{-1} - 2}{E}.$$  \hspace{1cm} (8)

If $R$ is about 0.1 and all Debye-like modes are fully excited below some critical frequency, the 6°C zero crossing in Fig. 3 would be shifted down to about 4°C.

An estimate of the energy density $D$ residing in Debye-like modes is given by:

$$D \approx \frac{4\pi kT}{\lambda_{\text{min}}}$$  \hspace{1cm} (9)

in which $\lambda_{\text{min}}$ is the Debye short wavelength limit and $k$ is Boltzman's constant. An estimate of the total thermal energy density $E$ is given by:

$$E = \rho_0 c_p T$$  \hspace{1cm} (10)

The ratio $D/E$ is found by inserting $k = 1.4 \times 10^{-16} \text{erg/}^\circ\text{C}, c_p = 4.2 \times 10^7 \text{erg/gm}^\circ\text{C}$ and $\rho_0 = 1 \text{ gm/cm}^3$ into Eqs. (9) and (10):

$$D \approx 4 \times 10^{-23} \lambda_{\text{min}}^{-3}$$  \hspace{1cm} (11)

If the ratio $D/E \approx 0.01$ as is called for by the experimental results of L. Sulak et al., this leads to a value of $\lambda_{\text{min}} \approx 10^{-7}$ cm. A crude check on the internal consistency of this result can be made by estimating directly the value of $\lambda_{\text{min}}$ from the formula:

$$\lambda_{\text{min}} \approx \left( \frac{4\pi}{9n} \right)^{1/3}$$  \hspace{1cm} (12)

in which $n$ is the number of water molecules per unit volume. The value $n = 3 \times 10^{22}$ leads to $\lambda_{\text{min}} \approx 10^{-7}$ cm, a satisfactory result.

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