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STABILITY OF THE PLASMA-SHEATH

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The existence, properties and structure of the sheath which forms adjacent to any surface in contact with a plasma have been studied in considerable detail in recent years. In the case of low density plasmas where the space charge dominated region is of the order of several Debye lengths thick and the charged particle motion collisionless, the requirement that the potential is monotonic leads to the Bohm criterion. This has been generalized to include an ion distribution. The requirement imposed relates to spatial and not temporal variations and so the stability of the solution is not investigated.

Stability analysis of the sheath is inherently complicated in comparison with situations in uniform plasma because, even in the simplest fluid model of ion motion, the charged particle densities and the ion speed are all functions of position. However, such analyses have in principle been carried out for some conventional situations. At high frequencies comparable with the electron plasma frequency studies of the Tonks-Dattner resonances both experimentally and theoretically have shown that electron plasma waves undergo absorption as they propagate into the sheath and are reflected. Corresponding work at frequencies of the order of and below the ion plasma frequency has also shown that ion modes propagating on the ion 'beam' which leaves the plasma and traverses the sheath are absorbed. One concludes that the conventional plasma sheath is stable.

There is a situation which arises in high temperature plasmas, e.g. fusion reactors or when the bounding surface is electron emissive which is a priori prone to instability. In this case an electron 'beam' leaves the surface and interpenetrates the ion 'beam' travelling in the opposite direction. The steady state analysis of this two stream situation has been given and is directly related to that for a hot cathode. Recent extensions have been made. The design of divertors depends upon whether these steady state solutions are physically significant, or as a result of instability, merely mathematical curiosities.

A complete analytical solution is impossible and so we examine an approximate model based on the following assumptions.

1. The situation is stable so that the steady state solutions for number density and particle velocities are meaningful.
2. The variation of the 'plasma' parameters is not so rapid as to render the use of uniform plasma theory corresponding to the local parameters invalid.
3. Instability will result at any frequency if a disturbance at that frequency grows sufficiently rapidly spatially as the sheath is traversed. Growth by a factor of or is taken as a criterion.

The following equations describe the steady state

$$n_{ep} = n_0 \exp \left(-\frac{\rho}{\lambda}\right) \quad \text{for the 'plasma' electrons}$$

$$n_{ip} = n_0 \left(1 + \alpha \right) \left(1 + \frac{2\gamma}{u_{io}^2} \right) \quad \text{for the 'plasma' ions}$$
\[ n_{eb} = \alpha n_0 \left( \frac{\Psi}{n_0} \right)^{\frac{1}{2}} \]

for the 'emitted' electrons where \( \Psi \) is the potential normalized to the electron temperature, \( n_0 \) is the density of plasma electrons and \( \alpha n_0 \) the density of surface-derived electrons in the plasma, \( u_{io} \) is the ion speed on leaving the sheath normalized to the ion acoustic speed \( c_s = \left( \frac{kT_e}{M_i} \right)^{\frac{1}{2}} \).

\( \Psi \) is the normalized potential difference between plasma and surface. The equations are closed by solving Poisson's equation with boundary conditions

\[ \frac{d\Psi}{r} = 0 \quad \text{and} \quad \frac{d^2\Psi}{dr^2} = 0 \quad \text{as} \quad r \to \infty. \]

For a given frequency \( \omega \) the imaginary part \( k_i \) of the wave number is determined by solving the fluid dispersion relation

\[ \frac{\omega_{peb}^2}{\omega_{pip}^2} - \frac{\omega_{pip}^2}{(\omega + kV_i)^2} - \frac{\omega_{peb}^2}{\omega - kV_e^2} = 0, \]

where \( \omega_{peb}, \omega_{pip} \) are the local plasma frequencies and \( V_i \) and \( V_e \) the speeds of the ion and electron 'beams'.

The range of parameters to be examined is determined by the fact that for any \( \Psi_c \) there is a limiting value of \( \alpha \) corresponding to zero field at the cathode. For \( \Psi_c = 10.0, \alpha^* = 0.175. \)

In general the growth rates of the fastest growing mode are low for small values of \( \Psi_c \) and of \( \alpha \) and the solutions are stable. A set of results for \( \alpha = 0.1 \) and \( \Psi_c \) is given in Figs. 1 - 3. Figure 1 shows the number densities and particle speeds as a function of the distance from the cathode. Fig. 2 gives dispersion diagrams for \( \Psi = 3.0 \) (dashed) and \( \Psi = 4.6 \) (solid). Fig. 3 gives spatial growth rates as a function of frequency and for these parameters they are sufficiently high for the situation to be classified as marginally unstable.

We conclude that for high emission currents instability can set in.

References.
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