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INTERACTION OF EM WAVES WITH A COMPRESSIBLE PLASMA COLUMN


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1. INTRODUCTION. The interaction of an electromagnetic wave with compressible plasma half-space or plasma column has been extensively studied in recent years (1/1-3/1). There have, however, been very few reports with numerical results about the interaction of electromagnetic waves by an infinitely long compressible plasma column. With the recent progress in space exploration, a problem of this kind has become an important one which has some relations with space-communication technology and radio astronomical problems. The present analysis investigates the scattering of an obliquely incident plane electromagnetic wave by an infinitely long lossless plasma column. Numerical results for the differential scattering cross section are obtained for a range of acoustic velocities in electron gas.

2. FORMULATION OF THE PROBLEM. We consider the scattering problem for the case where a plane electromagnetic wave is incident on an infinitely long homogeneous lossless compressible plasma column of radius a immersed in free space. The wave vector of an obliquely incident wave is assumed to be yz-plane and makes an angle φw with the negative y-axis. For an incident wave whose electric field vector of magnitude $E_0$ makes the angle $\Psi$ with the x-axis, the axial components take the form

$$E_z = \sum (-1)^n A_n H_n(k_{iy}r) F_n$$

$$H_z = \sum (-1)^n B_n H_n(k_{iy}r) F_n$$

where

$$A_n = \frac{B_n}{k_y} e^{ik_y a}$$

and $A_n$, $B_n$ are the Bessel functions of the first kind and order n. The axial components of the scattered wave in free space are

$$E_z = \sum (-1)^n A_n H_n(k_{iy}r) F_n$$

$$H_z = \sum (-1)^n B_n H_n(k_{iy}r) F_n$$

where $H_n(1)$ are the Hankel functions of the first kind and order n. $A_n$ and $B_n$ are scattering coefficients to be determined by the appropriate boundary conditions.

3. FORMAL SOLUTIONS. Supposing that the compressible plasma can be described by the small-signal theory and considering only the motion of electrons, the plasma column is governed by Maxwell's equations, the momentum equation and the combined equations of continuity and state (1/4). A straightforward manipulation of this equations gives the following coupled second-order differential wave equations:

$$\frac{d^2 E_z}{d^2 r} + \frac{1}{r} \frac{d E_z}{d r} + (p - \frac{1}{r^2}) E_z = -igP$$

$$\frac{d^2 H_z}{d^2 r} + \frac{1}{r} \frac{d H_z}{d r} + (p - \frac{1}{r^2}) H_z = 0$$

$$\frac{d^2 P}{d^2 r} + \frac{1}{r} \frac{d P}{d r} + (q - \frac{1}{r^2}) P = 0$$

with

$$p = \sin^2 \phi_0, \quad q = \cos^2 \phi_0, \quad g = \sin^2 \phi_0 \sin^2 \rho, \quad \frac{2}{\omega} = \frac{1}{\rho} + \frac{1}{\rho^2}$$

and $w$ and $w_p$ are, respectively, the circular frequency of the incident wave and electron plasma frequency, $P$ is the pressure deviation from the mean and $u_0$ is the acoustic velocity in electron gas. The axial components $E_z$ and $H_z$ of the field and $P$ deduced from eqn. (3) are

$$E_z = \sum (-1)^n A_n P J_n(p^2 r) + C_n R_n J_n(q^2 r) F_n$$

$$H_z = \sum (-1)^n N_n P J_n(q^2 r) F_n$$

$$R_n = \sum (-1)^n C_n R_n J_n(q^2 r) F_n$$

with

$$J_n = \frac{1}{\omega} \frac{e^{i \omega t}}{\sin \phi_0}$$

Using the transverse field components expressed in terms of their axial components and relation for the radial component of the average velocity of electron gas.
one can now to satisfy the boundary conditions at 
the free-space-homogeneous compressible plasma 
interface \( p_a k_0 a \). Matching the boundary condi-
tions gives

\[ X \cdot C = Y \]  

where \( X \) is a 5x5 square matrix, \( C \) and \( Y \) are 5x1 
matrices with the elements

\[
\begin{align*}
X_{12} &= X_{13} = X_{33} = X_{44} = X_{55} = X_{32} = 0 \\
X_{21} &= s H_0^0 (x_0) \\
X_{22} &= -p_j^0 (x_0) \\
X_{23} &= -q J_n (q_0) \\
X_{24} &= \delta H_n^0 (x_0) \\
X_{25} &= \frac{1}{\cos \phi_0} H_n^0 (x_0) \\
X_{32} &= \frac{1}{\cos \phi_0} H_n^0 (x_0) \\
X_{33} &= \frac{1}{\cos \phi_0} H_n^0 (x_0) \\
X_{34} &= \delta \left( \frac{1}{\cos \phi_0} H_n^0 (x_0) \right) \\
X_{35} &= \frac{1}{\cos \phi_0} H_n^0 (x_0) \\
X_{42} &= -q J_n (q_0) \\
X_{43} &= -q J_n (q_0) \\
X_{44} &= \delta H_n^0 (x_0) \\
X_{45} &= \frac{1}{\cos \phi_0} H_n^0 (x_0) \\
X_{52} &= \frac{1}{\cos \phi_0} H_n^0 (x_0) \\
X_{53} &= \frac{1}{\cos \phi_0} H_n^0 (x_0) \\
X_{54} &= \frac{1}{\cos \phi_0} H_n^0 (x_0) \\
X_{55} &= \frac{1}{\cos \phi_0} H_n^0 (x_0)
\end{align*}
\]

The matrix form (6) has been used for computation 
of the scattering coefficients \( A_n^S \) and \( B_n^S \).

4. NUMERICAL RESULTS. We consider only the 
case for an \( E \) incident plane electromagnetic wave 
and the distribution of scattered energy in cross-
polarized fields is expressed as follows

\[
E_n^S = \lim_{r \to \infty} \frac{1}{r} \left( E_n^S \times \hat{x} \right) \cdot \hat{r} \\
B_n^S = \lim_{r \to \infty} \frac{1}{r} \left( B_n^S \times \hat{x} \right) \cdot \hat{r}
\]

The scattered fields in above expressions are

\[
E_n^S = E_n^0 (x_0, E_0^S, E_0^S) \quad B_n^S = B_n^0 (x_0, E_0^S, E_0^S)
\]

Angular variation of a normalized cross-polarized 
components of scattering cross section for vari-
ous acoustic velocities in electron gas is plotted

\[\text{REFERENCES}\]