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A STATISTICAL THEORY OF LANGMUIR TURBULENCE

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Recent advances in the theory of "strong" Langmuir turbulence¹ have concentrated on the evolution of modulational instabilities and their relation to soliton solutions (in 1-D) and Langmuir collapse² in higher dimensions. Most of the nonlinear theories in this area have emphasized initial value or similarity solutions for the coherent nonlinear structures of interest. In many situations we can conceive of the nonlinear excitations as developing from the thermal (particle) fluctuations in the plasma in the presence of external beams or A.C. fields which drive Langmuir instabilities. Numerical studies³ have shown the birth and decay of "collapsons", new collapsons being generated from the residual fluctuation debris of previously decayed collapsons. It is not unreasonable to try to construct a statistical turbulence theory to describe such situations. The present work represents a preliminary attempt in this direction.

Our method is a generalization to the Zakharov equations¹ of the direct-interaction approximation (DIA) of Kraichnan⁴ which was originally applied to the Navier-Stokes equations. We begin with the Zakharov equations in Fourier transform space, which can be written in the compact form:

$$L_{12}N_2 = \frac{1}{2}X_{123}^{(2)}N_2N_3 + \eta_1, \quad (1)$$

where $N_1 = N_{\alpha_1}(\underline{k}_1, t_1)$ is a three component vector with the discrete indices α_1 taking on the waves +, 0, -. Repeated indices such as 2 and 3 are understood to be summed on discrete indices α_2, α_3 and integrated over the continuous variables $\underline{k}_2, t_2$, etc. The identification of the discrete indices is: $N_+(\underline{k}_1, t_1) = \phi^+(\underline{k}_1, t_1)$; $N_0(\underline{k}_1, t_1) = n(\underline{k}_1, t_1)$; $N_-(\underline{k}_1, t_1) = \phi^-(\underline{k}_1, t_1)$, (2)

where $\phi^+(\underline{k}_1, t_1)$, $\phi^-(\underline{k}_1, t_1)$ are the spatial Fourier transforms of the envelope function of the high frequency Langmuir potential and its complex conjugate $[\phi^+(\underline{k}, t)^* = \phi^-(\underline{k}, t)]$ and $n(\underline{k}, t)$ is the Fourier transform of the ion density deviation $[n(\underline{k}, t) = n(-\underline{k}, t)]$. The linear operator L_{12} is

$$\begin{bmatrix} \partial_{t_1} + v_e + ik_1^2 & 0 & 0 \\ 0 & \partial_{t_1}^2 + 2v_e \partial_{t_1} + k_1^2 & 0 \\ 0 & 0 & \partial_{t_1} + v_e - ik_1^2 \end{bmatrix} \cdot (2\pi)^3 \delta(\underline{k}_1 - \underline{k}_2) \delta(t_1 - t_2) \quad (3)$$

The only nonzero components of $X_{123}^{(2)}$ are

$$\begin{aligned} X_{+0-}^{(2)} &= -X_{-0+}^{(2)} = -i(\underline{k}_1 \cdot \underline{k}_3)k_1^{-2} \delta_{123}, \\ X_{++0}^{(2)} &= -X_{--0}^{(2)} = -i(\underline{k}_1 \cdot \underline{k}_2)k_1^{-2} \delta_{123}; \\ X_{0+-}^{(2)} &= X_{0-+}^{(2)} = -k_1^2 (\underline{k}_2 \cdot \underline{k}_3) \delta_{123} \\ \delta_{123} &= (2\pi)^3 \delta^3(\underline{k}_1 - \underline{k}_2 - \underline{k}_3) \delta(t_2 - t_3) \delta(t_1 - t_2). \end{aligned} \quad (4)$$

Equation (1) is a general quadratically nonlinear form to which the DIA can be immediately applied,⁵ giving a set of equations coupling three important objects: the ensemble averaged (mean) fields $\langle N_1 \rangle$, the infinitesimal response function⁵ (given by a functional derivative) and the covariance:

$$R_{11} = \delta \langle N_1 \rangle / \delta \langle \eta_1 \rangle; \quad C_{11} = \langle N_1 N_1' \rangle - \langle N_1 \rangle \langle N_1' \rangle \quad (5)$$

The equations connecting these are:

$$(L_{12} - \Sigma_{12})R_{21} = \delta_{11}, \quad \left[= \delta_{\alpha_1 \alpha_1} \delta^3(\underline{k}_1 - \underline{k}_1') \delta(t_1 - t_1') \right], \quad (6)$$

where,

$$\Sigma_{11} = X_{123}^{(2)} X_{231}^{(2)} R_{22} C_{33} + X_{123}^{(2)} \langle N_3 \rangle, \quad (7)$$

$$L_{12} \langle N_2 \rangle = \langle \eta_1 \rangle + \frac{1}{2} X_{123}^{(2)} (\langle N_2 \rangle \langle N_3 \rangle + C_{23}), \quad (8)$$

$$(L_{12} - \Sigma_{12})C_{21} = R_{12} S_{12}, \quad (9)$$

$$S_{11} = S_{11}^0 + \frac{1}{2} X_{123}^{(2)} X_{123}^{(2)} C_{22} C_{33}, \quad (10)$$

where S_{11}^0 is the correlation of the fluctuating part of the driving term η_1 which we can take, for example, to be white noise related to the damping decrements by a Nyquist theorem:

$$S_{11}^0 = \langle \delta \eta_1 \delta \eta_1' \rangle = \Gamma_{\alpha_1 \alpha_1} \delta^3(\underline{k}_1 - \underline{k}_1') \delta(t_1 - t_1'), \quad (11)$$

$$\Gamma_{+-} = \Gamma_{-+} = 4\pi T_e v_e k_1^{-2}; \quad \Gamma_{00} = 3k_1^2 (m_i/m_e) v_i;$$

others zero. (12)

This set of nonlinear, non-Markoffian coupled equations is currently being studied by numerical analysis. The general initial value problem is difficult because of the long time histories which

must be retained in an unstably evolving system. The DIA can be shown to have the general property that the mean constants of the motion of the Zakharov equations^{1,2} $\langle N \rangle$, $\langle P \rangle$ and $\langle H \rangle$ are independent of time. These conservation properties of the complete DIA give us some confidence that these equations contain at least some of the physics important for Langmuir collapse.²

We have investigated the steady state, spatially homogeneous solutions of these equations for zero mean field. Here the $R(C)$ matrices have a diagonal (anti-diagonal) form. The explicit equations for the various components are:

$$[-i(\omega_1 - k_1^2) + \nu_e - \Sigma_{++}(k_1, \omega_1)] R_{++}(k_1, \omega_1) = 1 \quad (13a)$$

$$\begin{aligned} \Sigma_{++}(k\omega) &= (2\pi)^{-4} \int d^3 k_2 \int d\omega_2 \{ i(\underline{k}_1 \cdot \underline{k}_2)^2 k_1^{-2} k_2^2 \\ &\quad \cdot C_{+-}(\underline{k}_2, \omega_2) R_{00}(\underline{k}_3, \omega_3) + (\underline{k}_1 \cdot \underline{k}_2)^2 k_1^{-2} k_2^{-2} \\ &\quad \cdot R_{++}(\underline{k}_2, \omega_2) C_{00}(\underline{k}_3, \omega_3) \} ; \end{aligned} \quad (13b)$$

$$\begin{aligned} [-i(\omega_1 - k_1^2) + \nu_e - \Sigma_{++}(k_1, \omega_1)] C_{+-}(k_1, \omega_1) \\ = R_{++}(k_1, \omega_1) S_{+-}(k_1, \omega_1) \end{aligned} \quad (14a)$$

$$\begin{aligned} S_{+-}(k_1, \omega_1) &= S_{+-}^0(k_1, \omega_1) + (2\pi)^{-4} \int d^3 k_1 \int d\omega_1 (\underline{k}_1 \cdot \underline{k}_3)^2 k_1^{-4} \\ &\quad \cdot C_{00}(\underline{k}_2, \omega_2) C_{+-}(\underline{k}_3, \omega_3) ; \end{aligned} \quad (14b)$$

$$[-\omega_1^2 + 2i\omega_1 \nu_i + k_1^2 - \Sigma_{00}(\underline{k}_1, \omega_1)] R_{00}(\underline{k}_1, \omega_1) = 1, \quad (15a)$$

$$\begin{aligned} \Sigma_{00}(\underline{k}_1, \omega_1) &= (2\pi)^{-4} \int d^3 k_2 \int d\omega_2 (-i)(\underline{k}_2 \cdot \underline{k}_3)^2 k_1^{-2} k_2^{-2} \\ &\quad \cdot [R_{++}(\underline{k}_2, \omega_2) C_{+-}(\underline{k}_3, \omega_3) - R_{--}(\underline{k}_2, \omega_2) C_{+-}(\underline{k}_3, \omega_3)] ; \end{aligned} \quad (15b)$$

$$\begin{aligned} [-\omega_1^2 + 2i\omega_1 \nu_i + k_1^2 - \Sigma_{00}(\underline{k}_1, \omega_1)] C_{00}(\underline{k}_1, \omega_1) \\ = R_{00}(\underline{k}_1, \omega_1) S_{00}(\underline{k}_1, \omega_1) , \end{aligned} \quad (16a)$$

$$\begin{aligned} S_{00}(\underline{k}_1, \omega_1) &= S_{00}^0(\underline{k}_1, \omega_1) + (2\pi)^{-4} \int d^3 k_1 \int d\omega_1 k_1^4 \\ &\quad \cdot (\underline{k}_2 \cdot \underline{k}_3)^2 C_{+-}(\underline{k}_2, \omega_2) C_{+-}(\underline{k}_3, \omega_3) ; \end{aligned} \quad (16b)$$

where throughout $\underline{k}_1 = \underline{k}_2 + \underline{k}_3$, $\omega_1 = \omega_2 + \omega_3$ and $R_{--}(\underline{k}_1, \omega_1) = R_{++}(-\underline{k}_1, -\omega_1)^*$ and $C_{+-}(\underline{k}_1, \omega_1) = C_{+-}(-\underline{k}_1, -\omega_1)$.

What usually is described as weak turbulence theory is obtained from equations (14) and (16) by replacing the response functions R and everywhere by their zero order values $R_{++}^0 = [-i(\omega - k^2) + \nu_e]^{-1}$, $R_{00}^0 = [-\omega^2 + 2i\omega \nu_i + k^2]^{-1}$, $C_{+-}^0 = W(\underline{k}) k^{-2} (2\pi) \cdot \delta(\omega - k^2)$, etc., where $W(\underline{k})$ is the Langmuir energy density. In this approximation, it is easily seen that the two terms in Σ_{++} and Σ_{00} describe induced decay processes of the form $L \rightarrow L+S$ which lead, for example, to the well-known cascade of wave energy toward lower k .

The DIA, on the other hand, treats the self-consistent renormalization of the response functions and this renormalization is necessary to preserve

the mean constants of the motion. This renormalization has extremely important consequences: For example, if in (15b) we approximate the quantities R and C by their zero order values and then substitute the resulting Σ_{00} into (15a) we find that R_{00} has complex poles corresponding to the roots of the well-known modulational dispersion relation, i.e., the quantity in square brackets in (14) is just this dispersion relation generalized to the case of a broad Langmuir spectrum.⁶ Thus, in addition to the modified ion sound poles in R_{00} (modified by the decay branch of the dispersion relation) the modulational instability introduces new modulational poles in R_{00} . When this renormalization R_{00} is put into (13b), it appears that flow of Langmuir energy toward higher k becomes possible. Furthermore, C_{00} now has large components due to the modulational instability which produce a significant source term (14b) on the right-hand side of (14a). The renormalized Langmuir modes then have a finite turbulent damping in contrast to the marginally damped modes found in the usual theory of the cascade of energy toward small k .

A fully self-consistent solution of the DIA equations requires computer analysis. We will present results of 1-D numerical solutions of the DIA equations as well as a comparison of the results with direct computer solutions of the Zakharov equations. Questions concerning the spontaneous generation of a mean field at $\underline{k} = 0$ to account for the Langmuir condensation and consequent modulational instability will also be addressed.

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