NONLINEAR DECAY INTERACTIONS FOR THE INSTABILITY OF BUNEMAN-FARLEY
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During the magnetic disturbances the small-scale irregularities occur in the polar and equatorial electrojet along the Earth's magnetic field with the cross-dimensions from 10 cm to 10 m. These irregularities are responsible for the intense VHF radio wave scattering and in the absence of sharp gradients arise from the Buneman-Farley instability \[1, 2\].

The experimental studies of auroral and equatorial radar echoes revealed the discrepancies between linear theory and observations and thus posed a problem of developing nonlinear theory.

In this paper we analyse one possible nonlinear process—Buneman-Farley wave decay interaction. When the amplitude of unstable waves grows to some level these waves intensively interact so as a result of the interaction a nonlinear flow of spectral energy from the region of linear generation to the damping region can be significant.

This process is possible if the resonance conditions \( \omega = \omega_2 + \omega_k \), \( \vec{R} = \vec{R}_1 + \vec{R}_2 \) are fulfilled. Taking into account the law of Buneman-Farley wave dispersion \[3\] these resonance conditions for the decay interaction in a three-dimensional case can be written as:

\[
\kappa \sin \Psi = \kappa_1 \sin \Psi_1 + \kappa_2 \sin \Psi_2
\]

\[
\kappa \cos \Psi \sin \Phi = \kappa_1 \cos \Psi_1 \sin \Psi_1 + \kappa_2 \cos \Psi_2 \sin \Psi_2
\]

\[
\kappa \cos \Psi \cos \Phi = \kappa_1 \cos \Psi_1 \cos \Psi_1 + \kappa_2 \cos \Psi_2 \cos \Psi_2
\]

\[
\frac{\kappa \cos \Psi \cos \Phi}{\sin \Psi_1 + \sin \Psi_2} = \frac{\kappa_1 \cos \Psi_1 \cos \Psi_1}{\sin \Psi_1 + \sin \Psi_1} + \frac{\kappa_2 \cos \Psi_2 \cos \Psi_2}{\sin \Psi_1 + \sin \Psi_2}
\]

\[
\vec{R} = \vec{R}_1(\vec{k}_1, \Psi_1) \quad \vec{R}_2 = \vec{R}_2(\vec{k}_2, \Psi_2)
\]

\[
\eta = \frac{\omega_k \omega_2}{(1 + \omega_1^2) \omega} \quad \xi = \frac{\omega_k}{(1 + R^2) \omega}, \quad \rho = \frac{\omega_k \omega_2}{\omega \rho_\infty \omega_\infty}
\]

Here \( \Psi \) is the angle between wave vector \( \vec{k} \) and the electrons drift velocity \( \vec{v}_0 \), and \( \Phi \) is the angle between the wave vector \( \vec{k} \) and the plane perpendicular to the magnetic field.

If \( \kappa^2 \eta \ll 1, \xi \sin^2 \Psi \ll 1 \) then the equations (1) - (4) admit the decay interaction of the Buneman-Farley waves and the above-mentioned parameters correspond to the following equations:

\[
\Psi_1 = \left( \frac{\Psi \cos(x + \Psi) \pm \sqrt{\Psi^2 + \Psi_0^2} \sin x}{\cos(x + \Psi)} \right)^\frac{1}{2}
\]

\[
\Psi_2 = \left( \frac{\Psi \cos(x + \Psi) \pm \sqrt{\Psi^2 + \Psi_0^2} \sin y}{\cos(x + \Psi)} \right)^\frac{1}{2}
\]

\[
\Psi_0 = \frac{\xi}{\eta} \cos \Psi \cos(x + \Psi) \left[ \sin x \sin y + 3 \cos x \cos y \right] \quad x = \Psi_1 - \Psi, \quad y = \Psi_2 - \Psi
\]

For two dimensional geometry \( \Psi = \Psi_1 = \Psi_2 = 0 \) using the equations (5-6) one can draw a conclusion that the decay process for the small angle waves \( \Psi_1 \) and \( \Psi_2 \) is impossible. Thus the studies of the resonance conditions with small but not equal zero angles \( \Psi, \Psi_1, \Psi_2 \) bring
about the conclusion that the decay process is possible if the parameters of interacting waves correspond to certain equations. In particular there is a possibility for the decay of the linear growing wave \( \gamma > 0 \) into two waves one of which spreads nearly perpendicular to the electrojet. A set of quasi-fluids equations for the electron and ion, equations of electrodynamic and equations of continuity results in the equations describing the decay process of the Buneman-Farley waves if the wave phase is fixed:

\[
\begin{align*}
\frac{\partial \eta_e}{\partial t} - \gamma_2 \eta_e &= -i s \sin \eta_e \eta_e, \\
\frac{\partial \eta_i}{\partial t} - \gamma_2 \eta_i &= -i s \sin \eta_i \eta_i, \\
\frac{\partial \eta_e}{\partial t} - \gamma_2 \eta_e &= -i s \sin \eta_e \eta_e
\end{align*}
\]

where the expansion of electron density takes the form:

\[
n(t) = \int \eta_e(\mathbf{r}, t) \exp\left\{-i(\omega(\mathbf{r}) - k \mathbf{v})\right\} d^2 \mathbf{r}
\]

here \( \omega(\mathbf{r}) = \Omega(\mathbf{r}) + i \gamma \) is the solution of the linear dispersion relation \( \mathcal{E}(\omega(\mathbf{r})) = 0 \), \( \gamma \) -growth rate and \( \eta_e(\mathbf{r}, t) = \eta_e \exp(iat) \) is the slowly changing amplitude of the electron density and \( \eta_e = \eta_e(\mathbf{r} - \mathbf{R}), \eta_i = \eta_i(\mathbf{r} - \mathbf{R}) \). The solution of the equations (7)-(9) determines the periodical transfer from one mode \( \eta_e \) to the modes \( \eta_e, \eta_i \). This process will continue without damping \( \gamma_2 = \gamma_2 = \gamma_2 = 0 \). However, the greatest interest for practical purposes is the case of the stationary state. As seen from Eqs. (7)-(9) the stationary state is possible with the decay of the wave having a vector \( \mathbf{R} \) into the waves dampening in the linear theory \( \mathbf{R} \), \( \gamma_2 < 0 \).

In this case the quasi-stationary turbulence level is determined by:

\[
\gamma_2 = \frac{n \eta_e}{\eta_i} = \frac{\eta_e \eta_e}{\eta_i \eta_i}, \quad \gamma_2 = \frac{\eta_e \eta_e}{\eta_i \eta_i}, \quad \gamma_2 = \frac{\eta_e \eta_e}{\eta_i \eta_i}
\]

Taking the interacting wave parameters according to (6), the stationary state is determined by:

\[
\frac{\partial}{\partial t} \frac{\eta_e}{\eta_i} = \frac{1}{k^2} \frac{\eta_e \eta_e}{\eta_i \eta_i} (\cos^2 \psi - \beta^2) (\cos^2 \psi - \beta^2) \sin(\mathbf{k} \mathbf{r} + k \mathbf{v} \mathbf{t})
\]

Taking the parameters for estimation: \( \psi = 0 \), \( \psi = 20^\circ \), \( \psi = -60^\circ \beta = 0.68, \gamma_2 = 10^{-3} \), we obtain. Thus, based on the estimates one can draw a conclusion that the above-mentioned process is quite effective for the stabilization. As a result of the stationary state the wave electrostatic turbulence regions can be formed with the directions of waves are almost perpendicular to magnetic field. The given estimates correspond to experimental observations of radio-echoes from the electrostatic wave turbulence regions.

References